

Divine music

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There is a general perception among the educators that our high school curriculum in science and mathematics is wanting; that is probably the reason why our students do not wish to pursue higher education in these streams. My submission is that we teach too much at high school. The students barely have any time to reflect on what they are learning. Besides, the curriculum stresses on the wrong end of the subjects. The stress is on packaged results rather than the conceptual foundations. I currently teach research students aspiring to become independent researchers carrying out their master's and doctoral work. I find myself teaching them high school mathematics in order to remedy their inadequacies. The inadequacies I notice, are not in their exposure to the topics. In almost every topic, their understanding and working competence is woefully inadequate.

In my high school days (1950s and 60s), there were two streams of mathematics. One was called general mathematics covering arithmetic and mensuration. The subject matter was limited to numbers, ratios and proportions, simple and compound interest calculations, and perimeter/area/volume calculations of regular curves/shapes/bodies. The other stream in mathematics was called composite mathematics. Students aspiring to go on to higher education in physical sciences used to opt for this stream. The subject matter was limited to arithmetic, algebra and plane geometry (theory and construction). The contact hours per week used to be respectively two and three hours. The total number of class hours in 11th and 12th class (in our times the 12th class was called pre-university and held in science/arts colleges) put together was probably about 200–300 h. The students had adequate time to absorb what was being taught; the teacher made the students practice what they understood in regular class hours. The practice of homework was not prevalent. Tuition and coaching classes were unheard of. There were quarterly, half-yearly and final examinations. Underperforming students repeated a year to reach satisfactory level of performance.

The current syllabus of 11th and 12th standard mathematics is given in Table 1. (<http://www.tn.gov.in/schoolsyllabus/>).

The syllabi for several of the national level entrance tests such as All India Engineering Entrance Examination (AIEEE) also reflect the same curriculum. I therefore, expect the mathematics curriculum in all the state boards to be the same. The present high school mathematics curriculum given in Table 1 saddens me.

Matrix algebra is introduced in isolation at the 11th standard syllabus. The syllabus elaborates on the topic as follows: 'Defining a matrix and identifying various types of matrices; computing order of a matrix; performing addition, scalar multiplication and finding the product of matrices'. Matrix is introduced as a stand alone animal. The student is not exposed to the fact that a matrix is only a notation. Learning matrices in isolation in school ahead of the geometrical ideas is counterproductive.

At school, I remember spending a lot of time in class drawing single lines, parallel lines, intersecting lines, measuring slopes, dividing lines into different proportions through simple construction techniques, drawing normals to lines, drawing equilateral, isosceles and right angle triangles, finding out the trigonometric ratios, measuring the angles, verifying the sum of angles in a triangle, finding out the centroid of a triangle by construction, drawing circles, diameters,

chords, tangents, concentric circles, intersecting circles, touching circles, non-touching circles and so on as part of the plane geometry lessons. I believe that these exercises to a great measure added to my skills of visualization. I did not find significant mention of any of these materials in the current syllabus.

In the current 12th standard curriculum, simultaneous equations are introduced. The stress is more on matrices and related ideas. The syllabus lists matrices, determinants, Cramer's rule and so on. There is not enough stress on the formulation of such equations, situations where such formulations are common, simple and long method of solving such equations, verifying if the answers are right or not, and so on. There is no mention of solving simultaneous equations of two variables by graphical method at all.

I remember that in our times, we were not exposed to vector algebra, complex variables, matrices, etc. till we were into the first or second year of the degree curriculum. Probability distributions, etc. had to wait much longer. By then a suitable foundation in engineering problems and their general structure was laid down so that a number of these mathematical formulations (matrix for example) and generalized tools and tackle (Cramer's rule, etc.) could be well appreciated in the context of typical applications. I classify complex variables also in the same category of tools and tackle convenient

Table 1. Syllabus of 11th and 12th standard mathematics

Subject	Number of periods	
	11th standard	12th standard
Number theory	20	
Binomial theorem, etc.	20	
Trigonometry	25	
Analytical geometry	15	30
		System of linear equations
		15
Matrix and determinants	15	15
		Vector algebra
		30
Functions and graphs	25	15
Differential calculus	30	40
Integration	20	15
Statistics	15	20
		Algebraic structures
		15
Total periods	185	195

to solve certain specialized problems (such as ac circuit theory). Therefore, learning about the tools ahead of the processes needing these tools is putting the cart before the horse.

The idea of straight lines being a family of linear functions of two variables can be very profitably exploited. The stress may be on constructivist method of learning; the student may soak in every advanced concept from the simple xy -plane and representation of lines in xy -plane. It is quite satisfying for the student to fully understand and get working competence in linear equations of two variables; the ideas of no solution, unique solution, infinite solutions can all be graphically seen and understood. With such a firm grasp of the basic concepts, when they start on the higher order equations, and matrix representations, the students will be able to quickly make the generalizations; higher dimensional mathematics (linear equations and matrices) can even wait till they are well into their degree programme.

The same arguments hold good for the topics of vector algebra and complex numbers as well. A mastery of the xy -plane and directed lines in the xy -plane, their addition, subtraction, multiplication are all less intimidating through the constructivist approach in the xy -plane. Applications such as projections, shadows, areas can all be graphically handled through plane geometry construction and measurement first; appropriate analytical follow-up can come next. Exhaustive ground work on the two-dimensional (plane) geometry with constructional approach will certainly pay rich dividends.

I remember that I learnt calculus only during my engineering degree education. Differential and integral calculus were introduced in the first and second year respectively. Solution to differential equations was covered in the third year concurrently with alternating circuits and

electrical machines. The role of calculus in the training of scientists and engineers is absolutely crucial. It is necessary to appreciate, understand and master all the cause-effect linkages to reach the leading edge of research. Every cause-effect relationship of the physical processes in nature is incremental and continuous. A full understanding of the accumulated past (integration) and the evolutionary future (rate or incremental change or derivatives) demands mastery of calculus. My submission is that good training in plane geometry through constructivist learning of slopes of functions and areas under functions will lead to natural mastery of calculus. Construction exercises are very helpful in enabling the students to develop their visualization and conceptualization skills. Consequently, learning differentiation and integration from the idea of slopes and areas is of great significance to the intuitive understanding of the processes. Additional time spent in mastering plane geometry at high school level will never be regretted by any engineer or scientist.

The last section in the current syllabus – algebraic structures – is an abstraction which has been done over several decades/centuries by learned mathematicians. I wonder if at the high school level, the student can understand and appreciate the same. These sections can certainly wait till the students get into graduate school.

In summary, I believe that the current curriculum is not giving enough depth to the conceptual base of the student; a lot of time is spent on generalized and abstracted ideas which may not be well appreciated till much later in the training of the student. I submit that the high school curriculum has to put in a very strong foundation of basic concepts. Such a strong foundation can be laid with a good mix of arithmetic, algebra, trigonometry and plane geometry compo-

nents; the stress has to be on constructivist method in each of these components as opposed to rote learning. The analytical aspect of geometry has to follow the constructional aspects appropriately. My further submission is that matrices, vector algebra, complex numbers, calculus and such topics may be pushed out to degree curriculum.

Such an approach will save the students from becoming memory machines. The school children will taste the joy of learning with understanding. We may recall the words of Leonardo Da Vinci, 'The noblest of pleasures is the joy of understanding'. Learning will become a positive joyful experience. The teachers will be rewarded with the pleasure of seeing the children learn and enjoy the experience. Abnormally long school hours and private tuition will disappear. The emotional health of the children will be restored. Parents will see both emotional and economic benefits. The students will look forward to higher education with eagerness and enthusiasm. Institutions of higher learning will have to spend less time on remedial courses. They can get back seriously into the business of advanced learning and research.

I suggest Table 2 as an alternative to the current curriculum. It may be seen that the total number of periods can be reduced substantially. The students may be given a firm grounding in the fundamental concepts. This will enable them to learn/appreciate/master the application tools and tackle readily at the university. A good base in plane geometry will make their later entry to calculus a simple, satisfying and enjoyable encounter.

This could be a curriculum which takes the student at an unhurried pace through the basic ideas. There will be adequate time to drill the student into total competence of whatever is being taught; internalization of the subject matter will become a habit. In closing, let me take strength from the (reported) sign at Plato's Academy in Athens which read 'Let no man ignorant of geometry enter' and the observation by Pythagoras 'There is geometry in the humming of the strings and music in the spacing of the spheres'. I might paraphrase the last observation as 'Plain [sic] geometry is divine music'.

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Table 2. Proposed curriculum

Subject	Number of periods	
	11th standard	12th standard
Number theory	40	
Algebra	Algebra	40
Trigonometry	Functions and graphs	40
Plane geometry	Plane geometry	40
	Analytical geometry	40
Total	160	160