Decoding coherent information in femtosecond shaped laser pulses

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We report here an experimental demonstration of a pulse decoding technique from spectral analysis of femtosecond pulses. This technique is based on a single-step Fourier domain inversion algorithm and shows the impact of the spectral window function on the retrieval of the signal pulse. Using two femtosecond laser pulses at 780 and 1560 nm from a fibre laser, we have shown that even when the spectral content of the reference pulse is far from a narrow band limit (as much as a quarter of its entire spectral content for our conditions), the single-step retrieval analysis using spectral windowing in the Fourier domain works efficiently. The enhanced signal levels possible due to the wider spectral window are critical in the unambiguous description of laser pulses, which would have potential application in the retrieval of comparatively weak and complex ultra-short pulses as is often needed in pulse shaping applications and coherent optical communications.

\textbf{Keywords:} Fourier domain, laser pulses, pulse decoding, spectral analysis.

Coherent optical communication, which is an integral part of most quantum information transfer and processing\textsuperscript{1}, involves encoding ultrashort laser pulses, transmitting and eventually retrieving their information content. Unfortunately, such coherent information encoding and retrieval are both technologically challenging, as they involve timescales that are faster than the response time of most electronics devices and instruments. Using Fourier transform to achieve spatial pulse shaping has resulted in the encoding of femtosecond laser pulses, which has made coherent optical encoding possible\textsuperscript{2,3}. However, decoding such coherent optically encoded pulses is also nontrivial. Much of the successful commercial techniques rely on iterative procedures\textsuperscript{4,5}, which could be quite sufficient for the pulse characterization process but not for information processing. Information processing channels require a single-step retrieval process such that an information train can be successfully decoded so as to be ready for the subsequent information train. In view of these requirements on coherent optical communication, which is a necessary and critical step towards quantum information transfer, we have looked at a specific decoding scheme that can reliably work in a single inversion process\textsuperscript{6-11}. This particular scheme is, in general, based on the spectral interference of a narrow window of the signal (in the theoretical limit of a delta function) with itself in the Fourier domain. The resultant interferogram when spectrally and temporally resolved provides complete information about the signal. Several variants of the approach have been attempted by us to test the robustness of this technique in its capability to retrieve the complete information content of the transmitted coherent information.

We have designed our experiments based at the C-band wavelength (1560 nm) of the communication channel given its advantages in fibre-optic transmission. Specifically, we use spectral windowing in the Fourier domain to decode the amplitude and phase from the analysis of the correlation spectra of two laser pulses, one at 1560 nm and the other at its second-harmonic wavelength of 780 nm. We gradually changed the width of the spectral window function to show that the inversion algorithm works efficiently even when the spectral content of the reference pulse is quite far from a narrow band limit (as much as a quarter of its entire spectral content for our experimental conditions). This technique is fast and provides good signal-to-noise ratio.

\textbf{Experiment}

Our experiments used a mode-locked femtosecond Er: doped fibre laser (Femtelite, IMRA Inc.), which provided two collinear weak femtosecond laser pulses, one at 1560 nm and the other at 780 nm, as a single beam with 50 MHz repetition-rate (http://www.imra.com). The 780 nm pulse is ~100 fs wide and has an average power of 20 mW, whereas the 1560 nm pulse was ~300 fs wide and has an average power of 40 mW. Such low power limit is desirable for the coherent pulsed communication as higher powers induce nonlinear processes which corrupt the communication channels. For our IMRA laser, though 780 nm has a relatively simple pulse shape, the 1560 nm pulse has a complex pulse shape with comb-like spectral features and comparatively a larger bandwidth (Figure 1). This gives us the advantage that we can directly
use it as an encoded pulse for our information retrieval experiments.

For decoding the information content in the pulses, we built a collinear Mach–Zehnder cross-correlator with the help of two dichroic beam splitters: one separates the two wavelengths, whereas the other recombines them. Each of the independent wavelength arms was transformed into the Fourier domain with the help of a grating and lens pair combination. This combination performs a forward Fourier transform, whereas the fold-mirror that retraces the path back through the grating–lens pair combination performs a back Fourier transform. Thus, the pulse would be unchanged as it goes through such geometry, often referred as the 4f configuration (f being the focal length of the corresponding lenses). Interestingly, the same geometry can be utilized for encoding the laser pulses through the Fourier domain pulse shaping. Experimentally (Figure 2), we achieved the 4f folded geometries using 600 grooves/mm gratings for the respective 780 and 1560 nm wavelengths (G1 and G2), two convex lenses (L1 and L2) of focal length 15 and 30 cm, and two modulators (which for decoding purposes were just spectral filters, such as a slit, and for simple phase encoding purposes, they can be glass plates of appropriate thickness) in front of the fold-mirrors M1 and M2. This particular geometry provides the capability to create and choose the reference and the signal pulses from both the wavelengths used.

In order to obtain the time-correlation information between the two arms of the interferometer, we used sum frequency generation (SFG) signal arising from the cross-correlation of 780 and 1560 nm using a type-I BBO (β-barium-borate) nonlinear crystal. As the SFG signal at 520 nm is in the visible region, it is an experimentally convenient feature of this cross-correlation procedure. To measure this cross-correlation signal in the time domain, we used a 500 MHz oscilloscope (LeCroy LT354M) interfaced to the computer with a National Instruments GPIB card. We performed the spectral analysis of the 520 nm SFG signal to decode the complete amplitude and phase information of the ultra-short pulses. To record 520 nm spectra, we used a high performance Peltier-cooled CCD-based spectrometer (SpectraPro-150 Monochromator, Acton Research Inc.; http://www.roperscientific.com). The designed experimental set-up enables retrieval of both the pulses (780 and 1560 nm), one with respect to the other, interchanging the reference and the signal pulse according to our requirement.
For the experimental slit width values in these experiments, we used an indirect measuring approach as it is difficult to measure them directly under the actual experimental conditions. In our approach, we first measured the beam diameter (Figure 3) of a Gaussian beam coming out from a commercial laser (Verdi, Coherent Inc.) using the knife-edge method\(^2\). Next, we took the same rectangular slit that we used in our actual experiments, and recorded the beam power coming out through the slit with a silicon photodiode (Thorlabs PD210) at various slit widths. Every time the slit width was changed, it was measured accurately (within \(\pm 0.02\) mm) using a vernier calipers with least count of 0.02 mm. In each case, the photodiode power measured was equivalent to the ratio of the area of the rectangle subtended by the slit to the area of the Gaussian beam passing through it (Figure 4). We fitted this ratio of area versus the slit width using a linear model. In our actual experiments, the area under each individual 520 nm cross-correlation spectra was available, which we matched with the previous linear curve and finally obtained the values of the slit widths for each of the experimental graphs (Figure 5).

**Results and discussion**

We now discuss how to retrieve (decode) the information content of the individual encoded pulse, one with respect to the other, using the Fourier domain single-step inversion algorithm. We will also show how the variation of the width of the spectrally windowed reference pulse affects the retrieval of the signal pulse. The experimental set-up enables us to interchange the reference and signal pulse according to our choice, which in turn provides an insight into understanding the effect of the spectral windowing of the respective reference pulses.

When the interferometer had identical wavelength in both the arms, we generated an autocorrelation signal. The autocorrelation trace of the 780 nm pulse and a cross-correlation between the 780 and the 1560 nm pulses (Figure 6) show that while the 780 nm pulse is close to Gaussian shape, the 1560 nm pulse deviates because it has comb-like spectral features. Using the pulse shaper, we left the 780 nm pulse unchanged (reference pulse) and phase-coded the 1560 nm pulse in such a way that it splits into a double pulse in the time-domain (encoded pulse)\(^3,14\).

Our experiment focused on retrieval of the signal pulse at different widths of the spectrally windowed reference pulse.

**Figure 3.** Measurement of beam diameter of the Gaussian beam coming out from the Verdi laser.

**Figure 4.** Area of the rectangle and of the Gaussian beam passing through the slit (in arbitrary units) as a function of slit width (mm).

**Figure 5.** Plot of model experimental value, which is the ratio of the area of the rectangular slit and the Gaussian beam passing through the rectangular slit for the corona laser, its linear fit and finally the actual normalized slit area (labelled as slit parameter used) as a function of slit width in our experiments.
pulse in two different experimental conditions: (i) 1560 nm pulse decoded using 780 nm pulse and (ii) 780 nm pulse decoded using 1560 nm pulse, when both the pulses travelled through the pulse shaper geometries. In both the cases, decoded information was the same in the absence of any spectral window on the respective reference arms. The difference arises only from the recorded delays on the signal arm in the opposite directions.

We retrieved the encoded 1560 nm signal using the single-step inversion algorithm. In this particular case, we used a spectral window (slit1, i.e. S1) in front of the mirror M1 on the Fourier plane of the lens L1 on the 780 nm arm to create spectrally windowed bright reference pulse. Though the doublet time feature of the 1560 nm signal pulse was observed for the narrowest possible spectra of the reference pulse (780 nm), the resolution of the time feature improved gradually as we slowly increased the width of the spectral window of the reference pulse (780 nm). The change in the 520 nm spectra resulting from the cross-correlation between 1560 nm as the signal pulse and 780 nm as the reference pulse for different widths of the spectral window is shown in Figure 7.

In contrast, we used phase-coded 1560 nm pulse as reference for the retrieval of the 780 nm pulse passing through the pulse shaper geometry. We achieved this with the spectral window (slit2, i.e. S2) in front of the mirror M2 on the Fourier plane of the lens L2 on the 1560 nm arm (Figure 2) to produce spectrally windowed 1560 nm reference pulse. We retrieved the two-channel spectral feature of the 780 nm signal pulse at the narrowest possible spectral width of the reference pulse (1560 nm), but the gradual widening of the spectral window on the reference pulse arm resulted in the doublet time feature in the 520 nm correlation spectra, which is due to the phase-code 1560 nm reference pulse. Different 520 nm spectra resulting from the different spectral widths of the 1560 nm reference pulse are shown in Figure 8.

In both the experimental conditions mentioned above, we observed two types of 520 nm spectral traces: the first type recorded at the narrowest possible limit (or slightly above the limit) of the spectrally windowed reference pulse, highlights only the features of the signal pulse, whereas the other type where the spectral window on the reference pulse is broad (or completely absent), highlights the features of the reference pulse as well as the signal pulse. We retrieved the signal pulse from all possible intermediate 520 nm spectral traces, to explore how the variation of the spectral window on the reference pulse affects the signals in the respective cases.

From the mathematical point of view, we express the electric field \( E_{wp} \) of the cross-correlation signal (520 nm) resulting from the cross-correlation between the spectrally filtered reference pulse and the signal pulse as:

\[
E_{wp}(\tau, \omega) \propto \int_{-\infty}^{\infty} A_s(\omega - \omega') e^{i\phi(\omega - \omega')}
\times A_r(\omega') e^{i\phi'\omega'} \text{rect}\left(\frac{\omega'}{W}\right) e^{-i\omega\tau} d\omega'.
\]

Here, \( A_s(\omega - \omega') e^{i\phi(\omega - \omega')} = E_s(\omega - \omega') \) is the signal pulse, \( A_r(\omega') e^{i\phi'\omega'} \text{rect}\left(\omega'/W\right) e^{-i\omega\tau} = E_r(\omega') \) is the spectrally filtered reference pulse. In the expression of the reference pulse, we considered \( \text{rect}(\omega/W) \) as the rectangular spectral filter (slit) function. In eq. (1), \( \phi' \) is the detuning, \( \tau \) the delay, \( \phi \) the dispersion and \( A_s \) and \( A_r \) are the pulse amplitudes.

Now the spectral window function \( \text{rect}(\omega/W) \) has value zero outside the range \([+W/2, -W/2]\) and hence the limit of the above integral can be changed to \([+W/2, -W/2]\). Within these limits, we can assume that

\[
A_s(\omega - \omega') = A_s(\omega), \quad A_r(\omega') = A_r(0), \quad \phi(\omega') = \phi(0).
\]

This assumption leads to constant product terms altogether in a parameter \( k \) like

\[
k = A_s(\omega)A_r(0)e^{i\phi(0)}.
\]
Now, if we assume that the variation of $\phi_i(\omega - \omega')$ in a Taylor series is limited to the first-order for narrow spectral filter (ideal delta condition), we can express it using a truncated Taylor series as

$$\phi_i(\omega - \omega') = \phi_i(\omega) - \left( \frac{d\phi_i}{d\omega} \right)_{\omega'} \omega'.$$

thus now we can write the up-converted field as

$$E_{\text{up}}(\tau, \omega) \propto k \int_{-(W/2)}^{+(W/2)} e^{i\omega\tau} \left[ \phi_i(\omega) - \left( \frac{d\phi_i}{d\omega} \right)_{\omega'} \omega' \right] d\omega'.$$

As the spectrometer detects the intensity of the spectrum, the profile detected by the spectrometer is given as

$$|E_{\text{up}}(\tau, \omega)|^2 \propto |k|^2 \left| \phi_i(\omega) \right|^2 \int_{-(W/2)}^{+(W/2)} |e^{i\omega\tau} \left( \frac{d\phi_i}{d\omega} \right)_{\omega'}|^2 d\omega'.$$
The modulus of the exponential outside the integral part is 1 and hence the up-converted field is given as

$$|E_{up}(\tau, \omega)|^2 \propto k^2 \left| \int_{-W/2}^{W/2} e^{j\omega \tau - \frac{\partial \phi}{\partial \omega} \tau} d\omega \right|^2. \quad (5)$$

Now the condition for the up-converted field to be maximum for a particular frequency $\omega$ is $(\partial \phi / \partial \omega)_{\omega = \omega_0} = \tau$ and we write the expression for the group delay as $\tau_{\phi} = (\partial \phi / \partial \omega)_{\omega = \omega_0}$.

Thus in the detected spectral trace, the location of the maxima along the delay (time) axis gives the value of the group delay and given the group delays of all the frequencies, the phase for a particular frequency $\omega$ is found using $\phi_{\omega}(\omega) = \int_{-W/2}^{W/2} \tau_{\phi} d\omega$. Thus we retrieve the phase of the signal pulse from the spectrally measured cross-correlation data.

Now eq. (5) derived for the up-converted field detected by the spectrometer is the power spectral density (PSD) in the $\tau - (d\phi/d\omega)$ domain, where PSD at $\omega$ is given by

$$P(\omega, \tau - (d\phi/d\omega)) = \left| \int_{-W/2}^{W/2} e^{j\omega \tau - \frac{\partial \phi}{\partial \omega} \tau} d\omega \right|^2. \quad (6)$$

So integration over $\tau$ provides the energy of the component of the signal at frequency $\omega$ within the integration limits. Since according to the above assumption, the amplitude of the signal does not vary much and is considered to be constant for all values of $\omega$, integration over $\tau$ results in

$$\int \left| E_{up}(\tau, \omega) \right|^2 d\tau \propto k^2 \int P(\omega, \tau - (d\phi/d\omega)) d\tau. \quad (7)$$
The term $\int_{-\infty}^{\infty} P(\omega, \tau) \exp\{i(\omega - (d\phi/d\omega)_{\omega})\} d\tau$ is a constant and independent of $\omega$ as explained above. We denote it by another constant $\kappa$ and hence we get

$$\int_{-\infty}^{\infty} \left| E_{\omega}^{\phi}(\tau, \omega) \right|^2 d\tau \propto |k|^2 \kappa,$$

which means that

$$\int_{-\infty}^{\infty} \left| E_{\omega}^{\phi}(\tau, \omega) \right|^2 d\tau \propto |k|^2.$$

Now recalling the fact that $k$ is proportional to $A_\kappa(\omega)$, we get

$$\int_{-\infty}^{\infty} \left| E_{\omega}^{\phi}(\tau, \omega) \right|^2 d\tau \propto \left| A_\kappa(\omega) \right|^2.$$

Hence by taking the square root of the integral we get the field of the signal pulse at frequency $\omega$. Thus by evaluating the field for all the values of $\omega$, we get the entire amplitude profile of the signal pulse.

However, if we use a wider spectral window, the contribution of the higher-order terms of the Taylor series of the phase becomes significant. Under this situation, the phase is correctly represented as:

$$\phi(\omega - \omega') = \phi(\omega) - \left( \frac{d\phi}{d\omega} \right)_{\omega} (\omega') + \frac{1}{2!} \left( \frac{d^2\phi}{d\omega^2} \right)_{\omega} (\omega')^2$$

$$- \cdots + \frac{(-1)^n}{n!} \left( \frac{d^n\phi}{d\omega^n} \right)_{\omega} (\omega')^n.$$

The inversion algorithm that we use here gives the delay considering only one term of the Taylor series. So for a wider spectral window, the higher-order terms of the Taylor series of the phase function, which are missing in eq. (3), result in error in the calculation. Hence as we gradually widen the spectral window, we expect an error in the retrieved group delay for higher detuning values. This error directly relates to the higher-order terms of the Taylor series, which make a considerable contribution for higher detuning terms. Here it may be important to point out that it might be possible to use an iterative algorithm with an error minimization technique to solve for the missing terms of the Taylor series, leading to a faster and more definite convergence.

Under the experimental condition, we managed to decode the pulses completely and accurately when the narrowest possible spectrally windowed reference pulse was used. With a gradual widening of the spectral window on the chosen reference arm (Figures 9 and 10), we can summarize our observations as follows:

(i) The narrowest possible spectral window (width of the window = 0.6 mm in both cases) allows only 5% of the total spectra of the respective reference arm. This corresponds to spectral widths of 1 and 2.3 nm in case of the 780 and 1560 nm pulses respectively.

(ii) When 25% of total spectra of the respective reference pulse was allowed to pass through the spectral window (width of the window = 3.2 mm in both the cases), we observed a noticeable change in the 520 nm spectral trace. This corresponds to the spectral widths of 5 and 12.3 nm in case of the 780 and 1560 nm pulses respectively.

(iii) For a smaller spectral window (less than 3.2 mm), the retrieved normalized spectral intensity in both cases was the same and deviated only when the width of the spectral window was 3.2 mm or more. The retrieved group delays and phases in both situations also followed the same trend. The results are shown in Figures 9 and 10. When we used phase-coded 1560 nm as the reference pulse.
pulses, the features (Figure 10) were comparatively noisier due to the complex nature of the 1560 nm pulse. In this case, it is difficult to generate a fine spectral component as the reference, but the trends in the results essentially reflect our observations in the other case.

For both the signal-decoding schemes, we observed that there was a small error in group delay and phase at the higher detuning values, as expected from our mathematical interpretation. But as the spectral amplitude is less for these higher detuning terms, the effect of the error on the retrieved pulse shapes obtained from the inverse Fourier transform can be negligible. On the other hand, when the spectral window is more than 3.2 mm (in each case), the error on the retrieved pulse shape will be more. Here the spectral profile becomes wide, which results in significant errors in the calculation of the group delay and phase even for lower detuning values. This establishes the fact that before the point where the spectral window is 3.2 mm, the inversion algorithm can predict the pulse characteristics almost accurately with smaller of the spectral window though the width is not in its narrowest possible limit.

This spectral domain Fourier retrieval as demonstrated here has potential applications in cases where: (i) better signal-to-noise ratio requirement is critical, such as, for a relatively weak or complex pulse retrieval process; (ii) iterative procedure cannot be used, and (iii) sign of the frequency modulation is required. Pulse shaping and/or coherent optical communication schemes thus effectively benefit from this technique. Our spectral windowing study shows that an almost exact retrieval of the unknown pulse information is possible from the single-step Fourier domain retrieval inversion algorithm, even when our reference pulse spectral window deviates significantly from the narrow band limit. Such robustness in the optical decoding scheme, therefore, leads to coherent information processing for weak and complex shaped pulses with sufficient signal-to-noise ratio avoiding complex experimental constraints.

Conclusions

We have demonstrated a specific decoding scheme, which retrieves coherent information from the laser pulses using a single-step Fourier domain inversion algorithm. We used 780 and 1560 nm pulses to build a collinear Mach–Zehnder cross-correlator, which retrieves the individual pulses, one with respect to the other, from the time and spectral domain measurements. The intensity cross-correlation (SFG) traces in addition to the intensity autocorrelation (SHG) of 780 nm provide time information, whereas spectral analysis of the SFG signals retrieves individual pulse shapes. Our experiment shows how the correlation spectral traces change with variation in the width of the spectral window on the reference pulse. This kind of Fourier domain signal retrieval study from a single-step inversion algorithm facilitates the complete retrieval of ultra-short pulses, which are specifically weak and complex shaped, by circumventing experimental ‘low signal’ constraints. In fact, we manage to show that in the experimental conditions we use, it is possible to retrieve pulse shapes from our Fourier domain single-step inversion approach as long as no more than 25% of the total spectral content of the reference pulse is used to measure the correlation spectra.

RESEARCH ARTICLES


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