Dynamic Pricing and Markdown Timing Policies for Fashion Goods with Strategic Consumers' Behavior

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In this paper, we discussed optimal pricing strategy and best markdown timing in the two-sale periods for a monopoly seller, which faced with strategic consumers. Based on the Stackelberg game theory, a mathematical model is constructed to maximize the seller's revenue when the markdown timing is certain or uncertain. Consumers are heterogeneous with different valuations for a same product. Moreover, after retailer decision-making, consumers would determine their purchase policies about the time and the price, through comparing the prices and the individual valuations in the two-sale periods. Finally, a numerical example is established to illustrate the optimal pricing strategy and the best purchase policy.

Key words: Strategic consumers, pricing strategy, markdown timing

The concept of perishable products was first proposed (Weatherford, 1992), such as vegetables, newspapers, food, fashion, airline tickets and electronic products. In this paper, we focused on the fashion goods, for which customers' reservation prices decrease over the sales season. Consequently, the seller should make different prices at different periods of the sales horizon to adjust demand, in order to maximize the profits of the seller. However, some consumers realize that the seller would carry out price reduction promotion. So they usually compare the consumer surplus in different sales seasons to determine the best purchase price and time. Accordingly, the seller should consider the effect of consumers' strategic behavior when making pricing policy.

Dynamic pricing and revenue management has gained an increasing popularity in retail settings, and has engendered a growing body of academic research in recent years (for a recent survey, see Bitran and Caldentey, 2003). The recent growth in internet-based marketing has stimulated widespread experimentation with dynamic pricing, the practice of varying prices for the same goods over time or across customer classes in an attempt to increase total revenue for the seller.

According to customer's willingness to wait and their sensitivity-price, consumers can be divided into two types: one is myopic customers called "angel" by the sellers, who will immediately buy a good when the current price is lower than their willingness to pay; the other is strategic customers called "devil", who predict the possibility of buying at a lower price in the future and select to wait the discounted price, even though the current price is lower than their reservation prices (Mc Williams, 2004).

There is an extensive literature on dynamic pricing research that includes mathematical analysis of optimization and game-theoretic models, empirical examination of field data, and laboratory experiments. Surveys appear in Bitran & Caldentey (2003), Elmaghraby & Keskinocak (2003), Chan (2004), Shen & Su (2007), Chen & Chen (2015) and more recently Papanastasiou & Savva (2016), Aviv et al. (2016), which focus on studies of dynamic pricing with strategic consumers. Elmaghraby (2008) studies optimal markdown mechanisms in the presence of strategic consumers who have fixed valuation throughout the selling season.

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Whether markdown timing is certain or uncertain, the arrival of myopic and strategic customers is random. In these circumstances, we discuss how a monopolist seller makes dynamic pricing policy at two sales-periods to gain the most revenue. Based on Stackelberg game theory, we first developed the optimal dynamic pricing policy model when markdown timing is certain. Next, the extension model is established to make the optimal joint decision about dynamic pricing and purchasing when markdown timing is uncertain. Then, an example analysis is employed to illustrate the optimal pricing and markdown timing of the seller and to find more interesting conclusions for the practices.

Base Model Descriptions

Problem Statement

This is a monopoly seller, which sells a single fashion product over a finite sales horizon [*0*, *H*] in the market without competition. The sales horizon is split into two periods of the sales season, [0, *T*] and [*T*, *H*]. The markdown time *T* is decided by the seller, when the seller will change the full price to a discounted price. The markdown time T is known by the consumers in the base model. In this game, the seller has a stock of Q units, and gives a fixed-initial price p_1 during a first period of the sales horizon, and the seller selects the markdown timing and offers a discounted price p_2 ($p_2 < p_1$) during the second part of the entire season (Yossi Aviv and Amit Pazgal 2008). The seller makes the optimal pricing and discount timing strategy in order to maximize the expected profits collected over the entire season.

In the market, there are such characteristic customers that are price-sensitive and time-sensitive over the selling season. There are two populations of consumers: a kind population of consumers who may purchase at a high price immediately when they arrive at the store early or they will wait until the time T to purchase at a discounted price, and another kind population of low-valuation consumers who purchase maybe at a deeply discounted price or give up buying no matter when they arrive at the store. The percentage of myopic consumers is β, and the percentage of strategic consumers is $1-\beta$. The purchasing decisions made by both of these populations are described in greater detail in below.

Customers continued to reach the store at a decided flow of constant rate λ arrivals per time unit (Su, 2007). We assume that each customer purchase only one item. Customers are heterogeneous with the different base valuations, which decline with the sales season. To reflect this, we use a multiplicative valuation function of the type:

$$V(t) = V * e^{-\alpha t}$$

In which, each customer's base valuation *V* is drawn from a given continuous density distribution form f (Yossi Aviv and Amit Pazgal 2008). The customer's valuation reduced at a rate α ($\alpha \geq 0$). The assumption that valuations decline over the course of the season seems to be prevalent in the sales of fashion and seasonal items (Desiraju and Shugan 1999). In this paper, we assume that the rate α of customers' valuation reduction is equal for the population of customers. For example, the case α=0 indicates that customers are not time sensitive in their valuations. In other words, customers' valuations for the products were not changed during the entire season. If the value of the rate α is greater, customers' valuations decrease faster. The second heterogeneity is the different arrival time of customers. Additionally, we assume that every consumer only buy a unit of the product and then leaves the store immediately.

Our model is characterized by the set of parameters $\{\lambda, \alpha, \beta, H, Q, p_1, f\}$, assumed to be known to the seller and all consumers. Additionally, each consumer only observes the current price, but he does not see other consumers or the current level of inventory.

Consumers' Purchasing Decision

In our model, myopic customers will buy a product immediately if their valuations at the arrival time are larger or equal to the current price announced by the seller. But strategic customers may decide to postpone their purchases if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. In this section, we will deeply analyze the purchase decision-making process of strategic consumers.

Strategic customers that arrive prior to time T behave according to the following lines: a given strategic customer i, arriving at time t, will purchase immediately upon arrival (if there is inventory) if two conditions are satisfied about his current surplus $V_i e^{-\alpha t} - p_1$: (1) it is non-negative; and (2) it is larger or equal to the expected surplus he can gain from a purchase at time T (when the price is reset to p_2). This decision has a premise that the customer believes that the stock will still be available at markdown time *T*. At time T, all existing customers take a look at the new price p_2 and if they can obtain a non-negative surplus, they request a unit of the remaining products (if any). If there are fewer units than the number of customers who wish to buy, the allocation is made randomly and equally. After time T new strategic customers buy according to whether or not the can gain a non-negative surplus. Nevertheless, myopic customer, arriving at time *t* before markdown time *T*, will buy a unit if his current surplus is non-negative, otherwise they will wait until time *T*. At or after time *T*, myopic customers make a purchase decision according to whether or not they can obtain a non- negative surplus.

When markdown time T is fixed, the seller must increase the consumer surplus gained at the first period or reduce the consumer surplus gained at the second period of the season, in order to attract more strategic consumers purchasing a unit at the first period to maximize his expected profits. Furthermore, there are two main strategies for the seller to reduce the initial price $p_1$or to postpone the markdown timing. We compare the seller's total revenue in the case of different markdown timing. (i)When markdown timing is early, the initial price p_1 will be higher. Only a few strategic consumers purchase at the first period. Most purchase at the second period. (ii) When markdown timing is later, the initial price p_1 will be lower. In this case, the total number of consumers who purchase will be more than (i). According to results in the later section of "A Numerical Analysis", we find that the seller's total revenue is lower when markdown timing is later. The reason is that the initial price p_1 and discount price p_2 are lower in (ii), although there are more consumers. Therefore, it is helpful to improve the economic performance of the seller by choosing the right time to cut down the price.

The seller at first announces the full price and the discount price and the markdown timing, and then consumers decide whether to buy or not, so a dynamic game of complete information is constructed. The game process is as follows: firstly, when entering the market, the seller announces the prices (p_1, p_2) in the two periods of sales season and markdown time *T*; second, consumers arrive at the market, and the arrival process according to a Poisson distribution with a mean of λ arrivals per time unit; third, at the first of the sales season, myopic customers decide whether to buy or not according whether their current valuation is larger or equal to the current price, and then leaves the market; nevertheless, strategic customers decide to buy or to wait through trading off the current and the expected future surplus in the second period of the sales horizon; finally, at the second period of the season, all population of consumers decide whether to buy or not by comparing the current valuation with the discount price.

This is a dynamic game in two periods, with backward induction method. We explore that consumers employ the purchasing strategy comparing the current and expected surplus, under the condition of a given seller's pricing and markdown timing strategy. Then, according to the analysis of the tradeoff results, the seller selects optimal pricing and markdown timing strategy.

In base model, given a markdown time *T*, the seller's strategy is to gives the full price p_1 and the discounted price p_2, in which the discounted price p_2 is dependent on the remaining inventory Q_T at the markdown time *T* and then forms the discounted price menu $\left\{ \{p_2(Q_T)\}_{Q_T=1}^{Q} \right\}$. There is a competitive situation among consumers, which arises due to the fact that an individual consumer's decision impacts the product availability for others. Theorem 1 below shows the existence of a threshold which is dependent on the price and the time. Then, consumers purchase the product immediately upon arrival if and only if their valuations are higher than this threshold.

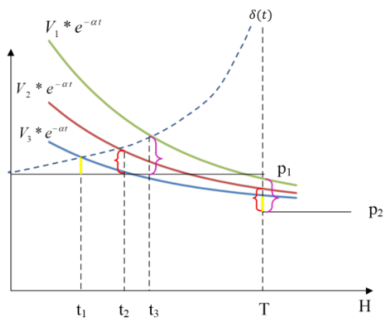

Fig 1. the decision model of strategic customers

Theorem 1 For any given pricing policy $\{p_1, p_2(Q_T)\}$ and markdown time *T*, it is optimal for all consumers to implement their purchasing decisions according to a threshold function $\theta(t)$. Actually, a consumer arriving at any time t of the sales horizon will buy a product immediately upon arrival if $V(t) \geq \theta(t)$. Otherwise, if $V(t) < \theta(t)$ and $t < T$, the consumers will revisit the store at time *T*, and buy an available unit if $V(t) \geq \theta(t)$.

For two different types of consumers, the threshold function $\theta(t)$ is defined in two different forms. The threshold function $\theta(t)$ for strategic customers (as see fig 1) is given by

$$\theta(t) = \begin{cases} \delta(t) & 0 \leq t < T \\ p_2 & T \leq t < H \end{cases} \quad (1)$$

Specifically, $\delta(t)$ in equation (1) is the unique solution to the implicit equation.

$$\delta(t) - p_1 = E_{Q_T}\left[max\left\{ \delta(t) e^{-\alpha(T-t)} - p_2(Q_T), 0 \right\} \cdot 1 \cdot Pr(N|Q_T) \right] \quad (2)$$

The left-hand-side of equation (2) represents the current surplus the strategic customer can gain by immediately buying a unit, whereas the right-hand-side of equation (2) represents the expected surplus that will be gained by postponing his purchase to time *T*. The value of the latter expected surplus takes into account two conditions. The first condition is that the discounted price $p_2(Q_T)$ must bring the customer to a non-negative surplus, that is $\delta(t)e^{-\alpha(T-t)} - p_2(Q_T) \geq 0$. The second condition is that the customer can be assigned to a product with the remaining inventory Q_T. The variable *N* represents the number of consumers strategically waiting to buy until time *T*. Given a specific inventory Q_T, the allocation probability depends on the distribution of the number of other consumers waiting to buy at markdown time *T*. In this paper, we assume that there is an equal probability for the customers to be allocated a unit. Moreover, the threshold function $\theta(t)$ in the rang $[0, T)$ will be defined as $\delta(t)$ in the following study.

Theorem 1 not only proves that the threshold policy is optimal for every consumer, but also takes into account the impact of the other customer purchase strategies.

Furthermore, the threshold function $\theta(t)$ for myopic customers is defined by

$$\theta(t) = \begin{cases} p_1 & 0 \leq t < T \\ p_2 & T \leq t < H \end{cases} \quad (3)$$

The Seller's pricing Strategy

In this section, we discuss the seller's optimal pricing policy in response to a given purchasing strategy and a given initial price p_1. We will divide our customers into four categories: the customers who buy immediately at the premium price, the customers who strategic wait for markdown time T to purchase a product at the discounted price, the customers who wait nonstrategic for the markdown time T to purchase a unit, the customers who purchase a product immediately at the discounted price. The word “purchase” means a desire to buy. Despite that, the desire is not sure to be implemented.

The first type is of the customers (denoted by ‘I’) that arrive during $[0, T)$ and purchase immediately at price p_1. The expected number of customers of this type can be calculated by the following equation (4).

$$N_I(\delta, p_1) = (1-\beta)\cdot\lambda\cdot\int_{t=0}^{T} P\left[V \geq \delta(t)e^{\alpha t}\right]dt + \beta\cdot\lambda\cdot\int_{t=0}^{T} P\left[V \geq p_1 e^{\alpha t}\right]dt \quad (4)$$

The second type is of the customers (denoted by ‘S’) that arrive during $[0, T)$ and strategically postpone their purchase to time T at price p_2 due to the anticipation of the higher expected surplus. The expected number of customers of this type can be calculated by the following equation (5).

$$N_S(\delta, p_1, p_2) = (1-\beta)\cdot\lambda\cdot \int_{t=0}^{T} P\left[\min\left\{\max\left\{p_1 e^{\alpha t}, p_2 e^{\alpha T}, \delta(t)e^{\alpha t}\right\}\right\} \leq V \leq \delta(t)e^{\alpha t}\right]dt \quad (5)$$

The third type is of the customers (denoted by ‘N_W’) that arrived during $[0, T)$ and waited for time T to buy a unit because their valuations upon arrival are lower than the initial price p_1. The expected number of customers of this type can be calculated by the following equation (6).

$$N_{N\_W}(p_1, p_2) = \lambda\cdot\int_{t=0}^{T} P\left[\min\left\{p_1 e^{\alpha t}, p_2 e^{\alpha T}\right\} \leq V \leq p_1 e^{\alpha t}\right]dt \quad (6)$$

The forth type is of the customers (denoted by ‘L’) that arrived during $[T, H]$ and have higher valuations upon arrival than the discounted price p_2. The expected number of customers of this type can be calculated by the following equation (7).

$$N_L(p_1, p_2) = \lambda\cdot\int_{t=T}^{H} P\left[V \geq p_2 e^{\alpha t}\right]dt \quad (7)$$

The total revenue collected from the four groups of customers is given by

$$\pi(p_1, p_2) = p_1\cdot\min\{N_I(\delta, p_1), Q\} + p_2\cdot \min\{N_S(\delta, p_1, p_2) + N_{N\_W}(p_1, p_2) + N_L(p_1, p_2), Q_T\} \quad (8)$$

The optimal discounted price p_2 are chosen to maximize the seller’s total expected revenue. Specifically,

$$\{p_1, p_2\} = arg \quad max_{p_1, p_2}\{p_1\cdot\min\{N_I(\delta, p_1), Q\} + p_2\cdot \min\{N_S(\delta, p_1, p_2) + N_{N\_W}(p_1, p_2) + N_L(p_1, p_2), Q_T\}\} \quad (9)$$

Extended model Analysis

In the extended model study, the markdown time T is the seller's decision variable. Then, we discuss the impact of the markdown time on the seller’s revenue. We verify each practical markdown time based on the total revenue function of the seller, and find the optimal markdown time T^* to maximize the seller's revenue.

The objective function of the seller based on the base model is given by

$$\max_{T}\left\{\max_{p_1, p_2}\{\pi(p_1, p_2, T)\}\right\} \quad (10)$$

The partial derivative of each variable is obtained by the extreme value theorem of multivariate function as the following equations.

$$\frac{\partial\pi}{\partial p_1} = 0 \quad \frac{\partial\pi}{\partial p_2} = 0 \quad \frac{\partial\pi}{\partial T} = 0 \quad (11)$$

Nevertheless, it is difficult to find the explicit expression of the two prices when the seller’s revenue is maximized because of the complexity of the solution. Then, we search the optimal joint decisions about equilibrium pricing (p_1^*, p_2^*) and markdown timing T^* through the analysis of numerical simulation in the next section.

A Numerical Analysis

In this section, a numerical study is employed to indicate the optimal joint decisions about pricing and markdown timing in the presence of heterogonous customers characterized by different base valuations. In this study, in order to represent the level of heterogeneity in the customers' base valuations, we assume that customers' base valuation follows a uniform distribution (Zhou et al., 2015; Fang et al., 2017). The probability density function of customers' base valuation is denoted as *f*, defined in (12).

$$f(\mathrm{V}) = \begin{cases} 1, & 0 < V < 1 \\ 0, & otherwise \end{cases} \quad (12)$$

Without loss of generality, the length of sales horizon *H* is normalized to 1. Therefore, *T* should be interpreted as fractions of the whole selling season. Similarly, let λ = 50, represents the average number of customer arrivals during the whole horizon.

Due to the complexity of the objective function, the expression of the optimal price (p_1^*, p_2^*) cannot be directly written, but all possible solutions can be calculated by Matlab to search for the maximum profit and the corresponding optimal price policy. In the numerical study, the five parameters are assigned to $\{\lambda = 50, \alpha = 0.2, \beta = 0.5, \mathrm{T} = 0.5, \mathrm{H} = 1\}$, and the inventory is always well stocked. According to the figure 2 below, We find that the total revenue of the seller increases first and then decreases with the increase of the discount price p_2 in the condition that the premium price p_1 is given; when the discount price p_2 is fixed, the profit of the manufacturer decreases with the increase of the premium price p_1. Therefore, the optimal prices is $p_1^* = 0.4524$, $p_2^* = 0.2353$, and then the maximum revenue is 39.1312. By the way, there is no dimension in all the parameters in the text.

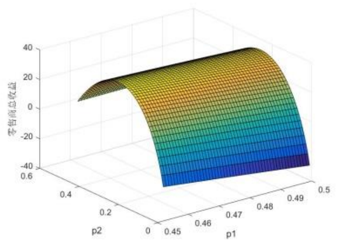

Fig 2. The impact of p_1 and p_2 on the seller's total revenue

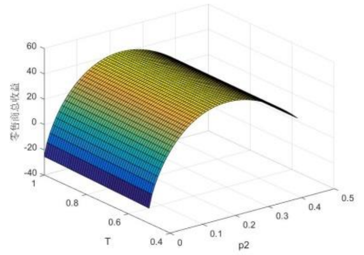

Fig3. The relationship between p_2, *T* and the seller's total revenue

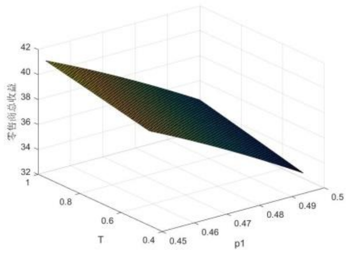

Fig4. The relationship between p_1, *T* and the seller's total revenue

These findings can be observed, through the analysis of Figure 3. when the markdown time *T* is a given value, the seller's revenue increases first and then decreases as the discounted price p_2 increases continually, in the case that the value of the perineum price p_1 is fixed to 0.4524. That is, the total revenue is a convex function of the discounted price p_2. In addition, the profit of the manufacturer increases with the delay of the time of the price reduction, when the discounted price p_2 is fixed as an arbitrary value, the profit of the seller increases with the delay of the discount timing *T*, that is, the objective function of the seller is strictly increasing about the markdown timing. In this numerical example, the seller revenue is maximized to 40.9346, when the markdown time T^* is 0.999.

In the contract, some different findings are obtained in the figure 4. In the case that the discounted price p_2 is equal to 0.2353, the seller's profit increases as the perineum price p_1 increases continually, that is, the objective function of the seller is increasing monotonous strictly. Moreover, the profit of the seller is decreasing with the delay of the markdown timing, when the original price p_1 is an arbitrary fixed value. We calculate that the seller revenue is maximized to 40.9349, when the markdown time T^* is 1.

CONCLUSIONS

This paper mainly studies the marketing strategy of perishable products considering both dynamic pricing and price reduction. Consumers arrive randomly in the selling period, while short-sighted and strategic consumers exist simultaneously. The markdown timing can be divided into two cases, namely, confirmation and uncertainty. The example shows that: with the increase of the proportion of strategic customers in the market, the perineum price, the discounted level in the second period and the profits of the seller are significantly reduced; no matter the proportion of strategic consumers, the profit will first increase and then decrease with the delay of the discount timing; if the proportion of strategic customers is far greater than myopic customers in the market, the more late the lower price is, the more advantageous to the seller.

This article assumes that the seller's inventory is unrestricted, but in general the model the initial inventory is determined by the customer demand and firm profits. In this paper, our model takes into account the uncertainty of the customer's arrival and the heterogeneity of basic valuation. However, the analysis of the heterogeneity of the base valuation is ignored in the numerical example. These need to be further studied. In addition, future research should also consider the asymmetry of information between the enterprise and the customer.

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