Entropy-based approach for estimation of natural recharge in Kodaganar River basin, Tamil Nadu, India

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An entropy-based approach has been developed for estimation of natural recharge in Kodaganar River basin, Tamil Nadu, southern India. Wells were located in a weathered aquifer which exhibits spatial variability in natural recharge. To determine the fractional amount of rainfall (called natural recharge), marginal entropy and transinformation of rainfall and depth to the water table at selected wells were calculated. Then a ratio of transinformation to marginal entropy of rainfall was used as a measure for assessing natural recharge. The mean natural recharge values at 28 wells distributed over the study area were computed. The average recharge rate was about 62.95 mm/yr or 14.48% of local average seasonal rainfall. The calculated annual input of rainfall to groundwater reserves was about 0.11×10^9 m^3/yr.

Keywords: Entropy, natural recharge, rainfall, unconfined aquifers, water table.

FOR management of groundwater resources in semi-arid regions, especially in hard-rock areas, it is essential to determine natural recharge. There are several methods for determining groundwater recharge, such as groundwater balance[1–3], lysimeters[4,5], piston-flow model[6–12], remote sensing (RS) and geographical information system (GIS) techniques[13–16], photogeological[17], hydrogeological[2,18] and geophysical methods[19], ^{14}C-age dating[20], chloride mass balance method[5], and regional groundwater models[21]. Among these methods, the tracer technique is the only direct method for estimation of groundwater recharge[6,7]. This technique estimates recharge on the basis of piston-flow model, and has been found useful[10,12,22–24]. Other methods are time-consuming and sometimes even uneconomical in developing countries like India, particularly when one has to deal with a large basin. Therefore, it is desirable to develop a simple method which can quickly provide estimates of natural recharge where further studies can be undertaken later. This study explores the use of entropy for developing such a technique and applies it in Kodaganar River basin of southern India.

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The Kodaganar River basin[25] lies in the Dindigul and Karur districts, Tamil Nadu (Figure 1) and covers about 1752 sq. km. It is essentially a drought-prone area. Rainfall in the area is erratic and vegetation is scarce[26]. Groundwater in the basin is heavily exploited for domestic, agricultural and industrial purposes. Indiscriminate exploitation of groundwater coupled with erratic rainfall has led to continuous decline of water level over a long period of time. During the last three decades the water level has declined by about 10–15 m in some places[27], resulting in the deepening of bore wells, depletion of groundwater resources and deterioration of groundwater quality in many parts of the area[28]. For effective management of groundwater resources, it is imperative to understand the replenishment by rainfall. Therefore, the

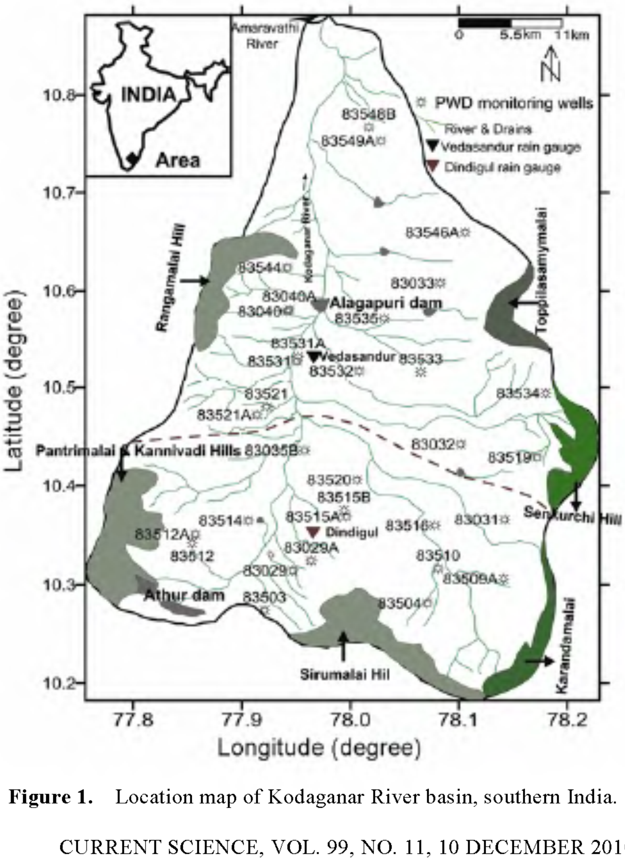

Figure 1. Location map of Kodaganar River basin, southern India.

objective of this study was to assess natural recharge using the concept of entropy, and calculate recharge rate and annual input of rainfall in the basin.

Concept of entropy

Entropy is a measure of the degree of uncertainty of a probability distribution and in turn the random variable. Since the reduction in uncertainty by more observations is equal to the amount of information gained, entropy indirectly measures the information content of a given series of data. Once the statistical distribution of a random variable is known, its entropy can be computed and expressed in specific units. The concept of entropy was first developed by Shannon[29] and has since been applied extensively in a variety of areas. Amorocho and Espildora[30] found that entropy yielded satisfactory results in comparing mathematical models and selecting the most appropriate model. Fiorentino *et al.*[31] analysed the basin morphological characteristics under the assumption that the only information available on a drainage basin is its mean elevation and the assumed connection between entropy and potential energy. For network assessment, Yang and Burn[32] have shown that entropy measures are more advantageous than other measures as they reflect a directional association among sampling sites. Singh[33] discussed the entropy concept and its application to the estimation of parameters of probability distribution, stream-flow forecasting, characterization of landscapes, evaluation of rainfall networks, reliability of water-distribution systems, aquifer parameter estimation, distribution of velocity in open channel and assessment of water quality and design of networks. Several workers[34–39] applied entropy to assess and optimize data-collection networks. So far the entropy concept has not been applied to estimate natural groundwater recharge.

Entropy measures

Entropy of a random variable is a measure of the information or uncertainty associated with it. Measures of information include marginal entropy, joint entropy, conditional entropy and transinformation. For a random variable x, the marginal entropy, $H(x)$, can be defined as the potential information of the variable. For two random variables x and y, the conditional entropy $H(x|y)$ is a measure of the information content of x that is not contained in the random variable y. The joint entropy $H(x, y)$ is the total information content contained in both x and y. The mutual entropy (information) between x and y, also called transinformation, $T(x, y)$, is interpreted as the reduction in uncertainty in x, due to the knowledge of the random variable y. It can also be defined as the information content of x that is contained in y. Entropy measures can be expressed using both discrete and analytical approaches[40,41]. Discrete forms of these entropies can be expressed as follows:

$$H(x) = -\sum_{i=1}^{n} p(x_i)\ln p(x_i), \quad (1)$$

$$H(y) = -\sum_{j=1}^{m} p(y_j)\ln p(y_j), \quad (2)$$

$$H(x,y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,y_j)\ln p(x_i,y_j), \quad (3)$$

$$H(x\,|\,y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,y_j)\ln p(x_i\,|\,y_j), \quad (4)$$

$$H(y\,|\,x) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,y_j)\ln p(y_j\,|\,x_i), \quad (5)$$

$$T(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,y_j)\ln\left[\frac{p(x_i,y_j)}{p(x_i)p(y_j)}\right], \quad (6)$$

where x and y are two discrete variables with values x_i, $i = 1, 2, \ldots, n$; y_j, $j = 1, 2, \ldots, m$, defined in the same probability space, each of which has a discrete probability of occurrence $p(x_i)$ and/or $p(y_j)$; $p(x_i, y_j)$ is the joint probability of x_i, y_j; and $p(x_i\,|\,y_j)$ is the probability of x_i conditional on y_j. Note that $H(x, y) = H(y, x)$.

To calculate information measures for more than one variable, the joint or conditional probability is needed, and this can be obtained using a contingency table. An example of a two-dimensional contingency table is given in Table 1. To construct a contingency table, let random variable x have a range of values consisting of u categories (class intervals), whereas random variable y is assumed to have v categories (class intervals). The cell density or the joint frequency of (x, y) represented by (i, j) is denoted by f_{ij}, $i = 1, 2, \ldots, u$; $j = 1, 2, \ldots, v$, where the

Table 1. Two dimensional contingency table (frequency)

	x(i)					
y (j)	1	2	3	…	u	Total
1	f_{11}	f_{12}	f_{13}	…	f_{1u}	$f_{1.}$
2	f_{21}	f_{22}	f_{23}	…	f_{2u}	$f_{2.}$
3	f_{31}	f_{32}	f_{33}	…	f_{3u}	$f_{3.}$
.	.	.	.	…	.	.
.	.	.	.	…	.	.
.	.	.	.	…	.	.
v	f_{v1}	f_{v2}	f_{v3}	…	f_{vu}	$f_{v.}$
Total	$f_{.1}$	$f_{.2}$	$f_{.3}$	…	$f_{.u}$	f_x or f_y

first subscript refers to the column and the second subscript to the row. Marginal frequencies are denoted by $f_{i.}$ and $f_{j.}$ for the column and row values of the variables respectively.

Transinformation $T(x, y)$ also can be expressed as[42]:

$$T(x,y) = H(x) - H(x \mid y), \tag{7}$$

$$T(x,y) = H(x) + H(y) - H(x,y), \tag{8}$$

$$T(y,x) = H(y) - H(y \mid x), \tag{9}$$

$$T(y,x) = H(y) + H(x) - H(y,x). \tag{10}$$

Rainfall is considered as an independent random variable (x) and the depth to water table for individual wells the dependent variable (y). Then, transinformation, $T(x, y)$ is interpreted as the reduction in the original uncertainty of depth to water table due to the knowledge of rainfall. It can also be defined as the information content of the water table which is also contained in the rainfall. In other words, it is the difference between the total entropy and the sum of marginal entropies of these two variables. This is the information repeated in both water table and rainfall, and defines the amount of uncertainty that can be reduced in one of the variables when the other variable is known. On the other hand, marginal entropy $H(x)$ is defined as the potential information of rainfall. Then, the ratio of $T(x, y)$ to $H(x)$ is simply a fraction of recharge due to rainfall. Therefore, the percentage of rainfall, R_e (%), contributing to the natural recharge of an unconfined aquifer is given as:

$$R_e(\%) = \frac{T(x,y)}{H(x)} \times 100. \tag{11}$$

Application in Kodaganar River basin (hard-rock area)

Study area

The study area is a drought-prone hard-rock terrain, and is located about 400 km southwest of Chennai, India. It lies between 77°45′32″E and 78°13′46″E long.; 10°11′10″N and 10°52′54″N lat. (Figure 1), and encompasses an area of about 1752 sq. km, covering parts of Dindigul and Karur districts. The area is characterized by undulating topography with main hills located in the southern (Sirumalai), southeastern (Karandamalai), eastern (Senkurchi and Toppilasamymalai) and western (Rangamalai) parts sloping towards north and northeast. The highest elevation (altitude) in the hilly area (Sirumalai Hill) is of the order of 1350 m amsl, whereas in the plains it ranges from 360 m amsl in the southern part to 120 m amsl in the northern part. Most of the tributaries of Kodaganar River originate from these hills, which enclose the basin from three sides, viz. south, east and west. Therefore, the entire run-off drains towards its confluence with Amaravathi River in the north. There are two surface-water reservoirs, one at Attur in the southern corner of upstream and another at Alagapuri, north of Vedasandur town. No perennial streams exist in this area, except for short-distance streams encompassing second and third order drainage[26]. The drainage pattern in the southern part is dendritic, whereas in the northern part it is sub-parallel. Run-off from rainfall within the area ends in small streams flowing towards the main Kodaganar River. Most of the rainfall occurs during the months of October–December. For the period of 2000–2007 annual average rainfall was about 875.8 and 607.6 mm in the Dindigul and Vedasandur rain gauge station areas respectively.

Geological and hydrogeological set-up

Granite and gneiss occupy most of the basin, except the hilly areas where charnockite hills form the drainage boundary[43,44]. The larger part is occupied by metamorphic crystalline rocks, which are highly folded, fractured and jointed[45]. Quartzite and pyroxenite occur in patches. Some dykes are present northeast of the Vedasandur area, which strike in NW–SE direction (Figure 2). There is

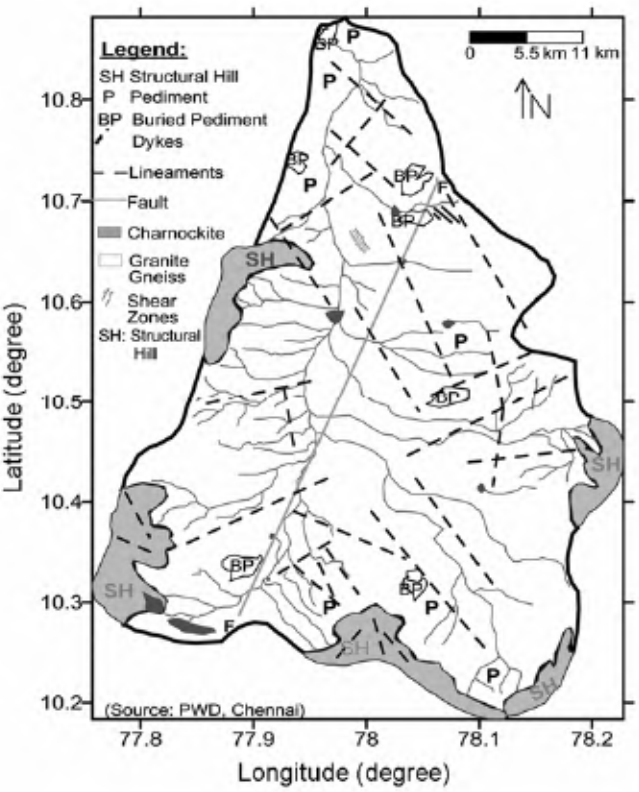

Figure 2. Geological map of Kodaganar River basin.

a major fault running in the NNE–SSW direction for several kilometres situated northeast of Dindigul town. Lineaments are found to a limited extent in the entire area, but are oriented mainly in the NNE–SSW, NEE–SWW and NW–SE directions. Shear zones are also found near Vedasandur. The denudational terrain surrounded by structural hills as described above, occurs in the form of pediments. Shallow pediments and buried pediments are major geomorphic units[26]. The thickness and intensity of this landform vary depending upon the slope and structural disturbances. The area covered by pediment (mostly in the northern part) exhibits rock outcrop with or without soil cover. These areas are basically run-off zones and groundwater potential in these areas is considered as poor[27]. In shallow pediment areas groundwater potential is rated as moderate. The areas of low relief comprising buried pediment are most favourable zones for good groundwater potential. Along the sides of the river or tributaries, flood plains of recent origin are formed potential aquifers. A limited extent of valley fills is also found in this basin.

Groundwater occurs in weathered portions and at depth in jointed and fractured zones[43]. It is being exploited through dug wells tapping the weathered zone, and bore and dug-cum-bore wells tapping fracture aquifers. The straight courses of nalas and streams indicate structural features such as lineaments; faults and joints have controlled sources. The weathered zone facilitates the movement and storage of groundwater through a network of joints, faults and lineaments. The area is interesting in that only a few dug wells function in the central and northern parts, whereas the presence of shear zones and lineaments control the groundwater system. Aquifer parameters, viz. transmissivity (T) and storage coefficient (S) were estimated at 28 existing dug wells through pumping tests. The pumping test data (both pumping and recovery phases) have been interpreted considering the field conditions and evaluated for aquifer parameters[46]. T values were found to vary from 4 to 1166 m^2/d, and S values from 0.00001 to 0.099 in the basin.

Data collection and analysis

Monthly rainfall data from Vedasandur and Dindigul rain gauge stations (Figure 1) were collected for the period from January 2000 to December 2007. During the same period monthly water-level data were also collected at 32 Public Works Department (PWD) monitoring wells. The missing water-level data were calculated using a moving average method[47]. Rainfall distribution in the study area may be non-uniform due to the presence of surrounding hills, undulating topography and other meteorological conditions. Rainfall was monitored by PWD at Vedasandur and Dindigul rain gauge stations for Lower and Upper Kodaganar River basin respectively. It was assumed that the measured rainfall at these two gauges was uniformly distributed in the Lower and Upper Kodaganar River basin.

Climate and rainfall patterns: Normally, sub-tropical climate prevails over the study area without sharp variations. Temperature increases slowly to a maximum in summer months up to May, after which it drops slowly. The mean of maximum temperature ranges from 36.5°C to 41.8°C and in hills it ranges from 7.9°C to 21.8°C. The mean of minimum temperature varies from 17.4°C to 24°C and in hills it varies from 6°C to 8.5°C (ref. 48). The season-wise normal rainfall values for the period from January 2000 to December 2007 are presented in Table 2. In the Lower Kodaganar River basin, 1.22% of the annual rainfall precipitated in winter (January and February), 18.10% in summer (March–May), 26.81% in the southwest monsoon period (June–September), and 53.87% in the northeast monsoon period (October–December), whereas these values were 2.61, 17.46, 25.59 and 54.34% respectively, in the Upper Kodaganar River basin.

As there were only two rain gauge stations, the whole study area was divided into two parts which are affected by the same recorded rainfall[10,12]. The average monthly rainfall values are illustrated in Figure 3, which indicates that the average monthly rainfall was comparatively more in the Upper Kodaganar River basin and in four different stretches. More rainfall, however, occurred in the last stretch each year. The average annual rainfall was estimated to be about 607.6 and 875.8 mm for Lower and Upper Kodaganar River basin respectively.

Hydrological provinces: The entire country (India) has been grouped into four main hydrogeological provinces based on natural recharge values[10]. They are granitic, basaltic, sedimentary and alluvial. The best-fit lines, obtained by the least square method, show a linear correlation between seasonal rainfall and natural recharge in each case. This linear relation between rainfall and natural recharge exists for all four major hydrogeological units. The regression equation, derived for each of the hydrogeological provinces, indicates a certain minimum rainfall requirement to initiate groundwater recharge. The minimum values are 255 mm/yr for granite, 355 mm/yr for basalt, 220 mm/yr for sediments, and 40 mm/yr for alluvial areas. The average natural recharge value in 15 granitic areas (in India) is 10.11% of rainfall during the rainy season, which was estimated using the tritium injection technique[10]. The seasonal normal rainfall during the NE monsoon in the proposed study area was 327.3 and 475.9 mm in the Lower and Upper Kodaganar River basin respectively (Table 2). Therefore, the rainy season and monthly rainfall values for the entire period were considered for the estimation of natural groundwater recharge using entropy.

Table 2. Season-wise normal rainfall at Vedasandur and Dindigul rain gauge stations

Season	Period	Rainfall (mm)	Percentage
Vedasandur rain gauge station			
Winter	January and February	7.4	1.22
Summer	March–May	110.0	18.10
Southwest monsoon	June–September	162.9	26.81
Northeast monsoon	October–December	327.3	53.87
	Total	607.6	100.00
Dindigul rain gauge station			
Winter	January and February	22.9	2.61
Summer	March–May	152.9	17.46
Southwest monsoon	June–September	224.1	25.59
Northeast monsoon	October–December	475.9	54.34
	Total	875.8	100.00

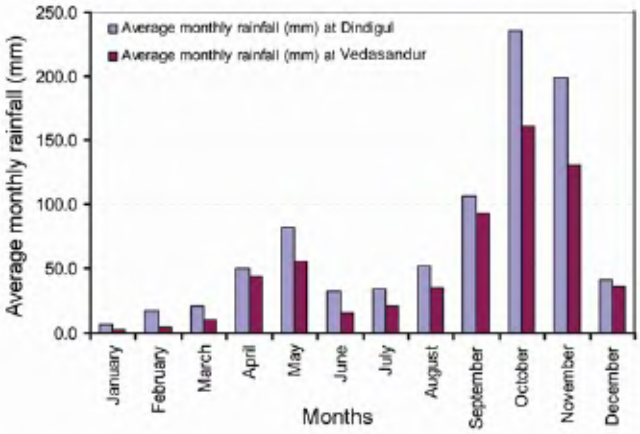

Figure 3. Average monthly rainfall (mm) at Dindigul and Vedasandur rain gauge stations.

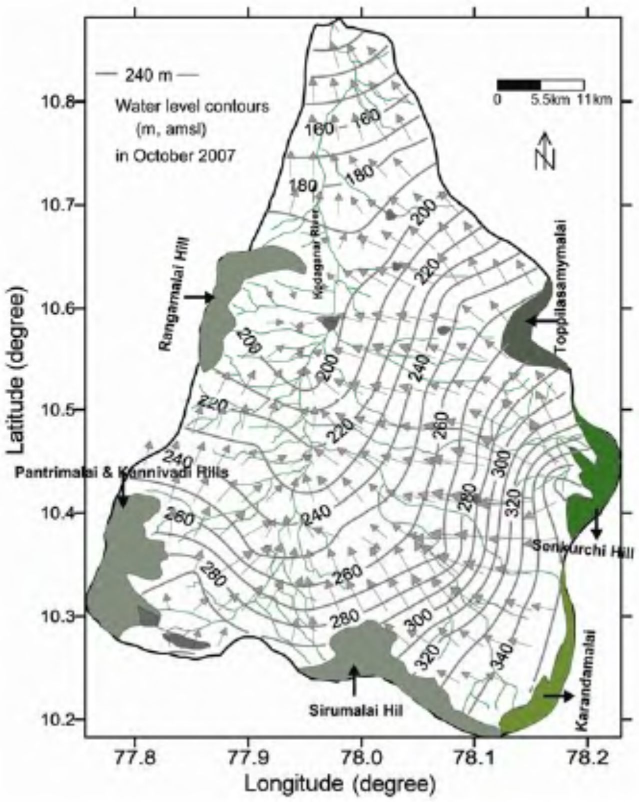

Figure 4. Water-level contour map and groundwater flow direction for October 2007.

Water-level measurements: Details of 32 PWD well inventories are given in Table 3. All the open wells were rectangular in shape, except for ten circular structures, with depth ranging from 10.60 to 28.50 m below ground level (bgl). The depth to water bearing zone varied from 1 m to 9.50 m bgl, and the thickness ranged from 1.54 to 17.45 m under phreatic conditions. Groundwater was extracted mainly through the bucket and pulley method for domestic and gardening purposes. Monthly water levels were monitored during the first week of every month[48] from January 2000 to December 2007 from these PWD wells, which are uniformly spread (Figure 1). The study of water-level fluctuation helps determine the time-wise depth to water level, recharge and discharge periods, hydraulic gradient, and rate of water-level increase or decrease[48]. The water level contours for October 2007 are shown in Figure 4. This exhibits regional groundwater flow direction, but not the micro-level characteristics of the aquifer.

When water-level hydrographs with rainfall were plotted, there was approximately one month time lag in the response of the water table to rainfall events. Typical well hydrographs at wells 83531 and 83533 and rainfall recorded near Vedasandur are shown in Figure 5 *a*, whereas Figure 5 *b* shows the well hydrographs at wells 83504 and 83520 and rainfall at Dindigul. The water-table fluctuations due to rainfall have been used to locate areas where there has been good response. The general trend of the hydrograph indicates that it closely follows the rainfall trend. In general, water level in most of the cases returns to its original position after good rainfall. This phenomenon may be due to rapid recharge that takes place due to heavy rainfall and also irrigation return flows.

Table 3. Inventory of Public Works Department (PWD) wells, Kodaganar River basin

Well ID	Village	Latitude	Longitude	Dimension (m)	Total depth (m bgl)	Lining depth (m)	Water bearing (m bgl)	Elevation of ground level (m amsl)	MP (m)	Depth to water level (m) (October 2007)
At Lower Kodaganar River basin										
83032	Vadamadurai	10°26′30″	78°06′08″	4.00 × 5.35	22.00	4.0	4.15–6.00	284.99	1.00	6.40
83033	R.Kombai	10°36′30″	78°04′58″	2.80 × 2.80	12.60	4.0	3.76–5.30	257.47	1.30	10.20
83040	Kaithienkottai	10°34′45″	77°56′32″	7.40 × 7.40	20.65	4.0	4.47–10.00	217.38	0.80	17.93
83040A	Kaithienkottai	10°34′53″	77°56′40″	2.14 × 2.48	18.40	5.0	5.15–8.00	220.45	0.72	10.98
83519	Puthur	10°25′45″	78°10′19″	2.17	11.50	2.0	1.00–8.63	372.37	0.89	10.40
83521	Alagupatti	10°28′48″	77°55′23″	4.80 × 4.59	11.25	3.0	3.45–7.00	231.54	0.44	8.06
83521A	Alagupatti	10°28′18″	77°54′50″	2.70 × 2.70	12.25	6.0	6.00–10.15	237.95	0.85	9.95
83531	Vedasandur	10°31′40″	77°57′03″	6.65 × 7.65	23.40	3.0	3.20–9.00	213.40	0.70	18.20
83531A	Vedasandur	10°31′57″	77°57′08″	2.40 × 2.20	17.50	2.0	1.65–14.04	211.72	0.85	16.82
83532	Marambadi	10°31′05″	78°00′30″	2.30 × 2.30	19.90	3.0	3.40–8.40	235.46	0.77	17.80
83533	Thennampatti	10°31′00″	78°03′55″	3.38 × 3.48	17.50	2.0	1.95–6.00	263.84	0.67	8.33
83534	Kollapatti	10°29′38″	78°10′48″	2.45 × 2.45	10.60	4.0	4.75–10.60	317.74	0.84	9.16
83535	Usilampatti	10°34′20″	78°01′50″	2.68 × 2.68	12.00	5.0	4.76–6.60	229.71	0.78	10.62
83544	Kalvarpatti	10°37′27″	77°56′30″	2.90	27.60	2.0	2.23–10.00	220.06	0.74	22.91
83546A	R.Kombai	10°39′35″	78°06′22″	2.67	20.50	6.0	4.74–8.12	242.76	1.10	17.15
83548B	Ayyampatti	10°46′00″	78°01′00″	2.38 × 1.87	22.00	2.0	2.14–5.80	156.19	0.72	19.98
83549A	R.Vellodu	10°45′12″	78°01′42″	2.05	19.15	2.0	1.70–19.15	191.55	0.85	12.75
At Upper Kodaganar River basin										
83029	A.Vellodu	10°18′50″	77°56′50″	3.02	22.00	3.0	2.60–8.10	279.76	1.00	16.05
83029A	A.Vellodu	10°19′26″	77°57′50″	3.93	28.50	3.0	3.00–7.00	282.80	0.45	20.33
83031	Ammapatti	10°21′58″	78°08′30″	2.00	12.50	6.0	6.00–10.80	341.16	1.00	12.05
83035B	Chettinaickenpatti	10°26′11″	77°57′28″	2.70	16.60	3.0	3.70–13.45	232.94	0.75	11.80
83503	Ambathurai	10°16′25″	77°55′14″	3.85 × 3.00	16.65	4.0	4.20–9.00	300.39	0.66	8.76
83504	Shanarpatti	10°16′53″	78°04′15″	4.76 × 5.40	16.80	4.0	4.33–12.39	330.79	0.96	12.34
83509A	Silvarpatti	10°18′23″	78°08′26″	4.60	15.95	2.0	8.50–15.50	353.21	1.15	14.75
83510	Ragalapuram	10°18′57″	78°04′53″	2.27 × 2.27	13.20	5.0	5.40–10.56	301.53	1.24	2.61
83512	Palayakannivadi	10°20′30″	77°51′15″	3.15	16.70	4.0	4.40–7.30	300.55	0.76	10.19
83512A	Palayakannivadi	10°21′05″	77°51′24″	5.65 × 6.80	19.55	5.0	9.50–19.55	286.86	0.60	5.15
83514	Sinthalakundu	10°21′55″	77°54′20″	2.41 × 2.47	14.00	5.0	5.00–8.00	258.60	0.90	12.40
83515A	Dindigul	10°22′10″	77°59′45″	7.36 × 6.08	14.65	3.0	2.54–11.10	258.44	1.05	9.35
83515B	Dindigul	10°22′30″	77°59′40″	3.50 × 3.25	18.00	4.0	5.85–17.50	258.97	1.25	2.45
83516	Madur	10°21′35″	78°04′40″	3.05 × 2.99	15.10	3.0	3.40–6.50	284.68	0.85	14.90
83520	Seelapadi	10°24′22″	78°00′23″	2.82 × 2.93	18.40	4.0	3.80–18.40	259.92	0.78	11.57

Well type: Dug well; Type of aquifer: Phreatic; Geology: Granite and gneiss; Stratigraphy: Archaean; Water-lifting device: Bucket and pulley method; bgl: Below ground level; MP: Measuring point.

Entropy-based analysis

In hard-rock areas in many countries, such as India, where groundwater occurs in shallow sparolite zones, the rise in groundwater table is a direct consequence of rainfall, particularly in the monsoon season, when groundwater withdrawal is minimum. The rise in water table at a particular place is a characteristic feature of the unsaturated zone[9,8,49–51]. Therefore, for a particular region, there exists a definite relationship between the depth to water table and rainfall. From an entropy perspective these two variables possess individual information, and some information is transmitted from rainfall to the depth to water table. This transmission of information is referred to as transinformation and can be hypothesized as proportional to recharge. This concept is employed to measure groundwater recharge characteristics or favourable recharge zones and their locations in hard-rock areas.

Data on the depth to water table and rainfall from Kodaganar River basin were analysed. Monthly water-level data, recorded at 28 monitoring wells for 7 years (during January 2000–December 2007) by PWD[48], Tamil Nadu, were considered along with rainfall recorded from Vedasandur and Dindigul rain gauge stations. The transformation values for rainfall and depth to water table for these PWD wells were determined using eq. (6) or (8). Then, the ratios between transformation of individual wells and marginal entropy of rainfall were calculated using eq. (11) for assessing the natural recharge.

For computation of information, joint or conditional probabilities were calculated using a contingency table for monthly data as well as NE monsoon data. A total 96 and 24 events were used for constructing contingency tables of monthly and NE monsoon datasets respectively. An illustration of a two-dimensional contingency table for monthly dataset of PWD well 83544 is given in Table 4.

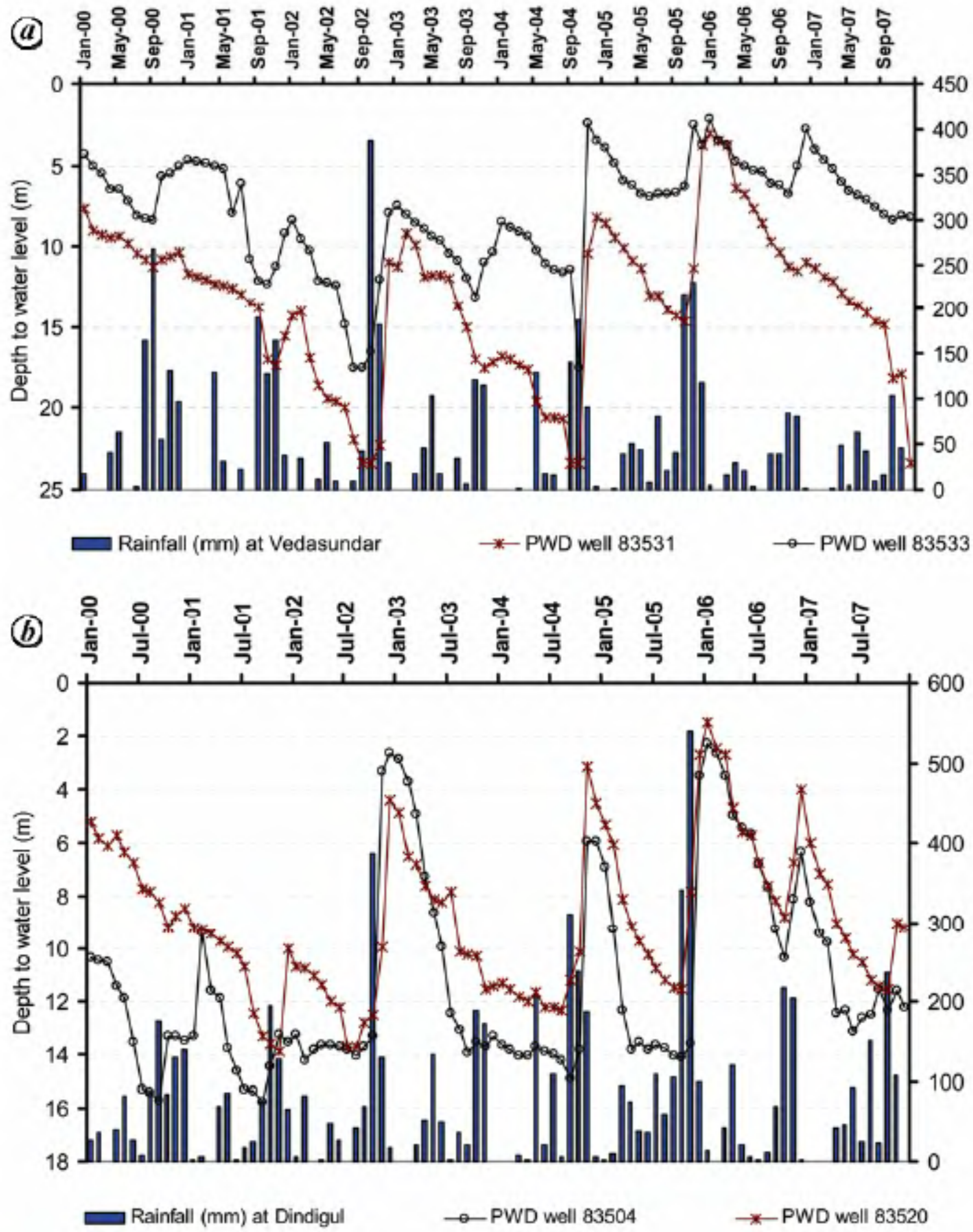

Figure 5. Well hydrographs at (***a***) wells 83531 and 83533 and (***b***) wells 83504 and 83520 from January 2000 to December 2007.

Rainfall was considered to have a range of values (0–100, 100–200, 200–300 and 300–400 mm) consisting of four categories (class intervals) and values (0–150, 150–300, 300–450 and 450–600 mm) consisting of four categories (class intervals) for the Lower and Upper Kodaganar River basin respectively, whereas the depth to water table was assumed to have five categories (class intervals) with the range of 5.00 m bgl. The cell density or the joint frequency for (i, j) was denoted as f_{ij}, $i = 1, 2, \ldots, 4$; $j = 1, 2, \ldots, 5$, where the first subscript refers to the column (rainfall) and the second subscript to the row (water table). Marginal frequencies were denoted by f_i and f_j for the column and the row values of these two variables respectively. Then the marginal entropies of rainfall and water table, and total entropy were calculated using eqs (1)–(3). Transinformation, $T(x, y)$, given by eq. (8), was also calculated for each PWD well. For better understanding, marginal, conditional and joint entropies, and transinformation of rainfall and water table at well 83544 for the period 2000–2007 are presented in Figure 6. Then, the fractional amount of natural recharge due to rainfall was estimated using eq. (11). The results are presented in Table 5 for the entire month and NE monsoon data respectively. Traninformation varied from 0.005 to 0.171

Table 4. Absolute frequency contingency table for rainfall (mm) and depth to water table (m bgl) at PWD well 83544 during January 2000–December 2007

		x (i), Rainfall (mm)								
y (j), Depth to water table		i = 1 0–100	i = 2 100–200	i = 3 200–300	i = 4 300–400	Total	p_i	ln p_i	p_i ln p_i	$H(Y)$ = 1.689 bits calculated from eq. (2)
j = 1	0–5	5	1	0	0	6	0.063	–4.000	–0.250	
j = 2	5–10	15	1	0	0	16	0.167	–2.585	–0.431	
j =3	10–15	11	2	1	0	14	0.146	–2.778	–0.405	
j= 4	15–20	2	1	0	0	3	0.031	–5.000	–0.156	
j = 5	20–25	44	10	2	1	57	0.594	–0.752	–0.447	
	Total	77	15	3	1	96				
	p_i	0.802	0.156	0.031	0.010	$H(X)$ = 0.898 bits calculated from eq. (1)				
	ln p_i	–0.318	–2.678	–5.000	–6.585					
	p_i ln p_i	–0.255	–0.418	–0.156	–0.069					
$p_{i,j}$		0.052	0.010	0.000	0.000					
		0.156	0.010	0.000	0.000					
		0.115	0.021	0.010	0.000					
		0.021	0.010	0.000	0.000					
		0.458	0.104	0.021	0.010					
$p_{i,j}$ ln $p_{i,j}$		–0.222	–0.069	0.000	0.000	$H(X, Y)$ = 2.546 bits calculated from eq. (3)				
		–0.418	–0.069	0.000	0.000					
		–0.358	–0.116	–0.069	0.000					
		–0.116	–0.069	0.000	0.000					
		–0.516	–0.340	–0.116	–0.069					
		$T(X, Y)$ = 0.041 bits calculated from eq. (8)								
		R_e (%) = 4.57 calculated from eq. (11)								

All entropies were calculated with base 2.

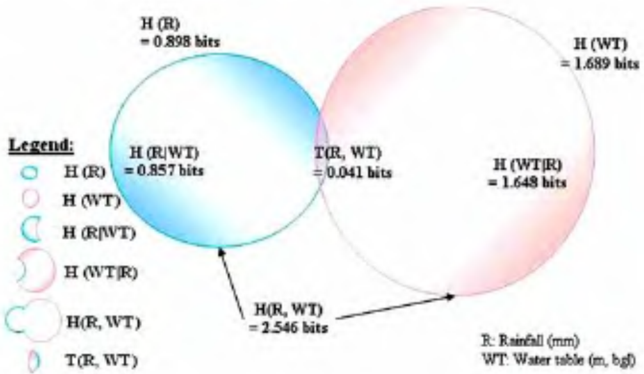

Figure 6. T (R, WT): Information common to rainfall (R) and water table (WT); H ($R|WT$): information only in rainfall; H ($WT|R$): information only in water table; and $H(R, WT)$: total information only in rainfall and water table together at PWD well 83544.

bits for the monthly dataset. For NE monsoon data, 0.030 to 0.397 bits of uncertainty reduced in rainfall when depth to water level was known in all the PWD wells, which indicates that the dependency between rainfall and water table was comparatively more in the NE monsoon period. It also implies that the natural recharge process was more dominant during this period when maximum rain occurred. The calculated seasonally natural recharge was 14.49% and 14.46% of rainfall for the Upper and Lower Kodaganar River basin respectively. The recharge rates ranged from 50.92 mm/yr in the Lower basin to 74.97 mm/yr in the Upper basin. Then the total annual groundwater replenishment in the basin for an average annual rainfall was calculated using the mean recharge rate of 62.95 mm/yr. The calculated annual input was about 0.11×10^9 m^3/yr.

Rangarajan and Athavale[10] estimated an average natural recharge ratio of 10.11% of seasonal rainfall for 15 granitic and gneiss areas in varying climatic and hydrogeological provinces of India using the tritium injection method. The average recharge ratio is about 9.90% for four-granitic and gneiss areas in Tamil Nadu. The estimated average natural recharge in the study area using entropy varies from 0.63% to 21.65%, and 2.00% to 26.12% during the entire period and NE monsoon respectively (Table 5). But the simple mean of these 28 recharge values estimated was 8.46% of rainfall in the entire period, which is acceptable. Further Table 5 shows that the estimated recharge proves that the fractional recharge due to rainfall depends on the magnitude, duration and intensity of rainfall. It can be noticed that the joint entropies of all the wells are not systematic during the different periods. This may be due to non-uniformity of precipitation and different well hydrogeological characteristics. If the precipitation is measured at each nearby well corresponding to the water table, measurements of natural recharge using this entropy will be more accurate.

Table 5. Estimated average natural groundwater recharge in the Lower and Upper Kodaganar River basin using entropy for monthly data and NE monsoon data during January 2000–December 2007

Lower Kodaganar River basin							Upper Kodaganar River basin							
PWD well	*H* (*x*)	*H* (*y*)	*H* (*x*, *y*)	*T* (*x*, *y*)	Natural recharge (%)	Average (%)	PWD wells	*H* (*x*)	*H* (*y*)	*H* (*x*, *y*)	*T* (*x*, *y*)	Natural recharge (%)	Average (%)	Average (%) in the basin
For monthly rainfall (mm) and depth to water level data (m bgl) from January 2000 to December 2007														
83032	0.898	1.747	2.530	0.115	12.81	8.38	83029	0.790	2.005	2.696	0.099	12.53	8.54	8.46
83033	0.898	1.354	2.212	0.040	4.45		83029A	0.790	1.055	1.771	0.074	9.37		
83040	0.898	1.986	2.755	0.129	14.37		83031	0.790	1.228	1.954	0.064	8.10		
83040A	0.898	1.147	1.934	0.111	12.36		83035B	0.790	1.534	2.153	0.171	21.65		
83521	0.898	0.650	1.524	0.024	2.67		83503	0.790	1.737	2.451	0.076	9.62		
83521A	0.898	1.034	1.898	0.034	3.79		83504	0.790	1.450	2.191	0.049	6.20		
83531	0.898	1.937	2.687	0.148	16.48		83509A	0.790	1.055	1.771	0.074	9.37		
83531A	0.898	1.754	2.598	0.054	6.01		83510	0.790	0.993	1.746	0.037	4.68		
83533	0.898	1.580	2.341	0.137	15.26		83512	0.790	1.775	2.508	0.057	7.22		
83534	0.898	1.472	2.319	0.051	5.68		83512A	0.790	1.875	2.604	0.061	7.72		
83535	0.898	1.312	2.173	0.037	4.12		83514	0.790	1.034	1.745	0.079	10.00		
83544	0.898	1.689	2.546	0.041	4.57		83515B	0.790	0.146	0.931	0.005	0.63		
83546A	0.898	1.560	2.362	0.096	10.69		83516	0.790	0.622	1.382	0.030	3.80		
83549A	0.898	1.423	2.284	0.037	4.12		83520	0.790	1.378	2.100	0.068	8.61		
For monthly rainfall (mm) and depth to water level data (m bgl) during NE monsoon in the same period														
83032	1.520	1.613	2.736	0.397	26.12	14.46	83029	1.497	1.892	3.034	0.355	23.71	14.49	14.48
83033	1.520	1.309	2.614	0.215	14.14		83029A	1.497	2.209	3.320	0.286	19.10		
83040	1.520	2.000	3.011	0.309	20.33		83031	1.497	1.189	2.538	0.148	9.89		
83040A	1.520	1.320	2.464	0.280	18.42		83035B	1.497	1.601	2.615	0.383	25.58		
83521	1.520	0.650	2.115	0.055	3.62		83503	1.497	1.692	2.887	0.202	13.49		
83521A	1.520	1.099	2.470	0.149	9.80		83504	1.497	1.158	2.491	0.164	10.96		
83531	1.520	1.867	2.918	0.296	19.47		83509A	1.497	0.209	3.320	0.286	19.10		
83531A	1.520	1.717	2.955	0.282	18.55		83510	1.497	0.650	2.014	0.133	8.88		
83533	1.520	1.786	2.820	0.286	18.82		83512	1.497	1.834	3.088	0.243	16.23		
83534	1.520	1.236	2.653	0.103	6.78		83512A	1.497	1.815	3.018	0.294	19.64		
83535	1.520	1.406	2.818	0.108	7.11		83514	1.497	0.785	2.115	0.167	11.16		
83544	1.520	1.206	2.531	0.195	12.83		83515B	1.497	0.095	1.562	0.030	2.00		
83546A	1.520	1.392	2.465	0.247	16.25		83516	1.497	0.497	1.916	0.078	5.21		
83549A	1.520	1.099	2.464	0.155	10.20		83520	1.497	1.528	2.757	0.268	17.90		

All entropies calculated with base 2; unit of entropy is bits.

Conclusion

Entropy is a potentially useful tool for the estimation of natural recharge using measured water table and rainfall. It has been applied in a hard-rock area, Kodaganar River basin in southern India. The results show that the average natural recharge rate is about 62.95 mm/yr or 14.48% of the local average seasonal rainfall. The annual input of rainfall to groundwater reserves is about 0.11×10^9 m^3/yr.

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ACKNOWLEDGEMENTS. N.C.M. had carried out this work, in part, under the BOYSCAST Fellowship funded by DST, New Delhi. We thank officials of PWD, Chennai for providing suitable data. We also thank the anonymous reviewers for constructive comments to improve the manuscript.

Received 15 December 2009; revised accepted 13 October 2010