The most influential speech ever presented in the history of mathematics

Rosalind Ezhil K.

In the year 1897, the very first 'International Congress of Mathematicians' (ICM) was convened in the city of Zurich. It was a time when Europe was at the forefront of mathematical study and the Congress was organized by leading French and German mathematicians of the day. It was decided at that meeting that the Congress would be a regular event in the coming years with one of the primary purposes being 'to promote personal relations between mathematicians of different lands. . .' Since then, except for the period corresponding to the two World Wars, the Congress has been a rather regular event. The next Congress is due to happen in 2010 (ICM 2010) and for the first time in its history, the Congress will take place in India.

In this article, we will look at a particularly significant event that took place in the second Congress ever to be held. This Congress was held at Paris in 1900. At this meeting, leading German mathematician of the day – David Hilbert, was invited to present a talk. And on the morning of 8 August 1900, during the section of the Congress on bibliography and history, David Hilbert presented his talk entitled, 'The future problems of mathematics'. He outlined some 23 problems in different fields of mathematics that he believed were a sample of what future mathematicians ought to be working at. (He had time during the lecture to present only ten problems – the rest were part of the published proceedings.)

It would be a daunting task to explain these problems even to a sophisticated mathematical audience – we will therefore not even attempt it here! However, to give a flavour of the talk, we present the list of the 23 problems here.

1. Cantor's problem of the cardinal number of the continuum.
2. The compatibility of the arithmetical axioms.
3. The equality of the volumes of two tetrahedra of equal bases and equal altitudes.
4. Problem of the straight line as the shortest distance between two points.
5. Lie's concept of a continuous group of transformations without the assumption of the differentiability of the functions defining the group.
6. Mathematical treatment of the axioms of physics.
7. Irrationality and transcendence of certain numbers.
8. Problems of prime numbers.
9. Proof of the most general law of reciprocity in any number field.
10. Determination of the solvability of a Diophantine equation.
11. Quadratic forms with any algebraic numerical coefficients.
12. Extension of Kronecker's theorem on abelian fields to any algebraic realm or rationality.
13. Impossiblity of the solution of the general equation of the 7th degree by means of functions of only two arguments.
14. Proof of the finiteness of certain complete systems of functions.
15. Rigorous foundation of Schubert's enumerative calculus.
16. Problem of the topology of algebraic curves and surfaces.
17. Expression of definite forms by squares.
18. Building up of space from congruent polyhedra.
19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?
20. The general problem of boundary values.
21. Proof of the existence of linear differential equations having a prescribed monodromic group.
22. Uniformization of analytic relations by means of automorphic functions.
23. Further development of the methods of the calculus of variations.

Did this talk have an instant impact? Apparently, British mathematician Charlotte Angas Scott who prepared a report on the proceedings for the *Bulletin of the American Mathematical Society* has recorded only 'a rather desultory discussion'[2] after the talk. And Hilbert himself was not very pleased with the conference – he wrote to his friend Adolf Hurwitz that 'the visit was not very strong in either the quantitative or in the qualitative regard'[2]. However, the importance of his talk was recognized rather quickly afterwards. The prediction of his close friend and fellow mathematician Hermann Minkowski 'Most alluring would be the attempt to look into the future, in other words, a characterization of the problems to which the mathematicians should turn in the future. With this, you might conceivably have people talking about your speech even decades from now'[3] was to come true many times over.

In fact, the impact of his talk on the course of mathematical research in the 20th century was extraordinary. Certainly none of the other ICMs that took place after the 1900 can boast of an event with consequences of such magnitude. Solving one of 'Hilbert's problems' is a matter of great prestige and has been the preoccupation and inspiration of a great number of mathematical minds. A large number of articles, papers, books have been written on the subject.

What is the status of the problems today? Some of the problems that Hilbert presented were solved fairly quickly; some were decided not to be problems at all whereas some continue to remain open to this day. In contrast to mathematicians of today, Hilbert was a 'universal' mathematician with a profound understanding of many unrelated areas of mathematicians. As Ben Yandell[4] says in his book (*The Honors Class: Hilbert's Problems and their Solvers*): 'Mathematics has come a long way. . . , expanding and fragmenting to the point that it is inconceivable that one person could survey the whole and make a similar list of problems. In the worst case such a list would be the work of a series of committees with no common members. The International Mathematical Union did better than this in 2000, when it published *Mathematics: Frontiers and Perspectives* ("It is inspired by the famous list of problems that Hilbert proposed") but still credited four editors, had thirty signed articles, and was 459 pages long'.

Born in 1864 in the town of Konisberg (then a part of the German Kingdom of

Prussia, now a part of Russia), Hilbert showed very early on, his extraordinary flair and passion for mathematics. At the time of his entry into the world of mathematics, the tenet of French philosopher Du Bois-Reymond that some scientific problems were unsolvable was much in vogue. However, this was something Hilbert completely rejected and fought against all his life.

At the age of 38 when he presented his bold talk, Hilbert ended his famous lecture thus 'There are absolutely no unsolvable problems. Instead of the foolish ignorabimus, our answer is on the contrary: We must know, we shall know'. A 'pure' mathematician at that stage in his life, Hilbert was not very interested in the applications of mathematics. His talk is considered to be a response to mathematician Poincaré's speech on the importance of applied mathematics at the previous ICM in Zurich. In fact there is some criticism of his talk as having '. . .surely swung too much the other way'[2] from the viewpoint Poincaré put forward.

We end with a tribute to David Hilbert in the words of his student Hermann Weyl, also a great mathematician, 'A great master of mathematics passed away when David Hilbert died in Göttingen on February the 14th, 1943, at the age of eighty-one. In retrospect it seems to us that the era of mathematics upon which he impressed the seal of his spirit and which is now sinking below the horizon achieved a more perfect balance than prevailed before and after, between the mastering of single concrete problems and the formation of general abstract concepts. Hilbert's own work contributed not a little to bringing about this happy equilibrium, and the direction in which we have since proceeded can in many instances be traced back to his impulses. No mathematician of equal stature has risen from our generation. . .'.

His Paris address on 'Mathematical problems' quoted above straddles all fields of our science. Trying to unveil what the future would hold in store for us, he posed and discussed 23 unsolved problems which have indeed, as we can now state in retrospect, played an important role during the following 40 odd years. A mathematician who had solved one of them thereby passed on to the honours class of the mathematical community[6].

1. Wikipedia.
2. Ivor Grattan-Guinness, *Notices of the Am. Math. Soc.*, 2000, **47**, 752.
3. Jeremy, G., *Newsletter 36 of the European Mathematical Society*, 2000, pp. 10–13.
4. Benjamin H. Yandell, *The Honors Class: Hilbert's Problems and Their Solvers*.
5. Sridharan, R., *Resonance*, 1999, **4**, 96.
6. Herman Weyl, *Bull. Am. Math. Soc.*, 1944, **50**, 612–654.

Rosalind Ezhil K. lives at 122, 3rd Main, Dollars Colony, RMV 2nd Stage, 1st Block, Bangalore 560 094, India.
e-mail: rosalindezhil@gmail.com

Edited and published by P. Balaram, Current Science Association, Bangalore 560 080.
Typeset by WINTECS Typesetters (Ph: 2332 7311), Bangalore 560 021 and Printed at Printek Printers, Bangalore (Ph: 2328 7763)