On the confirmation of the traditional unit of length measure in the estimates of circumference of the earth

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The prevalence of two different measures for the circumference of the earth in Sanskrit astronomical texts has been resolved by considering the concept of a constant *angulam* of 1.763 cm and the metrological table defined in Kautilya's *Arthasastra*. The traditional measures defined in the *Arthasastra* have been related to the measures defined in the *Ain-i-Akbari*, by considering the same issue of circumference of the earth. The study has established the connection of the traditional unit of measure, the *angulam*, of the Harappan Civilization (~2000 BC) with length measures of the Mughal period (AD ~1600), thereby confirming an unbroken tradition in the use of the *angulam* of 1.763 cm over a period of more than 3600 years in the Indian subcontinent. The possible measurement units which can be used to analyse the dimensions of most engineered structures of the subcontinent through the ages have been proposed and briefly confirmed by the dimensions of the marble platform of the Taj Mahal.

Keywords: Circumference, earth, length measures.

A comprehensive review of length measures in different parts of the Indian subcontinent was undertaken by Raju and Mainkar in the 1960s. They also analysed the measures assuming that the 'measure of 50 cm length' remained constant through the ages[1–5]. While this was useful in understanding the intermediate length measures, the interpretation of longer length measures posed severe problems. One notable example is the circumference of the earth. The Indian astronomical texts estimate the earth's circumference as either 3300 *yojanams* or 5000 *yojanams*[2]. The important texts that provided the value of 3300 *yojanams* were *Aryabhatiya, Panchsiddantika*, *Mahabhaskariya*, *Shisyadhvriddhida*, *Vateswarasiddhanta* and *Siddhantadarpan*, while the texts *Suryasiddhanta*, *Brahmasphutsiddhanta*, *Mahabhaskariya*, *Siddhantasekhara*, *Siddhantasiromani*, *Somasiddhanta*, *Shakalyokta* and *Bramasiddhantar* estimate the circumference as 5000 *yojanams*. It is indeed surprising that a natural constant number like the circumference of the earth was given by two different values, but it is clear that both of them implied the same value.

There are strong evidences to prove that people of the Indian subcontinent were very precise when it came to length measures because of their importance in the measurement of agricultural land and hence collection of revenue. For example, an inscription from Brihadesvara Temple in Thanjavur mentions that land was measured to an accuracy of 1/52,428,800,000 of a *veli* (which is about 2.6780 ha) during the rule of the Chola monarch Rajaraja I in early 11th century AD[6]. Therefore, it is reasonable to suppose that the circumference of the earth must have been estimated precisely.

This major problem of two values of circumference of the earth in the Indian astronomical texts has not been satisfactorily resolved till date. Raju and Mainkar specifically mentioned that 'considerable research is needed' to solve this problem[2]. This problem will be revisited in this article utilizing the novel concept of the constancy of the traditional unit of measurement, namely the *angulam*. In addition, the two estimates of the circumference of the earth mentioned in the *Ain-i-Akbari* will be analysed to obtain new insights on the metrology of the Mughal period.

Constancy of *angulam*

The traditional unit of measurement, the *angulam*, has been conclusively proved to have been in use over a long period of over at least 2400 years in the subcontinent. Specifically, the *angulam* of unit 1.763 cm derived from plans of Harappan Civilization (2500–1600 BC) settlement sites, without any a priori assumptions[7,8], was used to describe precisely the dimensions of the first engineered caves at the Barabar and Nagarjuni hills of the Mauryan Period (late 4th to early 2nd century BC)[9] and the Delhi Iron Pillar of the Gupta Period (AD 320–600)[10]. The constancy of *angulam* in later periods of Indian history will be explored by considering its association with the estimate of the earth's circumference in Indian texts.

The table of measures in Kautilya's *Arthasastra* (KA) will be the main focus as regards literary evidence, as this is the foundation of metrology of the Indian subconti-

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nent[11,12]. While references to certain names of some length units appear in the Vedic literature[1], their relationship has not been mentioned. It is *KA* that first describes the coherent system of length and area measures of the Indian subcontinent that was extant around the 3rd century BC. At the outset, it should be emphasized that literary evidences can be confusing because of the availability of different versions of the same text, apart from possible differences in interpretation and meaning of words. For example, even in the case of the *KA*, Shamasastry[11], who discovered the text in Mysore, mentions that a larger unit called *goruta* was equal to 1000 *dhanus*, while the edition by Kangle[12] mentions the *goruta* as 2000 *dhanus*. The difference is not due to interpretation alone, but due to readings based on different manuscripts. The current work has shown that a better match was obtained using Shamasastry's proposed value of a *yojanam* being equal to 1000 *goruta* rather than 2000 *goruta*. This does not imply that either one of the two readings has to be taken as final because the current approach has proposed a new original way of looking at the problem, independent of the manuscript readings. There is no ready solution to explain this discrepancy and this is left for other scholars to debate. However, it may be noted here that there is need to take recourse to scientific analysis of material evidences, which are far superior because the conclusions derived using them have firm support. In fact, in several cases, material evidences may help ease the confusion created by literary evidences.

Shamasastry[11] and Kangle[12] mention the division of scales in their respective editions. While the length measures at the smaller level are not relevant here because they originate from atomistic levels and are quite undefined, it is relevant to focus attention on intermediate (measures above the *angulam*) and larger measures. For example, the *KA* mentions that 12 *angulams* made one *vitasti* and further that two *vitastis* are equal to one *aratni*, which was also called a *hasta* (= 24 *angulams*). There are several other units defined in the *KA*. Raju and Mainkar[2] rightly pointed out that the *KA* was also responsible for laying the 'seeds of later confusion' in the interpretation of lengths and area units, because it specifies different measures with the same name. For example, there are three measures for *hasta* (of 24, 28 and 54 *angulams*), two for *kishku* (of 32 and 42 *angulams*), two for *danda* (of 96 and 192 *angulams*), two for *dhanus* (of 96 and 108 *angulams*) and two for *paurusha* (of 96 and 108 *angulams*).

In order to analyse the length measures of the subcontinent through the ages, Raju and Mainkar[1–5] assumed that the value of the *angulam* was changed with times in order to keep up with the different values of *hastas* that were in popular use. In particular, they assumed that the *hasta* of 28 *angulams* was unchanged throughout history at a value of 50 cm and therefore, went about analysing the other measures to fit their theory. It is not surprising that a change in the *angulam* unit was inferred because of this assumption. The procedure adopted by Raju and Mainkar is not precise because there is no supporting archaeological evidence for assuming the constancy of the 50-cm measure through the ages, unlike several (dated) archaeological material evidences[9,10] offering strong support for the constancy of the *angulam* of 1.763 cm.

The table of measures in the *KA* needs to be carefully analysed. For the sake of convenience, the different *hastas* in the *KA* will be defined so that there is no confusion. The *hasta* of 24 *angulams* will be called the P-hasta (P implying *prajpatya* as mentioned in the *KA*), the *hasta* of 28 *angulams* will be called the C-hasta (C standing for 'commercial', as the *KA* does not mention any unique name for this measure) and the *hasta* of 54 *angulams* will be called the F-hasta (F standing for 'forest', as the *KA* mentions that the *hasta* of this measure was used for measuring forests). In addition, we define a *hasta* of 42 *angulams* called the K-hasta (K implying *kishku hasta*) and a *hasta* of 32 *angulams* called the M-hasta (M implying *kamsa hasta*), because these measures appear in the *KA*[11,12] and were used in later periods in Indian history[1–5].

Considering the dimensions of the *angulam* to be unchanged at 1.763 cm, the dimensions of these five kinds of *hastas* can be precisely determined. This statement may appear trivial, but it is significant. Much of the confusion in later times was caused by different schemes of dividing the different *hasta* units. For example, the length of the C-hasta was divided into 24 divisions and 41 of these divided 'units' defined the Ilahi *gaz* in the *Ain-i-Akbari*[13]. This procedure will result in the apparent notion that the basic unit of measure did change with time, but this is really not the case. This will become clear later when the circumference of the earth will be estimated based on two values given in the *Ain*[13].

Utilizing a constant *angulam* of 1.763 cm, the basic measurements of the five types of *hastas* are 42.312 cm for P-hasta, 49.364 cm for C-hasta, 95.202 cm for F-hasta, 74.046 cm for K-hasta and 56.416 cm for the M-hasta. The larger measurement units defined in the *KA*, like the *danda*, *goruta* and *yojanam*, are all based on different multiples of the *hasta*. We shall focus attention on the five types of *hastas* defined above and estimate dimensions of the larger units for each of these *hasta*.

The *danda* is defined as four *aratinis* in the *KA*. Since the *aratni* was also called *hasta* in the *KA*[11,12], it is reasonable to multiply the *hasta* by four to arrive at the *danda*. The name *danda* is commonly used to define the measure that was four times the *hasta,* in later texts on metrology of the subcontinent[2]. We also note, for the sake of interest, that the *KA* mentions four names for the same measure of 'four times the *hasta*' (namely *danda, dhanus, nalika* and *paurusha*) and the possible origin for these names may be related to the way in which a particular *hasta* was used in the multiplication.

Next, the *goruta* is equal to 1000 *dandas* (*dhanus* is mentioned in the *KA*, but taken here as *danda* due to the

Table 1. Values in modern centimetres for the various larger measures defined in the *Arthasastra* using five different *hastas*

Measure	Multiple of *hasta*	P-hasta (24 A)	C-hasta (28 A)	F-hasta (54 A)	K-hasta (42 A)	M-hasta (32 A)
Hasta	1	42.312	49.364	95.202	74.046	56.416
Danda	4	169.25	197.26	380.81	296.184	225.664
Goruta	4000	169,248.00	197,456.00	380,808.00	296,184.00	225,664.00
Yojanam	16000	676,992.00 (6.77 km)	789,824.00 (7.90 km)	1,523,232.00 (15.23 km)	1,184,736.00 (11.85 km)	902,656.00 (9.03 km)

A, Angulam of 1.763 cm length.

use of this common term in later metrological texts[2]) and further, four *gorutas* make up one *yojanam*. These values in terms of the modern metric unit of centimetres are listed in Table 1. This table is fundamental in the understanding of measures of the Indian subcontinent through the ages. The important point to note is that the *goruta* has been taken as 1000 *dandas*, which was the view expounded by Shamasastry[11]. Kangle[12] suggested the unit of 2000 *dandas* to the *goruta*. The approach using the value of 1000 *danda* to the *goruta* provided a coherent understanding and analysis of the data.

At this juncture, we briefly digress to note that the *garhapatya dhanus*, which was also used for measurement according to the *KA*, contains 108 *angulams*[11,12]. This has not been considered separately, but it is easy to note that this is a simple double multiple of the F-hasta. We will, therefore, not add the *garhapatya dhanus* (= 190.4 cm) as a separate multiple, but note here that it offers a coherent explanation of the dimensions of several Harappan Civilization settlement sites[7,8] and the dimensions of the Delhi Iron Pillar[10]. Another 'double' measure that appears to have been popular through the ages is the measure of twice the P-hasta[9].

Circumference of the earth in Sanskrit texts

The circumference of the earth[14] is 40,075 km. For the case when the circumference was mentioned as 5000 *yojayams* in Indian texts, each *yojanam* is approximately close to 8 km. A *yojanam* of about 8 km is obtained while considering the measure based on the use of C-hasta, as can be seen from Table 1. Looking at this in another way, if the circumference of the earth was calculated based on the *yojanam* of 7.89824 km, the circumference of the earth comes to be 39,491.2 km. When one compares this with the known circumference of 40,075 km, the error is – 1.47%. The error is defined as the deviation of the proposed measure from the actual measure expressed in percentage of the proposed measure. This is indeed an excellent match, and a great tribute to the Indian astronomers who determined this number.

Let us understand the second value of 3300 *yojanams* for the circumference of the earth. In this case, the *yojayam* is approximately about 12 km. This is close to the *yojanam* of 11.8474 km, based on the use of K-hasta (Table 1). The earth's circumference is determined to be 39,096.4 km, with *angulam* taken as 1.763 cm and *yojanam* as defined by the K-hasta. The deviation from the exact circumference is –2.50%. Again, we notice that this gives an excellent match with the known value of circumference, which again goes to prove the accuracy of the calculations of the Indian astronomers.

The above analysis suggests that the astronomers who estimated the earth's circumference as approximately 3300 *yojayams* have utilized the measure based on the 42-angulam *hasta* (i.e. K-hasta), while those who estimated it to be 5000 *yojanams* have used the measure based on the 28-angulam *hasta* (i.e. C-hasta). The *angulam* remained the same at 1.763 cm.

The implicit assumption in the above argument is that the early Indian astronomers were able to measure the earth's circumference fairly precisely, since the actual value of the earth's circumference was used to work out the value of the *yojanam* that was in use. Recent statistical analyses of Aryabhata's tables of planetary longitudes[15–17] have clearly established the existence of a rich tradition of accurate observational astronomy in the Siddhantic period. Therefore, a precise measure of the earth's circumference would therefore have been well within the abilities of the savants of the time.

The confusion from astronomical texts is primarily caused because one does not know the precise length of the basic unit, based on which the measures are described. An example here will illustrate this point. In verse 7 of section I (*Gitikapada*) of *Aryabhatiya*, the diameter of the earth is mentioned as 1050 *yojanams*[18]. The circumference is to be obtained using the value 62,832/20,000 = 3.1416 for the ratio between the circumference and the diameter, as defined in verse 10 of *Ganitapada*. The circumference of the earth is, therefore, obtained as 3298.68 *yojanams* (which is quite close to the value of 3300 *yojanams* used in this article and has been explained based on the use of the K-hasta). The only clue to the measure of the *yojanam* that *Aryabhatiya* gives (in the same verse 7 of the *Gitikapada*) is that one *yojanam* is equal to 8000 measures of *nara*, the height of an average person. Using the value of *yojanam* as 11.847 km, according to the present article, the value of the *nara* can be estimated as 1.48 m. The problem has to be analysed further by searching out clues for the length of the *nara* defined in *Aryabhatiya*. This is beyond the scope of the present article.

At this juncture, it is important to note that the exact details available in the Sanskrit astronomical texts are not being specifically referred to. This is for the simple reason that differences will be found in the reasoning and in the measures that are specified. As pointed out earlier, literary evidences apart from the *KA* will not be critically analysed here. Much of the existing confusion is apparently due to different interpretations based on various textual sources. Therefore, the constant *angulam* (proved by material evidences) has been used to present a coherent argument by which the two measures of 3300 and 5000 *yojanams* for the earth's circumference can be reconciled satisfactorily. This is the novelty of the present work and there is no need to interpret the present results in terms of quotes from different historical texts.

The above proof has cleared the confusion regarding the two values for the estimate of the earth's circumference in traditional Indian astronomical texts. It is now clear that the two estimates resulted due to the kind of *hasta* that was used in defining the larger measurement units of the *yojanam*. There are several other major conclusions which are immediately obvious based on the above analysis. The first major conclusion is that the *angulam* remained the same. The second major conclusion is that the value of the *angulam* of 1.763 cm can explain the circumference of the earth with accuracy. This is indeed significant because this value of the *angulam* was derived from archaeological evidences without any a priori assumptions[7,8]. Another conclusion is that the *goruta* defined as 1000 *dandas* provided logical interpretation of data. More importantly, the present article has provided a new independent way of understanding historical aspects of metrology that is not dependent on manuscript readings.

A significant outcome of the present study is that the dimensions noted in structures of the Indian subcontinent must be in multiples of the five different kinds of basic *hastas*, namely 42.312, 49.364, 56.416, 74.046 and 95.202 cm. Analysis of dimensions of earliest human engineering caves (rock cut) at Barabar and Nagarjuni hills dated to the time of Ashoka (272–232 BC) has verified the use of 42.312 cm standard[9], while analysis of dimensions of the Delhi Iron Pillar dated to the time of Chandragupta II Vikramaditya (AD 375–414) has verified the use of the standard[10] of 95.202 cm.

Incidentally, the same multiples can also be used in the analysis of Islamic structures of the subcontinent. This specific issue will not be discussed here in detail. However, the above statement can be proved by the appearance of the same *angulam* and *hasta* units of *Arthasastra* in the *Ain-i-Akbari*. This will be of significance because, apart from establishing the continuity of the *angulam* over a longer period of Indian history, this will also validate the approach presented above to understand the earth's circumference.

Circumference of the earth in *Ain-i-Akbari*

It is of interest to understand how the traditional linear measures of the Indian sub-continent were viewed and stated in historical Islamic literature. The most critical and thorough analysis of Mughal linear and area measures is by Habib[19]. Attention here will be specifically focused on the *Ain-i-Akbari*, because the specific connection between the traditional unit of measurement and the unit of measure standardized during Akbar's period has been stated, and this is with particular reference to the circumference of the earth. Surprisingly, Habib[19] has completely neglected this important piece of evidence while establishing the relationship between various measurement units in use during the Mughal period. The analysis by Habib, however, clearly establishes the widespread use of Akbar's measurement units, especially the area units, up to the pre-modern period in Indian history, till the adoption of the British units of measurement.

The *Ain-i-Akbari* is a significant document because, like the *Arthasastra* it gives a fairly detailed account of the state of affairs, especially in matters related to administration, of the Mughal empire in its heydays under the rule of Akbar (AD 1555–1605). Of equal significance is the continuation of Akbar's measures till the adoption of the British units[3,19]. There is a period of almost 1900 years separating the *Arthasastra* (~300 BC) and the *Ain-i-Akbari* (AD ~1600). It will be of interest to decipher the *Arthasastra hasta* measures in the *Ain*, to further understand the constancy of the traditional *angulam* unit of 1.763 cm.

The system of measures is mentioned in *Ain*[13] and the relevant portions will be consulted to gain further insights. The circumference of the earth and the linear measures are also discussed in the book[13].

The *Ain* specifically mentions that there were two values of the circumference of the earth and the number depended on 'old' school and 'new' school of geographers (astronomers).

The *Ain* mentions that according to the 'old' school, the earth's circumference was 8000 *farsakh*, with each *farsakh* being equal to 3 *koz*. Further, *Ain* specifies that each *koz* contains 4000 *gaz* of 24 'old digits' or 3000 gaz of 32 'old digits'. It is clear that the multiplication of 4000 by 24 will yield the same number as the multiplication of 3000 by 32, which the *Ain* also notes. We shall therefore stick to the former (i.e. 4000 *gaz* of 24 'old digits') for further analysis. The *Ain* mentions the term 'digit' (specifically, *angusht*) for the division units, but we have specifically called it the 'old digit' to differentiate it from the 'new digit' of the new school.

Therefore, according to the *Ain*, the 'old' school of astronomers obtained the value of the circumference as 8000 ×

$3 \times 4000 \times 24$ 'old digits'. Utilizing the known value of the circumference of the earth (40,075 km), the old digit can be calculated to be 1.739 cm. It is immediately obvious that this is remarkably close to the value of the traditional *angulam* of the Harappan Civilization[7,8]. The match must be considered excellent, with an error of only about –1.38% from the Harappan Civilization *angulam* of 1.763 cm. Since the old digit in the above formula for the circumference of the earth is the traditional Indian *angulam*, then the factor $24 \times$ old digit is precisely the P-hasta. Therefore, the *Ain* describes the circumference of the earth according to the old school in terms of the P-hasta of the *Arthasastra.* We can also recalculate the circumference as given in the *Ain* using the traditional *angulam* (i.e. old digit) taken as 1.763 cm. With this approach, the circumference will now be equal to 40619.52 km, which is away from the actual value by +1.34%.

The *Ain* also quotes that the earth's circumference according to the 'new' school is 6800 *farsakhs*. This 'new' school came about apparently because Akbar noticed that there was considerable confusion in the use of length measures in his empire and in order to systematize the measurement unit, he standardized it. Going by the same argument as before, the circumference will now be given by $6800 \times 3 \times 4000 \times 24 \times$ new digit. Assuming that the earth's circumference is 40,075 km, we can estimate this new digit to be 2.046 cm.

The physical significance of the relation between new digit and old digit can be understood. We can write the relationship for the earth's circumference as $6800 \times 3 \times 4000 \times 24 \times$ (old digit $\times$ 28/24), since the new digit is related to the old digit by the multiplication factor 28/24, namely $(28/24) \times 1.763$ cm = 2.057 cm (which is off by a low error margin of +0.54% from the value estimated from the circumference of the earth of 6800 *farsakhs*, namely 2.046 cm)[2]. This leads us to the conclusion that the new school determination utilizes the measure of 28 *angulam* C-hasta of the *Arthasastra*, but divides it into 24 equal parts, giving an apparent notion that the basic unit of measure had increased, whereas the basic measure (the *angulam*) was very much fixed at 1.763 cm. More important, this analysis proves the continuity of length measures from the time of Kautilya to Akbar, with the *angulam* remaining fixed at 1.763 cm.

As another example, the *Ain-i-Akbari* also states that an 'Akbar Shahi *gaz* of 46 fingers was used . . .'. It would be interesting to understand what this 46 finger (apparently new digit) measure is[2]. The 46 new digits will give us 46×2.057 cm = 94.622 cm. It is clear from Table 1 that this is the F-hasta of *Arthasastra* measuring 95.202 cm. The error is –1.06%.

The direct connection between the traditional *angulam* and the basic unit of measure used during Akbar's period has been established here. Its connection with the other units used during the Mughal Period can be derived, since the units of Akbar's period are related to those under other Mughal monarchs[19]. This has been done and the results are available elsewhere[20].

It should be possible to understand the dimensions of significant Islamic engineering structures of the Indian subcontinent using the measures defined in the *Arthasastra*. One of the well-known engineered structures of the Mughal Period is the Taj Mahal, located in the Taj Mahal Complex at Agra. It is one of the wonders of the modern world. Examination of the plan of the Taj Mahal Complex reveals that it was planned based on ordering of grids. It is indeed surprising that the first detailed scholastic examination of how the various dimensions of the Taj Complex might fit into a pattern was undertaken only in 1989. Using several accounts of the 17th century description of the Taj Mahal Complex in terms of the *gaz* or *zira* (Mughal linear measures), two different grid patterns have been proposed to account for the planned dimensions of the Complex. Begley and Desai[21] concluded that a 400-*gaz*, simple, fixed grid and its sub-divisions were used. In contrast, Barraud[22] utilized a generated grid system to explain the layout of the complex. He suggested that the Taj Mahal Complex was planned as a tripartite rectangle of three 374-*gaz* squares. Both these analyses, unfortunately, are not accurate and the various discrepancies have been simply listed as being due to errors in the contemporary descriptions, rounding-off errors, inaccuracies of reporting from third persons and errors in workmanship[21,22]. The apparently peculiar numbers for the dimensions (when expressed in terms of *gaz* or *zira*) were simply explained as being part of geometric understanding[21,22], whereas it is impossible to understand the use of illogical numbers when the dimensions are expressed as *gaz* units.

It should be possible to apply the results of the analysis presented in this article to the dimensions of the Taj Mahal Complex. Detailed analysis has revealed that a coherent pattern emerges when units described in the *Arthasastra* are used[23]. This is not the theme of the present article, but a simple example here will shed some insights.

The most detailed measurements of the Taj Mahal Complex were made by Hodgson and his assistants in 1825. Hodgson was the then Surveyor General of India. The main purpose of his study was to determine the precise length of the linear measure *gaz* that was in popular use at that time all over North India. The emerging British power had to fix land records so that income can be generated by taxing the lands under their control. After a detailed analysis of the measurements obtained by his team, Hodgson[24] stated that the square marble platform (on which the Taj Mahal mausoleum has been erected in a geometrically well-defined systematic plan) was the most convenient for measurement and simplest for explanation. The length of each side determined by Hodgson[24] was 9587.79 cm. The direct connection of this dimension with 100 times F-hasta units of the *KA* is evident from

Table 1. This 100 F-hasta measurement can also be represented as 50 *dhanus*, with each *dhanus* measuring 108 *angulams*. Therefore, it can be seen that the length of each side of the marble platform on which the Taj Mahal rests can be reconciled as 50 *dhanus* (i.e. $50 \times 108 \times 1.763$ cm = 9520.20 cm). This estimate is off from the measured value of the length of marble platform by –0.71%, low margin of error. In a similar manner, the entire dimensions of the Taj Mahal Complex and the Taj Mahal can be analysed[23] in terms of *Arthasastra* units, using a constant *angulam* of 1.763 cm. This implies continuity in the civil-engineering traditions of the Indian subcontinent, especially in matters related to metrology.

Conclusion

This study has shed further insights into the constancy of the basic unit of measurement the *angulam* through the ages of the Indian subcontinent. Utilizing the concept of a constant *angulam* of 1.763 cm and the metrological table defined in the *KA*, the confusion regarding the prevalence of two different measures for the circumference of the earth in Sanskrit astronomical texts has been resolved. The apparent difference is due to the type of *hasta* unit used for defining larger measures like *yojanam*. Additionally, the relationship between the traditional *angulam* and *hasta* measures of the *Arthasastra* and the measures in the *Ain-i-Akbari* has been explained by considering the same issue of circumference of the earth.

The connection of the *angulam* of the Harappan Civilization (~2000 BC) is therefore established with length measures of the Mughal Period (AD ~1600). This implies an unbroken tradition in the use of the *angulam* of 1.763 cm over a period of more than 3600 years in the Indian subcontinent.

Finally, the possible measurement units which can be used to analyse the dimensions of most engineered structures of the subcontinent through the ages are proposed to be 42.312, 49.364, 56.416, 74.046 and 95.202 cm, based on the five *basic* hasta units mentioned in the *Arthasastra*.

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