Yves Meyer, the 2010 Gauss Prize winner

Yves Meyer at ICM 2010 on 19 August 2010 (courtesy: Rahul V. Pisharody).

Yves Meyer is a French mathematician who has been awarded the 2010 Gauss Prize, given jointly by the International Mathematical Union and the German Mathematical Union, 'for fundamental contributions to number theory, operator theory and harmonic analysis, and his pivotal role in the development of wavelets and multiresolution analysis'. He is currently a Professor Emeritus at the École Normale Supérieure de Cachan, France.

In a conversation with *Current Science*, Meyer throws light on French mathematics, and the significance of awards and exciting areas of research in the field. He illustrates his work that has been acknowledged with the Gauss Prize and leaves a message for the young.

About mathematics in France

The French have been making their mark in the field of mathematics. What is the reason for this?

We have in France a strong mathematical tradition which began with François Viète (1540–1603) and did not stop since. In France, more than in other countries, we believe that mathematics is the language in which all the scientific knowledge shall be written eventually. Our educational system has been putting a strong emphasis on mathematics up to now. I will later comment on the new and dangerous changes which were recently introduced in high schools.

How is the mathematics school education in France now? Could you compare it with your schooling days?

Everything is so different. I did not study sciences at high school. I took humanities, Latin and Greek. The number of children attending high school in France has been multiplied by ten since I was a child and the way of teaching has evolved accordingly. I was attending high school in Tunis (Tunisia), a place where the level of education was stimulated by the competition between different ethnic groups. It is clear that what is taught nowadays at high school is negligible both in quality and quantity, compared to what we were learning in the fifties. But the number of teenagers attending high school is much larger. People are right saying that the educational level has increased on the average. People are also right saying that the smartest of our children are not stimulated the way they should be.

About the Gauss Prize

Could you explain your work for which you have been awarded the Gauss Prize by way of an example?

The jury of the Gauss Prize liked the absence of clear frontiers between pure and applied mathematics in my research. The most striking illustration is my discovery of some geometrical structures which anticipated quasicrystals. Quasicrystals were later found as specific organizations of atoms in certain alloys in chemistry. I will comment now on these findings.

After completing my Ph D, I became fascinated by number theory and by the work achieved by Tirukkannapuram Vijayaraghavan (1902–1955), an Indian mathematician from the Madras region. He worked with G. H. Hardy (a famous English mathematician) on what are now called Pisot–Vijayaraghavan numbers. He did this work when he went to Oxford in the mid-1920s. Vijayaraghavan was a fellow of the Indian Academy of Sciences, Bangalore, elected in the year 1934. Charles Pisot (1910–84) was a French mathematician whom I knew and admired.

I am returning now to my discovery of what was going to become quasicrystals. In 1969, I discovered some new configurations of points in the plane or in the space. Like stars in the sky the relation between these configurations and the Pisot–Vijayaraghavan numbers is quite exciting. If L is an ordinary lattice in the plane, then the dilated lattice $2L$ is contained in L, the same way that even integers are integers. For my configurations of points in the plane or in the space (configurations which I will still name L), we have something similar: θL is contained in L when θ is a Pisot–Vijayaraghavan number. These configurations have been found independently by Roger Penrose (Penrose pavings, 1976) and next by D. Shechtman, I. Blech, D. Gratias and J. W. Cahn in chemistry (1984). The corresponding quasicrystal is the set of vertices of this paving.

The positions of atoms in certain alloys exactly obey the mathematical rules I had discovered 15 years ago. This striking example proves that mathematicians can be prophetic. It suffices to type 'quasicrystal' or 'Meyer sets' in Google to get some of these beautiful configurations (Figure 1).

The story does not end there. Indeed quasicrystals have been used as decorative patterns in medieval Islamic art (Figure 2). Peter Lu, Harvard University, made this outstanding discovery while he visited a madrassa at Boukhara, Uzbekistan.

I love this story where number theory, pavings, chemistry and Islam are reconciled. I always liked moving from one field of mathematics to another, from pure to applied mathematics and founding new and unexpected connections.

The Gauss Prize is awarded for the application of mathematics. What do you think is the most striking/ fascinating application of mathematics?

In my opinion the most striking application of mathematics is the digital revolution. The digital revolution is affecting our everyday life. When I was 15 years

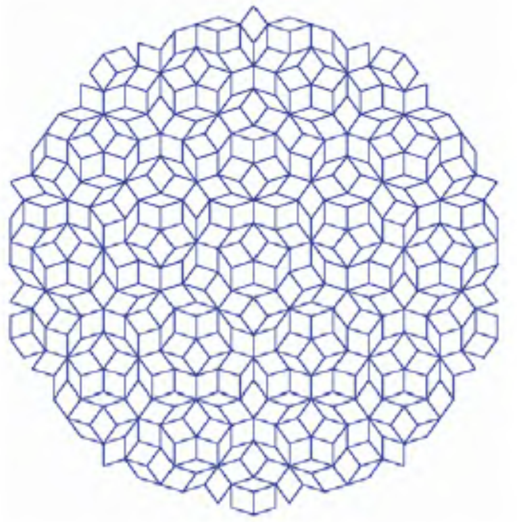

Figure 1. A Penrose paving (courtesy: Meyer).

Figure 2. Quasicrystals used as decorative patterns in medieval Islamic art (courtesy: Meyer).

old, it was almost impossible to hear a telephone conversation from Tunis to Paris correctly (I spent my childhood and adolescence in Tunis). The crackles were much stronger than the voice of the person speaking. Nowadays everything is fine as your voice is converted into a string of 0s or 1s. Your voice is not transmitted. What is travelling is this string of 0s or 1s which is a 'copy' of your voice. Strings of 0s or 1s are not affected by the noise on the line (the transmission channel). For achieving this digital transmission you need to compress the signal to be transmitted. Compressing means simplifying and sometimes destroying some aspects of the message. Clever and efficient compression algorithms have been designed by mathematicians since the pioneering work of Claude Shannon. Sophisticated mathematical tools are being used in the digital revolution. The space mission, Herschel is transmitting images of the early universe to earth. Transmitting these images from a telescope located at one more than a million kilometres from the earth requires compression.

The prizes bring along with them recognition. Has the Gauss Prize changed the way your professionally close ones deal with you?

I am 71 years old and my colleagues know me quite well. They know my qualities and my weaknesses. I do not think this prize will change their opinion.

How do you plan to use your prize money?

I do not know yet.

About mathematics and mathematicians

How has the research in mathematics evolved/changed over the years?

I am happy to answer this question. One of the greatest changes in the last 20 years is the increasing relevance of mathematics in our industrial societies. Let me just give a trivial example. My brother-in-law is a German architect. He told me how much his field has evolved by using mathematics. Nowadays architects are constructing buildings which seem to defy common sense in terms of stability. These incredible constructions are checked on a computer with software which incorporate all the laws of mechanics together with all the existing knowledge on the mechanical properties of glass, iron and concrete. In this example mathematics is replacing the traditional experience. It works beautifully and these new buildings are fine. It is clear that mathematics is much stimulated nowadays by the needs of the society.

A second striking fact is extraordinary vitality of the traditional parts of mathematics, as number theory, algebraic geometry, topology of manifolds, etc. These fields were viewed for a long time as completely irrelevant in applied problems. Nowadays people understand that the converse is true. Let us single out the role of mathematical logic in the certification of some large industrial softwares. Using abstract tools coming from mathematical logic, Gérard Berry proved that the softwares embarked on the Airbus airplanes were safe.

The vitality of mathematics could be compared to one of a living organism. As Raphy Coifman (Yale) used to say, many key fields in mathematics are still in their infancy. His favourite example is the study of differentiable functions of more than a thousand real variables. Coifman says that our intuition on such functions is wrong as it is coming from the experience of functions of two or three variables. A new science has to be built for dealing with functions of a thousand real variables. Studying such functions is a key ingredient for understanding the functioning of the human brain. Every progress in mathematics is widening the power of our brain. In other words mathematics is providing mankind with an extended and more powerful brain. Therefore, mathematics should belong to mankind, which is far from being the case.

What are the current exciting areas of research in mathematics?

There are so many of them that I will focus on two examples only, nonlinear partial differential equations (PDEs) and number theory.

Navier–Stokes equations are nonlinear PDEs governing fluid dynamics. Navier–Stokes equations tell us what kind of turbulence an airplane is developing around its fuselage. Navier–Stokes equations are seminal in weather prediction. Seventy years after a major breakthrough achieved by Jean Leray, these equations are still completely mysterious. We are unable to solve the most basic questions, as uniqueness and stability of solutions. This part of mathematics is close to what Cédric Villani (Fields Medallist 2010) is doing.

Number theory is, in my opinion, one of the most beautiful parts of mathematics. The Langlands program bridges the gaps between number theory, automorphic forms, algebraic geometry and group representations. It is one of the most fascinating intellectual constructions in mathematics. It is what Ngô Bảo Châu (Fields Medallist 2010) is doing. One should also mention Riemann hypothesis which governs the distribution of prime numbers. The Riemann hypothesis is related to many problems in mathematical physics, and these incredible connections were understood and studied by Alain Connes.

What is the most fascinating aspect of today's mathematics is the cross-fertilization between distinct fields. When I was preparing my Ph D, people interested in the topology of manifolds (pure mathematics) were ignorant of the advances in PDEs (viewed as applied mathematics by the Bourbaki team). But the proof of the Poincaré conjecture by Grigori Perelman is based on some deep results in nonlinear PDEs. The Witten program and string theory offer another example of cross-fertilization between geometry and physics. Today mathematics strongly interacts with other scientific fields. An example is image processing where the interaction with mathematics is particularly intense. In signal processing, the recent results on compressed sensing by Emmanuel Candès and David Donoho are based on deep mathematics and are raising some exciting problems in the field. Applications are immense in medical imaging (functional magnetic resonance imaging).

Here is a personal opinion. I feel that one of the ultimate goals of mathematics is to understand the functioning of the human brain. One will discover that large pieces of mathematics are encoded in our brain. As geometry is concerned, this was already checked by the Nobel Prize laureate, David Hubel. But more is going to be discovered. This would explain the Plato myth of knowledge as memory, a theory which is developed in his dialogue named Menon.

How do you think teaching in mathematics can be improved?

I taught mathematics as a high-school teacher for three years. Afterwards I taught undergraduate and graduate courses at several universities. I did that for 47 years. I have now an accumulated experience of 50 years of teaching. For a child, doing mathematics at the high-school level should mean making progress in reasoning, in thinking. To me the greatest advantage of a mathematical education is the ability to think by oneself, without any guidance. Children should know that they can discover truth by themselves, which is in my opinion the beauty of doing mathematics. Nowadays, one is stressing the relevance of mathematics in technology. It is a completely distinct story. Every child should feel that he is so important, that his opinions are relevant, and that some nugget of gold is always lying inside his seemingly absurd talk. Then he will suddenly make progress.

Mathematics is not very popular among the public at large. What do you think could be done to popularize it?

Populism and demagogy are infecting France nowadays. Most of our great intellectual traditions are increasingly despised. A football player is a hero nowadays in France, while a scientist is a person you should not mix with. All sciences are affected by these prejudices against learning. The number of students in sciences is dropping at a dangerous rate in France. High-school teachers acknowledge that the French author, Michel de Montaigne is too difficult for their students. Poetry is not popular in France. In many high schools, a boy who likes reading is ostracized as being effeminate. The number of hours devoted to mathematics in French high schools is steadily decreasing, while hours devoted to sports have increased. Is there a remedy? It is clear that our present government is anxious to raise popular approval in lowering the educational load on children.

What is the success mantra for a mathematician?

I am not sure if I understand this question correctly. I am inclined to translate this question into 'What spiritual preparation do you need to be a successful mathematician?' I will answer that question. The success mantra of a successful mathematician is the complete lack of prejudices about a problem, the willingness to accept completely crazy ideas. Your mind has to be flexible. David Hilbert said that one of his students did not have enough imagination to become a mathematician. Instead, he became a poet. In my opinion great successes rely on cross-fertilization of ideas coming from unexpected sources. This has been the case when I solved Calderon's conjectures. Alan Mc Intosh understood that these conjectures could be incorporated into a larger program raised by Tosio Kato in operator theory and this new light on Calderon's conjectures was decisive.

Meyer in the light of mathematics

What is your approach in doing mathematics? Do you like working on specific problems?

I love working on specific problems. I spent about 10 years (1974–84) working on Calderon's conjectures, which I eventually solved. I also loved working (1967–70) on a problem raised by Raphaël Salem on the role of Pisot numbers in spectral synthesis. I solved this problem and discovered quasicrystals on the way. Working on hard problems forces one to create new and unexpected intellectual tools.

Is the application targeted first and then the problem chosen to work on?

I am not an applied mathematician and I am not able to build mathematical models for problems raised by industry or finance. Building models is necessary if you want to solve a practical problem. The ability to build models is a remarkable talent. Jacques-Louis Lions had this talent at an extreme level. This talent lies outside mathematics. I always worked on models built by other applied mathematicians.

Do you continue to teach? What are the important things a school teacher must keep in mind for teaching mathematics? Is it different from teaching other subjects?

I am teaching a graduate course at École Normale Supérieure de Cachan. Teaching mathematics is quite distinct from teaching other fields. In mathematics, the student can state that the teacher is wrong and prove his claim. You cannot prove to your physics teacher that the electron does not exist. What is fascinating in teaching mathematics is that a student can be smarter than you. It is the only field in science where this equality between you and the student exists. The student has to be a believer in all other fields. A student in physics cannot reproduce the experience by Michelson and Morley, which opened the gate to Einstein's work on relativity. The student has to take for granted the results of this experience. It is a kind of religious attitude. This religious attitude does not exist in mathematics. You can check everything by yourself.

About prizes in mathematics

Why do you think there is no Nobel Prize for mathematics?

There exist several legends about the conflicting relations between Alfred Nobel, the chemist who founded the Nobel prizes, and the Swedish mathematician Mittag Leffler. It seems that Mittag Leffler was irritating Nobel. Many people say that the conflict between these two strong personalities was the problem.

Is it in your view appropriate to compare the Fields Medal to the Nobel Prize? Why/why not?

It is clear that the two prizes are completely distinct. The Fields Medal was not awarded to gigantic mathematicians as Vladimir Arnold, Alberto Calderon, Lennart Carleson, Alexander Kolmogorov, Jean Leray, Louis Nirenberg and André Weil, who truly reshaped mathematics. If a Nobel Prize in mathematics existed, these seven mathematicians would have got it. A Fields medallist is, by definition, a precocious mathematician. Some Fields medallists were problem-solvers and did not have a lasting influence on mathematics. Comparing a Fields medallist to a Nobel prize winner is like comparing a sprinter to a marathonian.

What is the role of these prizes in mathematics?

I am hesitant about the role of prizes in mathematics. Sometime ago I belonged to the Salem Prize committee. The Salem Prize is awarded to a young mathematician under 30 working in the same areas as Raphël Salem did. In my opinion this prize should be used to detect talented mathematicians who have not yet obtained international recognition. This was perhaps an extreme viewpoint and was questioned by the Salem family who insisted that the prize should be given to the most famous mathematician under 30. This conflict could not be solved and I resigned from the committee. Your illusions about the meaning of prizes fade when you work in a prize committee. During my experience in the Salem Prize committee, I was always convinced that the recipient deserved the prize, but it was often clear that other mathematicians would have deserved it as well. That remark applies to me and to the Gauss Prize.

As an outsider, why do you think the Fields Medal has not come to India so far?

India has an outstanding mathematical tradition and I would like to mention a few names. I already stressed my connection to Vijayaraghavan. From 1974 to 1984, I used to visit the University of Chicago every year and I liked discussing with Raghavan Narasimhan, who is now emeritus. As every mathematician, I am fascinated by Srinivasa Ramanujan (1887–1920). Ramanujan worked in analytic number theory. He was a genius and raised some important conjectures. A famous Ramanujan conjecture describes the growth of coefficients in the power series expansion of what is named the Ramanujan tau-function. This was answered by Pierre Deligne in 1974. Deligne won the Fields Medal for this spectacular achievement. I would like to mention the book *Souvenirs d'apprentissage* by André Weil. André Weil studied Sanskrit with much care before deciding to move to India and collaborate with Indian mathematicians. In *Souvenirs d'apprentissage*, he tells us how much he loved India. Later on, André Weil designed the program which was completed by Deligne and which led to the solution of the Ramanujan conjecture. This deep relation between Ramanujan, André Weil and India is divine.

Harish-Chandra (1923–83) was another outstanding Indian mathematician. He did fundamental work in representation theory, especially harmonic analysis on semi-simple Lie groups. The Harish-Chandra Research Institute in Allahabad is named in his honour.

Let me now answer your question about Fields medallists. There are so few Fields medals that you cannot judge the mathematical level of a country on the number of its Fields medallists. Statistics on such small numbers is wrong, as every statistician knows. For example, the Vietnamese mathematician Ngô Bảo Châu got the Fields Medal and it is obvious that India has a stronger mathematical tradition than Vietnam.

Message for the young

What is so intriguing about mathematics, knowing which would interest young students in mathematics?

Kids can be attracted to mathematics for two reasons. It is a magic field because you can do it by yourself, without the help of other people. It suffices to think. You are the king of your kingdom. But today mathematics is also magic because it is the key opening the digital world. Listening to music on your iPod is made possible because mathematicians designed compression algorithms. This is unique in the history of mankind.

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