

Learning about the Universe from Cosmic Microwave Background Radiation

T. R. Seshadri

Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India

The Cosmic Microwave Background Radiation (CMBR) is a nearly isotropic radiation that carries information about the state of the Universe in its early phase and about different physical processes that occur at different times during the course of its evolution. The high degree of isotropy is a compelling evidence of the inflationary model of the Universe. The nature of specific features on the CMBR like the minute level of temperature anisotropy and polarization can narrow down the parameters which govern the evolution of the Universe. In these features the information about the initial conditions, evolution, geometry as well as the material content of the Universe is encoded. I have given in this review an overview of the physics involved in CMB studies and what it tells us about the Universe. I have also discussed the observations from 5-year data of WMAP and their implications.

Keywords: Cosmic microwave background radiation, isotropy, polarization, Universe.

Introduction

THE Cosmic Microwave Background Radiation (CMBR) has by now become an important source of information about the nature, evolution and the constituents of the Universe. Its prediction and its subsequent discovery has provided compelling evidence that our Universe is described by the Big Bang model. It gives us direct evidence that our Universe was nearly homogeneous and isotropic when it was about 300,000 years old. It also gives us a lot of indirect information about the physics of early Universe and about the dominant processes operating at that time. This has helped in building a model of the Universe and its evolution. The physical processes in more recent epochs also leave their imprints in the form of characteristic signatures on the CMB. Once we have a reasonably well-motivated and reliable model of the early Universe, we can predict with reasonable confidence the features which should be present in the CMB. These features can get modified due to more recent processes. By observing and analysing the features of the CMB, and hence knowing the deviations from these models of the early Uni-

verse, we get a handle to understand the nature of these physical processes in more recent times, which could have induced these deviations. In other words, CMB is an encyclopedia (*albeit* an encoded one) containing information about the nature and evolution of our Universe and the constituents it is made up of. The physics of CMB today essentially deals with decoding of the information to gain a more detailed understanding of our Universe.

In the standard Big Bang model, it was predicted that we should be immersed in a radiation bath. The temperature of the radiation bath depends on the model we choose. The prediction and observations about the background radiation have a long history and various researchers have contributed to this in both theoretical as well as observational fronts¹. The existence of such a background radiation and that its intensity is the same from all directions was conclusively shown by Penzias and Wilson². The intensity of this radiation from all directions is the same. This observation was made in 1965 for which they got the Nobel Prize in 1978. They made a measurement of this radiation at a wavelength of 7.3 cm. If the radiation spectrum is assumed to be of blackbody, then the observed intensity at the above wavelength corresponds to a Planck spectrum of temperature 3.5 ± 1.0 K. However, this interpretation was under the assumption that the radiation is Planckian, which at that point of time was only a theoretically motivated speculation. However, an important information which one got from this observation was that this radiation is isotropic. In other words the intensity of this radiation received from all directions is the same. As we will see later, this is a compelling evidence of the homogeneity of the Universe.

We see today that there are structures on various scales. The Universe is not completely homogenous over all scales. It is believed that the structures we see today started out as small inhomogeneities in the distribution of matter in the early Universe. The gravitational collapse of these small-density perturbations led to their growth³. These structures lead to inhomogeneities which manifest in the form of anisotropy in the CMBR today. There can be ripples in the geometry, called gravitational waves, which is a prediction of Einstein's theory that can leave imprints on the CMBR. Effects on the CMBR can also be due to magnetic fields on cosmological scales and the presence of free charges during different stages of evolution of the Universe.

e-mail: seshadri.tr@physics.du.ac.in

There are broadly three different features of CMBR.

1. Angular dependence: Intensity of radiation is almost isotropic, but can have small dependence in direction.
2. Spectrum: The radiation is almost Planckian, but can have slight deviations from it.
3. Polarization: Degree of polarization can be slightly different in different directions.

The processes discussed in the previous paragraph leave their imprint in the above characteristics to varying degrees.

The primary feature about the CMBR is that it is isotropic to a great extent. The angular variation⁴ is one part in about 10^5 – 10^6 . Apart from more intricate features, even the fact that CMBR is almost isotropic, strongly indicates that the Universe underwent a brief period of rapid expansion, known as inflation. We do not have any other viable model that can explain the isotropy.

Beyond the basic overall isotropy, finer details of the anisotropy also have information about the Universe encoded in them. This anisotropy is in the form of peaks and troughs in the angular power spectrum over different angular scales. The position of these peaks and their heights (Figure 1) tells us a detailed story of the history of our Universe and about dominant processes at different epochs. It also tells us about the material contents of the Universe and their relative proportions. By analysing these features, we can get an idea about the evolution of the geometry of the Universe. Similarly, the ionization history of the Universe, nature and strength of cosmological magnetic fields and gravitational waves have implications for the polarization of the CMBR. Compton scattering from free electrons can lead to deviations from Planck spectrum. The topic of microwave background is discussed in a number of reviews^{5,6} and textbooks^{3,7–9}.

In this article, I first describe the overall nature of the gross features of the microwave background and the infor-

mation it gives about the Universe. I then discuss two specific features, namely that of temperature and polarization anisotropy. I will discuss how these originate and how we can characterize them. I will end the article with the implications of the recent WMAP 5-year data.

The isotropic background

Thermal history in a homogeneous an isotropic background

The origin of CMBR is intricately related to the thermal and ionization history of the Universe. It has to do with the fact that the Universe was very hot in the early phase and subsequently cooled with time as it expanded. By the expansion of the Universe we mean that the distance between any two points (which are at rest with respect to their local environment) increases with time. Since this aspect is central not just in the context of the CMBR but any issue related to cosmology, we will discuss this. The expansion of the Universe is quantified by a parameter called the scale factor, which is a function of time. Since the Universe is assumed to be homogeneous and isotropic, this parameter can only depend on time and not on spatial coordinates.

In a flat static (Minkowski) spacetime, the spacetime line-element is given by,

$$ds^2 = dt^2 - (dx^2 + dy^2 + dz^2). \quad (1)$$

The spatial distance is defined as the value of $\sqrt{-ds^2}$ at a constant time. Similarly, time interval is defined as $\sqrt{ds^2}$ at constant spatial coordinates.

For an expanding Universe one generalization of this line-element turns out to be,

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2], \quad (2)$$

which in spherical polar coordinates takes the form,

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2]. \quad (3)$$

The time-dependent function, $a(t)$, which multiplies the spatial intervals is called the scale factor. The expansion of the Universe would mean that $a(t)$ is an increasing function of time. It turns out that there are other possibilities too for an expanding, homogenous and isotropic Universe. In general, the line-element is given by,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - k/r^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \right]. \quad (4)$$

The constant $k = \pm 1, 0$. It turns out to be more useful to define a new time coordinate,

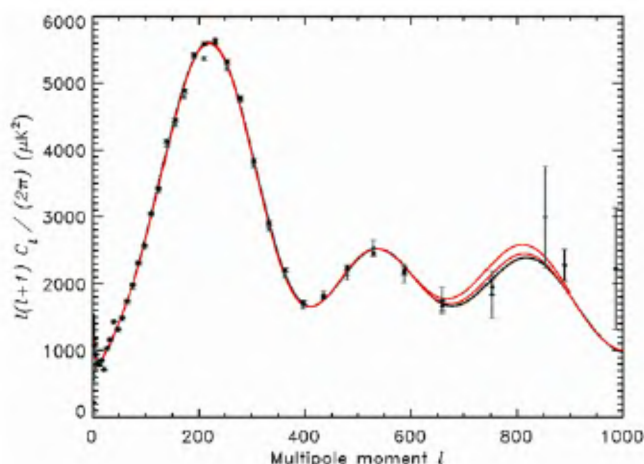


Figure 1. Temperature anisotropy power spectrum for WMAP5 (after Komatsu *et al.*²⁰).

$$d\eta = dt/a(t), \quad (5)$$

called the conformal time. The line-element in terms of these coordinates is given by,

$$ds^2 = a^2(t) \left[d\eta^2 - \frac{dr^2}{1-k/r^2} - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2 \right]. \quad (6)$$

For photons we have $ds^2 = 0$ and for any material particle with non-zero mass, $ds^2 > 0$. Clearly, for a photon that travels radially (so that without loss of generality we may take $\theta = 0, \phi = 0$), we have $d\eta = \pm dr/\sqrt{1-k/r^2}$. The evolution of the scale factor depends on the nature of the constituents of the Universe. This in turn dictates the time evolution of density and pressure of these constituents. The volume element varies with time as a^3 . If ρ and P are the density and pressure, the energy conservation equation becomes,

$$\frac{d(\rho a^3)}{dt} + P \frac{d(a^3)}{dt} = 0. \quad (7)$$

Further, if the equation of state is given by,

$$P = w\rho, \quad (8)$$

we conclude from the above equations that the density varies with the scale factor as,

$$\rho \propto a^{-3(1+w)}. \quad (9)$$

For radiation, the equation of state parameter, $w = 1/3$. With this value of w , we see from eq. (9) that for radiation, $\rho \propto a^{-4}$. This behaviour may be understood in a simple way. The number density of photons drops as the inverse of volume and hence as a^{-3} . Further, the energy (and hence the frequency) of every photon drops as a^{-1} . Both these effects combine to give the above variation of energy density with scale factor. The evolution of the scale factor at any point of time will primarily be governed by the constituent whose energy density dominates the Universe at that time. For most kinds of matter the scale factor evolves with time t as a power-law, $a \propto t^n$ with $0 \leq n < 1$. Hence, we conclude that the energy density of radiation is more in the past, and earlier the epoch we consider, higher is the density.

The CMBR which we observe today is believed to be this relic radiation of the early Universe. The spectrum of the CMBR is a blackbody spectrum. The energy density of the blackbody spectrum is proportional to the fourth power of temperature, $\rho \propto T^4$. Together with the fact that $\rho \propto a^{-4}$, we find that the temperature of radiation is inversely proportional to the scale factor a . Since $a \propto t^n$, we see that the temperature of radiation in the past was higher. Further, the earlier the epoch we consider, higher

is the temperature. Hence, it is expected that at a sufficiently early era, the temperature of the Universe was high enough to ionize the atoms and maintain them in the ionized state.

Origin of CMB and the homogeneity of the Universe

The nature of the CMBR is closely related to the ionization history of the Universe, which in turn is related to the temperature of radiation in the past. The 'normal' matter in the Universe consisted of about 75% hydrogen and 25% helium. As we saw in the previous section, the temperature of radiation is expected to have been more in the past. The earlier the epoch, higher was the temperature. At some epoch in the past the temperature of the radiation must have been high enough to ionize the matter. The temperature which is just sufficient to sustain matter in the ionized form depends on the ionization energy of matter. For hydrogen, for instance, the ionization energy is 13.6 eV. From the time-temperature relation of the radiation, this fixes an epoch when the radiation temperature is sufficient to ionize the matter. The epoch when this happened is called the epoch of recombination. We denote this by t_r and the corresponding scale factor by a_r . The temperature when this happens is about 3000 K. Prior to this epoch, the temperature would have been even higher, so that the matter could be sustained in the ionized form.

The degree of ionization at a given temperature is given by the Saha ionization formula¹⁰,

$$\frac{X_e^2}{1-X_e} = \frac{1}{n_e + n_H} \left[\left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-(m_e + m_p - m_H)/T} \right]. \quad (10)$$

Here, X_e is the free electron fraction and is equal to,

$$X_e = \frac{n_e}{n_e + n_H}, \quad (11)$$

where n_e and n_H are the number densities of free electrons and hydrogen atoms respectively. m_e, m_p and m_H are the masses of electron, proton and hydrogen atom respectively. When the temperature is very high, the degree of ionization X_e is almost unity. With cosmological expansion, when the temperature drops to a particular point called the ionization temperature, the degree of ionization drops rapidly to zero. It is important to note that this transition is fairly well defined, although it is not instantaneous. Since the temperature drops with time due to cosmological expansion, the matter in the Universe makes a transition from plasma state to a state of neutral atoms.

Since photons couple strongly to charged particles, during the plasma phase the photons undergo significant

scattering. Thus before the epoch of recombination, the photons undergo a random walk due to scattering from charged particles. Thus any direct information they carry from the early Universe gets washed out. However, after the epoch of recombination, the electrons and nuclei combine to form neutral atoms. Hence, the scattering of photons becomes negligible. Thus after this epoch, most of the photons can travel unhindered in a straight line. In order to capture the basic essence of the process, the situation in $1 + 2$ dimensions is shown in Figure 2. During the pre-recombination epoch, the photons keep getting scattered into random directions. After a scattering event, the photon moves freely along a light cone till it undergoes the next scattering at a different location at a future time. After this scattering again the photon goes in a random direction, but still along the light cone. The photon thus undergoes a Brownian motion. This process of alternate scattering and free flight continues till the epoch t_r , which is the epoch of recombination. As a crude approximation, we can consider the recombination as an instantaneous process. The last scattering launches the photon as before in a random direction, but since there are no more free charged particles available after this epoch, the photon does not get re-scattered. (As stated this is only a crude approximation. In reality, although the transition from the plasma phase to a neutral phase is fairly sharp, it is not instantaneous. This approximation will suffice for our purpose here.) Consider a point P as shown in Figure 2 at the epoch of recombination. From P there will be photons emitted in all directions. The photon scattered into a particular direction will reach the

observer today. Photons from a point P_1 would have reached the observer's location and gone past at an earlier epoch. Similarly, a photon which got last scattered for the last time at a point P_2 has not reached the observer at time t_0 . The locus of the points from where the photons would have reached the observer at t_0 forms a circle. Thus the observer perceives an apparent circle from where the photons are reaching him/her. Since this is the circle on which the photons got scattered for the last time, this can be called the 'circle of last scatter'. For the case of the $1 + 3$ dimensional Universe that we inhabit, instead of the circle of last scatter we will have a spherical surface of last scatter with the observer at its centre. The photons when they started out from this surface must have had the temperature corresponding to the ionization temperature of matter. This turns out to be about 3000 K. However, while the photons travel through space in the post-recombination era, the temperature drops due to the expansion of the Universe. As a result, we receive these photons at a temperature of 2.73 K today. For a black-body radiation (from Wein's displacement law), this corresponds to the microwave region. It comes from all directions around us and it is interpreted to be of cosmological origin. It is hence called CMBR.

Blackbody nature of CMBR and proof of Big Bang cosmology

The instrument called FIRAS (Far-InfraRed Absolute Spectrophotometer) flown in the COBE satellite, has made precise measurements of the spectrum of the CMBR^{11,12}. In particular, this instrument looked for deviations of the CMBR spectrum from a blackbody spectrum. The deviations were measured to be less than 50 parts per million¹³. This is a strong indication that the CMBR was in thermal equilibrium in the past. (Even if it is not in equilibrium today, as long as it was in equilibrium in the past the expansion of the Universe will not spoil the blackbody nature of the spectrum³. The spectrum will evolve into a blackbody spectrum of a lower temperature. This is however true only if there is no injection of photons.) Hence, it implies that the Universe was very hot in the past. This is one of the most concrete evidences for the Hot Big Bang model.

Isotropy of CMBR as an evidence of cosmological inflation

The observation that the CMBR is isotropic, brought to focus a paradox in the standard model of the Universe. In the late 70s and early 80s a new model was suggested to resolve this problem. This model and its variants are collectively referred to as inflationary models. The fact that these are the only models that accommodate the observed isotropy of the CMBR, implies that this is a strong evi-

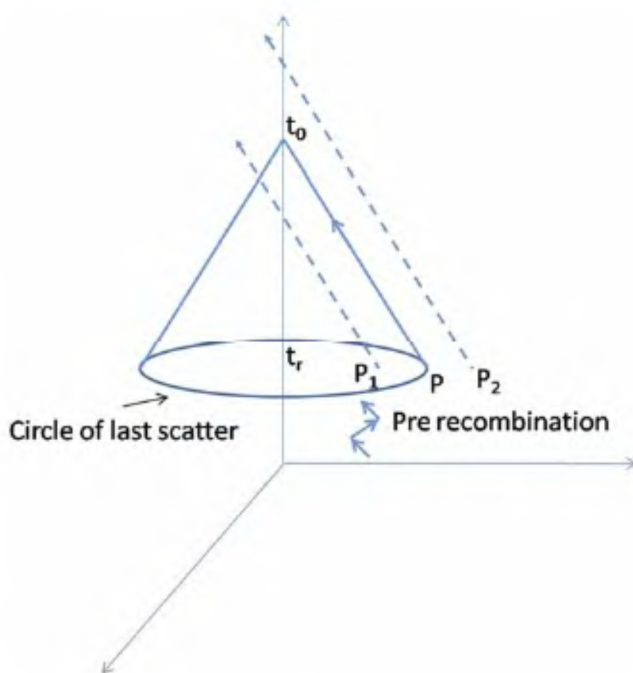


Figure 2. Circle of last scatter in a $1 + 2$ dimensional Universe.

dence in favour of an inflationary Universe. In this section we discuss the origin of this problem in standard cosmology and address the issue of its resolution in inflationary models.

Once again let us go back to the $1+2$ dimensional Universe and then generalize to the real Universe of $1+3$ dimensions. Consider the $1+2$ dimensional spacetime diagram in Figure 3. Consider two portions A_1A_2 and B_1B_2 on the circle of last scatter. Isotropy implies that all points on the circle should be similar. In particular, the radiation coming from the region A_1A_2 and B_1B_2 should be at the same temperature. We, however, see that each of these regions is inside different forward light cones. The region A_1A_2 cannot be affected or influenced by the point B , and the region B_1B_2 cannot be affected by point A . As a result within the framework of standard (pre-inflationary) model of the Universe, the issue of why the CMBR is isotropic and hence, the Universe homogenous does not have a resolution. This problem is referred to as the horizon problem.

In the late 70s and early 80s, it was noted that if the Universe had undergone rapid expansion (now called inflation) in its early phase, many of the outstanding problems in standard cosmology (including the above-mentioned horizon problem) have a resolution¹⁴⁻¹⁶. In these models, the Universe is believed to have undergone an exponential expansion in which the scale factor increases by a factor of about 10^{28} within a time interval of about 10^{-34} s. In the simplest models this is supposed to have taken place about 10^{-36} s after the Big Bang. The net effect is that the photon trajectories or so modified in the

very Universe (much before recombination) that the entire circle of last scatter (and similarly, the spherical surface of last scatter in $1+3$ dimensions) is engulfed in a single causally connected domain. With this mechanism it is no more a surprise that the CMBR is isotropic. Thus the observed isotropy of the CMBR tells us that the Universe must have undergone a phase of inflationary expansion.

Anisotropy of the CMBR

It was recognized after the discovery of the CMBR that the formation of structures in the Universe should have left an imprint in the form of anisotropies in the CMBR¹⁷. The successful detection of the anisotropy in the CMBR by the DMR [4] on COBE satellite heralded a new era in cosmology. The observed anisotropy at the level of 1 part in 10^5 narrowed down the list of acceptable candidates for dark-matter in the Universe. While it was found that cold dark-matter models could be accommodated, hot dark-matter candidates were ruled out. This small level of anisotropy also ruled out several baryon-dominated models^{18,19}. After that many experiments have observed the angular anisotropy over different scales. The most recent of these is the 5-year data release of WMAP²⁰. Later in this article the results and the interpretation of these results will be discussed.

Characterizing temperature anisotropy

In order to understand the origin of anisotropy, it will be useful to once again start with an illustrative example of a $1+2$ dimensional Universe. In such a case we had seen that the photons we receive are from a circle of last scatter located at the epoch of recombination. If the Universe were homogeneous at that time, the nature of the photons received from all the points on the circle would have been similar. However, if there were inhomogeneities, all the points on the circle of last scatter would not be at the same temperature. The photons we receive from different points on the circle of last scatter would be different. Thus we would see the radiation to be anisotropic. Let us go further and discuss how to analyse this anisotropy in temperature. First of all we can define an average temperature as

$$T_0 = \frac{1}{2\pi} \int_0^{2\pi} d\theta T(\theta). \quad (12)$$

We may then define a fractional deviation of temperature from this mean value as

$$\epsilon(\theta) = \frac{T(\theta) - T_0}{T_0}. \quad (13)$$

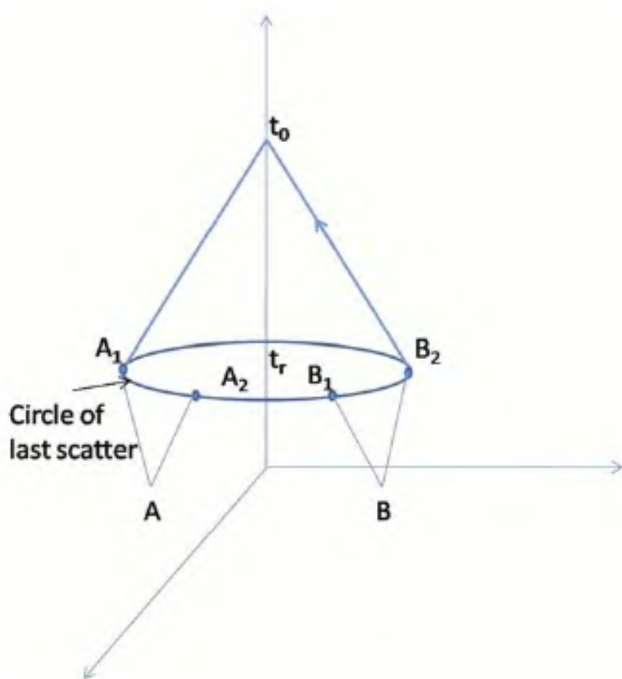


Figure 3. The horizon problem.

Since the fractional deviation is defined on a circle, it can be expanded as a Fourier series, and it is often more convenient to work in terms of these Fourier coefficients.

In the real 1 + 3 dimensional Universe, the temperature in any direction is denoted by $T(\theta, \phi)$. We define a mean temperature,

$$T_0 = \frac{1}{4\pi} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin(\theta) d\theta d\phi T(\theta, \phi). \quad (14)$$

We express the temperature in any direction as,

$$T(\theta, \phi) = T_0[1 + \Theta(\theta, \phi)], \quad (15)$$

where $\Theta(\theta, \phi)$ is the fractional perturbation in the temperature $\Delta T/T_0$. As in the case of 1 + 2 dimension, it will prove more useful to expand Θ in terms of a set of basis functions, which in this case are the spherical harmonics $Y_{lm}(\theta, \phi) = Y_{lm}(\hat{\mathbf{n}})$, which is a set of complete basis functions defined on the celestial sphere.

$$\Theta_{lm} = \int d\Omega Y_{lm}^*(\hat{\mathbf{n}}) \Theta(\hat{\mathbf{n}}). \quad (16)$$

The anisotropy is specified by the angular two-point correlation function $\langle \Theta(\hat{\mathbf{n}}) \Theta(\hat{\mathbf{m}}) \rangle = C(\alpha)$. This is a function only of α , the angle between $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$. The reason for this is that due to isotropy, what matters is only the angle between the two directions and not the direction in which this set is pointing. If the temperature anisotropy has a Gaussian behaviour, the anisotropy should be completely describable by the power spectrum

$$C_l \delta_{ll'} \delta_{mm'} = \langle \Theta_{lm}^* \Theta_{lm'} \rangle. \quad (17)$$

The fact that the power spectrum C_l is independent of m has just to do with the fact that the quantity $C(\alpha)$ depends only on the angle between $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$. One may look at it in the following way. Since no particular direction is special, we can take the direction $\hat{\mathbf{n}}$ to be the z -direction. Thus α becomes the polar angle. Further, since the direction $\hat{\mathbf{m}}$ is only constrained to make an angle α with $\hat{\mathbf{n}}$ but is otherwise arbitrary, the azimuthal angle has no role.

From the point of view of measurement, even if we have perfect instrumentation without error, there is a basic error called cosmic variance which we cannot escape. It arises from the fact that for every l mode there are $2l + 1$ number of m values which contribute. Hence, for any given l , there is an uncertainty in the value of C_l which is given by,

$$\Delta C_l = C_l \left(\frac{2}{2l+1} \right)^{1/2}. \quad (18)$$

The mean square temperature anisotropy is $\langle (\Delta T)^2 \rangle = T_0^2 [C(\alpha)]_{\alpha=0}$. In terms of C_l s we have,

$$\sqrt{\langle (\Delta T)^2 \rangle} = T_0 \left(\sum_l C_l \frac{2l+1}{4\pi} \right)^{1/2}. \quad (19)$$

In the large l limit this becomes,

$$\sqrt{\langle (\Delta T)^2 \rangle} = T_0 \left[\int d(\ln l) \frac{l(l+1)C_l}{2\pi} \right]^{1/2}. \quad (20)$$

The quantity usually plotted to show the behaviour of temperature anisotropy for various l values is either the power per unit logarithmic interval in l , $\langle \Delta T_l \rangle^2$, or its square root, where

$$(\Delta T_l)^2 = T_0^2 \frac{l(l+1)C_l}{2\pi}. \quad (21)$$

Origin of temperature anisotropy in the CMBR

The Boltzman equation that describes the evolution of the distribution function is given by,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{dx^i}{dt} \frac{\partial f}{\partial x^i} + \frac{dp_j}{dt} \frac{\partial f}{\partial p_j} = C[f]. \quad (22)$$

In the case of the CMBR photons, the collision term on the right-hand side ($C[f]$) is primarily governed by the Compton scattering process between the electrons and photons. From the moments of the Boltzman equation we can arrive at the equations for the evolution of excess temperature of the CMBR. The nature of Coulomb interactions dictates the form of the collision term. With these two in eq. (22), we can derive the evolution equation of the radiation temperature. Using the moments of the Boltzman equation, the equation governing temperature anisotropy turns out to be,

$$\dot{\Theta} + ik\Theta = -\Phi - ik\mu\Psi - \tau \left[\Theta_0 - \Theta + \mu v_b - \frac{1}{2} P_2(\mu) \Pi \right]. \quad (23)$$

(For details of the derivation the reader is referred to Dodelson¹⁰ and Subramanian²¹). Θ is the Fourier transform of the temperature anisotropy and v_b is the velocity field of baryons in Fourier space. The term Π is the sum of quadrupole terms of temperature anisotropy, polarization anisotropy and the polarization monopole term. Φ and Ψ are perturbations in the metric and are related to the gravitational potential perturbations. These perturbations are dominant typically over angular scales of more than 1° . The factor, τ is the optical depth. The tempera-

ture anisotropy over the largest angular scales arises from the perturbations in the gravitational potential on the surface of the last scatter. This is referred to as the Sachs–Wolfe effect.

As the photon travels from the surface of the last scatter, it passes through structures which are in the process of collapse and hence in a time-dependent gravitational potential that is deepening. This leads to further anisotropies and the effect is called the integrated Sachs–Wolfe effect. The baryon and the photon fluid are tightly coupled to each other. This mixture behaves like a single fluid. Perturbations in this fluid undergo acoustic oscillations. These oscillations at the SLS manifest themselves as crests and troughs in the anisotropy power spectrum. The angular size (or alternatively the corresponding l value) for a physical length scale depends on the background geometry through which the photons have travelled to reach us. However, the correspondence is not one-to-one. This is because due to projection effect, a range of physical length-scales will correspond to the same angular scale. The contribution to the angular power spectrum for a certain l comes mainly from $l = k(\eta_0 - \eta^*)$. Here η_0 and η^* are the conformal times corresponding to today's epoch and the epoch of recombination respectively. Because of the projection effect, however, this value of l is not the only contribution but only the predominant one.

We will now summarize the information we can get about the parameters in cosmology from the structure of the angular power spectrum curve²¹. The angular power spectrum has a series of peaks. The values of l at which the peaks occur are

$$l = n\pi \frac{\eta_0 - \eta^*}{r_s(\eta^*)}.$$

Here $r_s(\eta^*)$ is the comoving sound horizon at the epoch of recombination. The curvature of the Universe has a bearing on the location of the first peak. The odd peaks tend to have a larger value than the even ones. This is due to the non-zero value of the baryon density.

Characterizing the nature of the CMB polarization anisotropy

Several features and processes in the Universe can induce a polarization of the CMBR. Polarization, however, is a much weaker signal compared to temperature anisotropy. However, with the development of newer experimental techniques one has started seeing signals of polarization anisotropy, and the situation is expected to improve with the satellite-borne experiments that are planned. Hence, analysis of the CMBR polarization has now become more important and interesting than before. Here we review these characteristics in brief. For details, there are good reviews and textbooks available^{8–10,22,23}.

Conventionally polarization is described in terms of Stokes' parameters, I , Q , U and V . Here, I is the total intensity. Its information is encoded in the temperature anisotropy. As we will see in the next section, polarization in the CMBR is caused due to Thomson scattering of anisotropic radiation from free electrons. Such a process can lead to a non-zero degree of plane polarization, but cannot produce circularly polarized light. The Stokes' parameter V is a measure of circular polarization. Hence, in polarization processes in the context of cosmology, this parameter is taken to be zero. Thus we may assume that to describe polarization in the context of cosmology, we could work with the two Stokes parameters, Q and U . The Stokes parameter Q measures polarization in the vertical and horizontal directions. U measures polarization at $\pm 45^\circ$ with the horizontal. It is immediately clear that a pure Q -type polarization goes to pure U -type just by rotation of the coordinate system. Ideally one would like to have a description which does not depend on the coordinate system. One might naively think that this is possible if we define a particular direction as universal. This is true in a two-dimensional Euclidean surface, but not on the celestial sphere as it is non-euclidean. This can be seen through the following example. If we have a horizontal plane described by a two-dimensional Euclidean geometry, then one can define a universal reference axis and specify orientations of straight lines with respect to this. The specification of this direction is unique, no matter where the straight line is located.

The situation is not simple when we are describing orientations on the surface of a sphere. Two small line segments at well-separated locations on a spherical surface, both of which are pointing north-south, will have two different orientations. The situation is similar in the case of polarization. The polarization directions are specified on the celestial sphere, which obviously is not Euclidean. To measure/compute polarization anisotropy, we need to correlate polarization direction at two different locations. Due to the complication arising out of non-Euclidean geometry, this exercise is non-trivial if we simply use the Stokes' parameters Q and U . If we were to correlate the polarization orientation in two directions \hat{n} and \hat{m} , that subtend a small angle with each other, then of course one can make a flat-sky approximation over a small region on the celestial sphere. (By small angle we mean $\cos^{-1}(\hat{n} \cdot \hat{m}) \ll 2\pi$.) Since in this small region the deviation from Euclidean nature is small, we can follow the same procedure as in the case of a Euclidean two-dimensional plane. We can define a direction in the flat patch on the celestial sphere. The polarization orientation correlation at two different points in this nearly flat patch will not have any significant artefact due to the non-Euclidean nature. However, if we need to measure or describe the correlation at two points that are well separated, we cannot make a flat-sky approximation.

We know that a spinor, which is invariant under a rotation of 4π radians, is described by a spin-1/2 field. Similarly, vectors which are invariant under a rotation by 2π are described by a spin-1 field. Polarization is invariant under a rotation by π and is described by a spin-2 field.

Following Durrer⁸, we define the dimensionless Stokes' parameters,

$$Q = \frac{Q}{4I}, \quad U = \frac{U}{4I}. \quad (24)$$

Under rotation by an angle ψ , these quantities transform as

$$Q' \pm iU' = \exp^{\pm i2\psi} (Q \pm iU), \quad (25)$$

$$I' = I, \quad (26)$$

$$V' = V, \quad (27)$$

and hence,

$$Q' \pm iU' = \exp^{\pm i2\psi} (Q \pm iU). \quad (28)$$

Just as we expanded the temperature on the celestial sphere in terms of the spherical harmonics, a spin-2 field can be expanded on a spherical surface in terms spin-weighted spherical harmonics, ${}_{\pm 2}Y_{lm}$.

$$(Q \pm iU)(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^l a_{lm}^{\pm 2} {}_{\pm 2}Y_{lm}, \quad (29)$$

$$= \sum_{l=2}^{\infty} \sum_{m=-l}^l (e_{lm} \pm ib_{lm}) {}_{\pm 2}Y_{lm}. \quad (30)$$

Under parity transformation, $a_{lm}^{+2} \leftrightarrow a_{lm}^{-2}$, $e_{lm} \leftrightarrow e_{lm}$ and $b_{lm} \leftrightarrow -b_{lm}$.

Our aim is to describe polarization anisotropy in terms of variables, which like temperature anisotropy are invariant under rotation. Directly in terms of Stokes' parameters, it is not possible as they are components of a spin-2 field. However, one can construct spin-0 fields by the following procedure. One can define spin raising (ρ) and lowering (ρ^*) operators. Operating this twice on spin-2 fields, one can construct spin-0 fields which have this invariance property. The spin raising and lowering operators are similar to the raising and lowering operators one comes across in the context of angular momentum in quantum mechanics.

The spin-0 fields thus constructed are much like temperature anisotropy and can be expanded in terms of the usual spherical harmonics. To this end one constructs these operators which are defined in terms of their action on the spin-weighted spherical harmonics.

$$\rho^2 ({}_{-2}Y_{lm}) = (\sqrt{(l+2)!/(l-2)!}) Y_{lm}, \quad (31)$$

$$\rho^{*2} ({}_{+2}Y_{lm}) = (\sqrt{(l+2)!/(l-2)!}) Y_{lm}. \quad (32)$$

We can then define polarization parameters denoted by \mathcal{E} and \mathcal{B} as,

$$\mathcal{E} = \frac{1}{2} [\rho^2 + \rho^{*2}] Q, \quad (33)$$

$$\mathcal{B} = -i \frac{1}{2} [\rho^2 - \rho^{*2}] U. \quad (34)$$

Using these raising and lowering operators on both sides of eq. (29) we have,

$$\mathcal{E}(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^l e_{lm} Y_{lm}(\hat{n}), \quad (35)$$

$$\mathcal{B}(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^l b_{lm} Y_{lm}(\hat{n}). \quad (36)$$

These two variables describe polarization, while at the same time are invariant under rotation. We have already mentioned the transformation properties of e_{lm} and b_{lm} under parity transformation. Using this we find from the above expression that \mathcal{B} has negative parity and \mathcal{E} has positive parity. Due to this property, the former is called magnetic-type and the latter electric-type polarization.

The electric- and magnetic-type polarization anisotropies are measured in terms of the power spectrum of the respective quantities. As in the case of temperature anisotropies, we define the power spectrum in electric- and magnetic-type polarization anisotropies as,

$$C_l^{\mathcal{E}} = \langle |e_{lm}|^2 \rangle, \quad (37)$$

$$C_l^{\mathcal{B}} = \langle |b_{lm}|^2 \rangle. \quad (38)$$

We can also define the cross-correlation $C_l^{\Theta\mathcal{E}}$ as,

$$C_l^{\Theta\mathcal{E}} = \langle \Theta_{lm} e_{lm} \rangle. \quad (39)$$

$C_l^{\mathcal{E}\mathcal{B}}$ should be zero because of \mathcal{E} and \mathcal{B} are of opposite parity. Similarly, $C_l^{\Theta\mathcal{B}}$ should also be zero. However, if there were parity-violating processes, then they would also be non-zero. Thus measurement of $C_l^{\mathcal{E}\mathcal{B}}$ and $C_l^{\Theta\mathcal{B}}$ can put bounds on parity-violating processes.

Origin of CMB polarization anisotropy

The possibility that the CMB could be polarized was realized²⁴ in 1968. Consider the following situation shown in Figure 4. Light is incident from point A to point P , where

there is a free electron. Let AP be parallel to the y -axis. Point P is vertically above a point O , where the observer is located. OP direction is defined as the z -axis. For incident light, the direction of electric field oscillations is in a plane parallel to the x - z plane. Hence, when this light disturbs the electron at P , it will oscillate only in the x - z plane, i.e. perpendicular to the y -axis. This oscillation will lead to radiation. Consider the ray of light travelling from P to O . Since the electron oscillation does not have a component along the y -axis, the electric field vector does not have a component in that direction. The oscillations along the z -axis do not contribute to radiation. This implies that the radiation along PO has electric oscillations which are parallel only to the x -axis. Hence we find that although the incident light is unpolarized, the scattered light in the PO direction is plane-polarized. If at sunrise or sunset, we observe the light from the zenith through a polarizer, we will find that the light has a non-zero degree of polarization. The free electrons in the ionosphere scatter the light coming from the horizon. When we look at the zenith at this time, the direction of scattered light that reaches the observer from the zenith is perpendicular to the direction of the incoming light. If the incident light was from all directions with equal intensity, it is easy to see that the scattered light along PO is unpolarized. This implies that the incident light needs to be anisotropic. It turns out that if the incident light has dipole anisotropy, this is still insufficient to produce polarization after Thomson scattering. In order to produce polarization we need light (with a quadrupole component of anisotropy) to be Thomson scattered off free charged particles.

The Universe is believed to have undergone re-ionization in the post-recombination era. With a quadrupole component in the anisotropy of the CMBR, this is

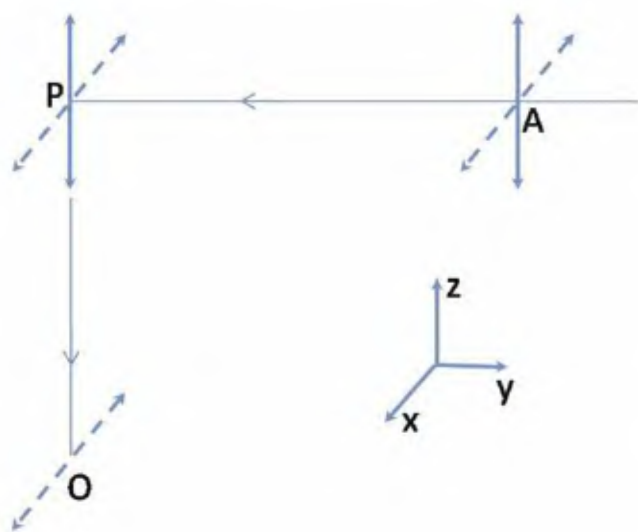


Figure 4. Polarization from Thomson scattering.

one possible scenario for the origin of polarization. Another situation that can cause polarization is the finite thickness of the surface of last scattering. The electron that is responsible for the last scattering of a photon sees a quadrupole anisotropy from the last but one scattering. Thus even without re-ionization, one does expect a polarization signal in the CMBR. We had earlier discussed the electric- and magnetic-type polarization. Scalar modes (arising due to density and potential perturbations) lead only to the electric-type perturbations. Cosmic magnetic fields produce solenoidal velocity fields that lead to vector-type perturbations. These perturbations produce magnetic-type polarization^{25–27}. Models of inflation produce tensor perturbations in addition to scalar ones. The magnetic type polarization can be produced by such tensorial perturbations.

Recent developments and present status of CMBR observations

Since the first detection of temperature anisotropy by the COBE-DMR in 1992, several experiments have measured anisotropy over different angular scales. We now are beginning to get information on the polarization properties of the CMBR also. The most recent results on the CMBR are the WMAP5.

Details of the angular power spectrum have a wealth of information. The scales that leave the Hubble radius during the early phase of inflation are expected to affect the large angle (low l) region of the power spectrum. A model called punctuated inflation^{28,29} which fits with the low value of quadrupole angular power has been proposed. Here a two-stage slow rollover inflation model has been considered.

The vortical velocity perturbations are produced by tangled magnetic fields over small angular scales. For $l \sim 10,000$, a nearly scale-invariant spectrum of such fields which have got redshifted today to 3×10^{-9} Gauss can produce temperature anisotropy^{30,31} of about 0.3–0.4 μK . These also are expected to produce B -type polarization anisotropy³² of a strength of about 0.4 μK .

The fifth-year data release of WMAP has further advanced our knowledge of the Universe and put stronger bounds on the cosmological parameters. The angular power spectrum derived from WMAP5 is given in Figure 1. Various parameters leave their imprint in the features of this curve. Further, the parameter space can be narrowed down by analysing this curve in conjunction with baryon acoustic oscillations and supernova data [20].

Density parameter and dark energy: There is now fairly stringent constraint on the dark energy equation-of-state and the density parameter in the Universe. We find that the density parameter of the Universe is constrained as $0.9010 < \Omega < 1.0179$, with the equation-of-state parameter constrained as $-1.14 < w < 0.88$. This implies that a spa-

tially flat Universe with cosmological constant as dark energy is perfectly consistent with observations.

It has also been suggested by a number of authors that the dark energy equation-of-state could be a function of time, or alternatively, a function of the scale factor. In the absence of any theoretical handle on the possible variation of the equation-of-state parameter for dark energy, it is important to constrain the behaviour of w . The reanalysis of WMAP5 gives us some handle by placing a constraint on the value of w today. The constraint imposed is $-1.33 < w < 0.79$.

Limits on non-Gaussianity: In the simplest models, temperature anisotropy follows a Gaussian statistics. In such models, the power spectrum or alternatively, the angular two-point correlation function, specifies the temperature distribution completely. However, non-Gaussianity of the temperature distribution can arise in certain situations. As stated earlier, cosmic magnetic fields, induce non-Gaussian temperature anisotropy. Another source of non-Gaussianity is nonlinear terms in the gravitational potential function. The simplest way to characterize non-Gaussianity is to calculate the bispectrum, which is related to the three-point angular correlation function $\langle \Theta_{l_1 m_1} \Theta_{l_2 m_2} \Theta_{l_3 m_3} \rangle$. In Fourier space, the gravitational potential $\Phi(\vec{k})$ can be expressed as $\Phi(\vec{k}) = \Phi_L(\vec{k}) + \Phi_{NL}(\vec{k})$. These nonlinear terms in the gravitational potential produce non-Gaussianity in the CMB. Denoting the reduced bispectrum by $b_{l_1 l_2 l_3}$ for large angular scales, we get $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \sim 4 \times 10^{-18} f_{NL}$, where f_{NL} is a measure of the contribution of nonlinear terms in the gravitational potential perturbations. The WMAP5-year results constrain the values of f_{NL} in two limits. One of the limits is the local limit, where two of the l values are much larger than the other. The other limit is the equilateral limit where all the l values are almost same.

$$-9 < f_{NL}^{\text{local}} < 111, \quad (40)$$

$$-151 < f_{NL}^{\text{equil}} < 253. \quad (41)$$

The bispectrum arising out of primordial magnetic fields has been calculated^{33–35}. For the bispectrum induced by tangent magnetic fields, we have $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \sim 10^{-22}$.

We had earlier seen that the cross-correlations $C_l^{\mathcal{EB}}$ and $C_l^{\mathcal{EB}}$ should be zero if there are no parity-violating processes. Bounds on such processes can be specified by the rotation of the polarization angle in the post-recombination era. WMAP observations put the constraints on this rotation as $-5.9^\circ < \Delta\alpha < 2.4^\circ$.

Concluding remarks

In this article, I have attempted to give a brief overview of what kind of information is hidden in the CMB. I have also tried give an idea of the method to quantify the features in the CMB. COBE, WMAP and several other experiments which have helped us to get precise data on the CMB. With the launch of the Planck satellite, the subject of CMB is likely to get a further boost.

1. <http://astrophysics.arc.nasa.gov/mway/AMES-CMB.pdf>
2. Penzias, A. A. and Wilson, R. W., *Astrophys. J.*, 1965, **142**, 419–421.
3. Padmanabhan, T., *Structure Formation in the Universe*, Cambridge University Press.
4. Smoot, G. F. et al., *Astrophys. J. Lett.*, 1992, **396**, L1–L5.
5. Challinor, A. and Peiris, H., *Lecture Notes on the Physics of Cosmic Microwave Background Anisotropies* (astro-ph/0903.5158).
6. Samtleben, D., Staggs, S. and Winstein, B., *The Cosmic Microwave Background Radiation for Pedestrians* (astro-ph/0803.0834)
7. Padmanabhan, T., *Theoretical Astrophysics, Galaxies and Cosmology*, Cambridge University Press, Cambridge, 2002, vol. III.
8. Durrer, R., *The Cosmic Microwave Background*, Cambridge University Press, Cambridge, 2008.
9. Giovannini, G., *A Primer on the Physics of the Cosmic Microwave Background*, World Scientific, Singapore, 2008.
10. Dodelson, S., *Modern Cosmology*, Elsevier, 2003.
11. Mather, J. C. et al., *Astrophys. J.*, 1994, **420**, 439–444.
12. Mather, J. C. et al., *Astrophys. J.*, 1994, **420**, 450–456.
13. Fixen, D. J. et al., *Astrophys. J.*, 1996, **473**, 576.
14. Sato, K., *Phys. Lett.*, 1981, **B99**, 66.
15. Kazanas, D., *Astrophys. J.*, 1980, **241**, L59.
16. Guth A., *Phys. Rev. D*, 1981, **23**, 347.
17. Sachs, R. K. and Wolfe, A. M., *Astrophys. J.*, 1967, **147**, 73.
18. Efstathiou, G., Bond, J. R. and White, S. D. M., *MNRAS*, 1992, **158**, 1P.
19. Padmanabhan, T. and Narasimha, D., *MNRAS*, 1992, **259**, 41P.
20. Komatsu, E. et al., *Astrophys. J. Suppl.*, 2009, **180**, 330.
21. Subramanian, K., *Curr. Sci.*, 2005, **88**, 1068.
22. Kosowsky, A., *New Astron. Rev.*, 1999, **43**, 157.
23. Hu, W. and White, M., *New Astron.*, 1997, **2**, 323.
24. Rees, M. J., *Astrophys. J.*, 1968, **153**, L1.
25. Sunyaev, R. A. and Zeldovich, Y. B., *Commun. Astrophys. Space Phys.*, 1972, **4**, 173.
26. Seshadri, T. R. and Subramanian, K., *PRL*, 2001, **87**, 101301.
27. Mack, A., Kashniashvili, T. and Kosowsky, A., *PRD*, 2002, **65**, 123004.
28. Jain, R. K., Chingangbam, P., Gong, J. O., Sriramkumar, L. and Souradeep, T., *JCAP*, 2009, **0901**, 009; arXiv:0809.3915 [astro-ph]
29. Jain, R. K., Chingangbam, P., Sriramkumar, L. and Souradeep, T., *JCAP*, 2009; arXiv:0809.3915 [astro-ph].
30. Seshadri, T. R. and Subramanian, K., *PRD*, 2005, **72**, 023004.
31. Lewis, A., *PRD*, 2004, **70**, 043011.
32. Subramanian, K., Seshadri, T. R. and Barrow, J. D., *MNRAS*, 2003, **344**, L31.
33. Giovannini, M., *PMC Phys. A*, 2007, **1**, 5.
34. Paoletti, D., Finelli, F. and Paci, F., *MNRAS*, 2009, **396**, 523.
35. Seshadri, T. R. and Subramanian, K., *PRL*, 2009, **103**, 081303.