On the confirmation of the traditional unit of length measure in the estimates of circumference of the earth

R. Balasubramaniam
Department of Materials and Metallurgical Engineering, Indian Institute of Technology, Kanpur 208 016, India

The prevalence of two different measures for the circumference of the earth in Sanskrit astronomical texts has been resolved by considering the concept of a constant angulam of 1.763 cm and the metrological table defined in Kautilya’s Arthasastra. The traditional measures defined in the Arthasastra have been related to the measures defined in the Ain-i-Akbari, by considering the same issue of circumference of the earth. The study has established the connection of the traditional unit of measure, the angulam, of the Harappan Civilization (~2000 BC) with length measures of the Mughal period (AD ~1600), thereby confirming an unbroken tradition in the use of the angulam of 1.763 cm over a period of more than 3600 years in the Indian subcontinent. The possible measurement units which can be used to analyse the dimensions of most engineered structures of the subcontinent through the ages have been proposed and briefly confirmed by the dimensions of the marble platform of the Taj Mahal.

Keywords: Circumference, earth, length measures.

A comprehensive review of length measures in different parts of the Indian subcontinent was undertaken by Raju and Mainkar in the 1960s. They also analysed the measures assuming that the ‘measure of 50 cm length’ remained constant through the ages1-3. While this was useful in understanding the intermediate length measures, the interpretation of longer length measures posed severe problems. One notable example is the circumference of the earth. The Indian astronomical texts estimate the earth’s circumference as either 3300 yojanams or 5000 yojanams4. The important texts that provided the value of 3300 yojanams were Aryabhatiya, Panchsidantika, Mahabhashkariya, Shisyadhvirdhida, Vatesvarasiddhanta and Siddhantadarpan, while the texts Suryasiddhanta, Brahmasphutisiddhanta, Mahabhashkariya, Siddhantasekhara, Siddhantasiromani, Somasiddhanta, Shakalyovkta and Bramasiidh Antar estimate the circumference as 5000 yojanams. It is indeed surprising that a natural constant number like the circumference of the earth was given by two different values, but it is clear that both of them implied the same value.

There are strong evidences to prove that people of the Indian subcontinent were very precise when it came to length measures because of their importance in the measurement of agricultural land and hence collection of revenue. For example, an inscription from Brihadisvara Temple in Thanjavur mentions that land was measured to an accuracy of 1/52,428,800,000 of a veli (which is about 2.6780 ha) during the rule of the Chola monarch Rajaraja I in early 11th century AD5. Therefore, it is reasonable to suppose that the circumference of the earth must have been estimated precisely.

This major problem of two values of circumference of the earth in the Indian astronomical texts has not been satisfactorily resolved till date. Raju and Mainkar specifically mentioned that ‘considerable research is needed’ to solve this problem6. This problem will be revisited in this article utilizing the novel concept of the constancy of the traditional unit of measurement, namely the angulam. In addition, the two estimates of the circumference of the earth mentioned in the Ain-i-Akbari will be analysed to obtain new insights on the metrology of the Mughal period.

Constancy of angulam

The traditional unit of measurement, the angulam, has been conclusively proved to have been in use over a long period of over at least 2400 years in the subcontinent. Specifically, the angulam of unit 1.763 cm derived from plans of Harappan Civilization (2500–1600 BC) settlement sites, without any a priori assumptions7, was used to describe precisely the dimensions of the first engineered caves at the Barabar and Nagarjuni hills of the Mauryan Period (late 4th to early 2nd century BC)8 and the Delhi Iron Pillar of the Gupta Period (AD 320–600)9. The constancy of angulam in later periods of Indian history will be explored by considering its association with the estimate of the earth’s circumference in Indian texts.

The table of measures in Kautilya’s Arthasastra (KA) will be the main focus as regards literary evidence, as this is the foundation of metrology of the Indian subconti-
RESEARCH ARTICLE

While references to certain names of some length units appear in the Vedic literature, their relationship has not been mentioned. It is K.A. that first describes the coherent system of length and area measures of the Indian subcontinent that was extant around the 3rd century BC. At the outset, it should be emphasized that literary evidences can be confusing because of the availability of different versions of the same text, apart from possible differences in interpretation and meaning of words. For example, even in the case of the K.A., Shamasrastry11, who discovered the text in Mysore, mentions that a larger unit called goruta was equal to 1000 dhanus, while the edition by Kangle12 mentions the goruta as 2000 dhanus. The difference is not due to interpretation alone, but due to readings based on different manuscripts. The current work has shown that a better match was obtained using Shamasrastry’s proposed value of a yojanam being equal to 1000 goruta rather than 2000 goruta. This does not imply that either one of the two readings has to be taken as final because the current approach has proposed a new original way of looking at the problem, independent of the manuscript readings. There is no ready solution to explain this discrepancy and this is left for other scholars to debate. However, it may be noted here that there is need to take recourse to scientific analysis of material evidences, which are far superior because the conclusions derived using them have firm support. In fact, in several cases, material evidences may help ease the confusion created by literary evidences.

Shamasrastry11 and Kangle12 mention the division of scales in their respective editions. While the length measures at the smaller level are not relevant here because they originate from atomistic levels and are quite undefined, it is relevant to focus attention on intermediate (measures above the angulam) and larger measures. For example, the K.A. mentions that 12 angulams made one vitasti and further that two vitastis are equal to one aratni, which was also called a hasta (≈ 24 angulams). There are several other units defined in the K.A. Raju and Mainkar rightly pointed out that the K.A was also responsible for laying the ‘seeds of later confusion’ in the interpretation of lengths and area units, because it specifies different measures with the same name. For example, there are three measures for hasta (of 24, 28 and 54 angulams), two for kishku (of 32 and 42 angulams), two for danda (of 96 and 192 angulams), two for dhanus (of 96 and 108 angulams) and two for paurusha (of 96 and 108 angulams).

In order to analyse the length measures of the subcontinent through the ages, Raju and Mainkar1−5 assumed that the value of the angulam was changed with times in order to keep up with the different values of hastas that were in popular use. In particular, they assumed that the hasta of 28 angulams was unchanged throughout history at a value of 50 cm and therefore, went about analysing the other measures to fit their theory. It is not surprising that a change in the angulam unit was inferred because of this assumption. The procedure adopted by Raju and Mainkar is not precise because there is no supporting archaeological evidence for assuming the constancy of the 50-cm measure through the ages, unlike several (dated) archaeological material evidences9,10 offering strong support for the constancy of the angulam of 1.763 cm.

The table of measures in the K.A needs to be carefully analysed. For the sake of convenience, the different hastas in the K.A will be defined so that there is no confusion. The hasta of 24 angulams will be called the P-hasta (P implying prajapatya as mentioned in the K.A), the hasta of 28 angulams will be called the C-hasta (C standing for ‘commercial’, as the K.A. does not mention any unique name for this measure) and the hasta of 54 angulams will be called the F-hasta (F standing for ‘forest’, as the K.A mentions that the hasta of this measure was used for measuring forests). In addition, we define a hasta of 42 angulams called the K-hasta (K implying kishku hasta) and a hasta of 32 angulams called the M-hasta (M implying kamsa hasta), because these measures appear in the K.A.1,12 and were used in later periods in Indian history.1−5

Considering the dimensions of the angulam to be unchanged at 1.763 cm, the dimensions of these five kinds of hastas can be precisely determined. This statement may appear trivial, but it is significant. Much of the confusion in later times was caused by different schemes of dividing the different hasta units. For example, the length of the C-hasta was divided into 24 divisions and 41 of these divided ‘units’ defined the Ilahi gaz in the Ain-i-Akbari.13 This procedure will result in the apparent notion that the basic unit of measure did change with time, but this is really not the case. This will become clear later when the circumference of the earth will be estimated based on two values given in the Ain13.

Utilizing a constant angulam of 1.763 cm, the basic measurements of the five types of hastas are 42.312 cm for P-hasta, 49.364 cm for C-hasta, 95.202 cm for F-hasta, 74.046 cm for K-hasta and 56.416 cm for the M-hasta. The larger measurement units defined in the K.A, like the danda, goruta and yojanam, are all based on different multiples of the hasta. We shall focus attention on the five types of hastas defined above and estimate dimensions of the larger units for each of these hasta.

The danda is defined as four aratinis in the K.A. Since the aratni was also called hasta in the K.A., it is reasonable to multiply the hasta by four to arrive at the danda. The name danda is commonly used to define the measure that was four times the hasta, in later texts on metrology of the subcontinent. We also note, for the sake of interest, that the K.A mentions four names for the same measure of ‘four times the hasta’ (namely danda, dhanus, nalika and paurusha) and the possible origin for these names may be related to the way in which a particular hasta was used in the multiplication.

Next, the goruta is equal to 1000 dandas (dhanus) is mentioned in the K.A, but taken here as danda due to the
Table 1. Values in modern centimetres for the various larger measures defined in the Arthasastra using five different Hastas

<table>
<thead>
<tr>
<th>Measure</th>
<th>Multiple of hastas</th>
<th>P-hasta (24 A)</th>
<th>C-hasta (28 A)</th>
<th>F-hasta (54 A)</th>
<th>K-hasta (42 A)</th>
<th>M-hasta (32 A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hasta</td>
<td>1</td>
<td>42.312</td>
<td>49.364</td>
<td>95.202</td>
<td>74.046</td>
<td>56.416</td>
</tr>
<tr>
<td>Danda</td>
<td>4</td>
<td>169.25</td>
<td>197.26</td>
<td>380.81</td>
<td>296.184</td>
<td>225.664</td>
</tr>
<tr>
<td>Goruta</td>
<td>4000</td>
<td>169.248.00</td>
<td>197.456.00</td>
<td>380.808.00</td>
<td>296.184.00</td>
<td>225.664.00</td>
</tr>
<tr>
<td>Yojanam</td>
<td>16000</td>
<td>676.992.00</td>
<td>789.824.00</td>
<td>1,523.232.00</td>
<td>1,184.736.00</td>
<td>902.656.00</td>
</tr>
<tr>
<td></td>
<td>(6.77 km)</td>
<td>(7.90 km)</td>
<td></td>
<td>(15.23 km)</td>
<td>(11.85 km)</td>
<td>(9.03 km)</td>
</tr>
</tbody>
</table>

A. Angulam of 1.763 cm length.

use of this common term in later metrological texts\(^5\) and further, four gorutas make up one yojanam. These values in terms of the modern metric unit of centimetres are listed in Table 1. This table is fundamental in the understanding of measures of the Indian subcontinent through the ages. The important point to note is that the goruta has been taken as 1000 dandas, which was the view expounded by Shamasasrty\(^1\). Kangle\(^17\) suggested the unit of 2000 dandas to the goruta. The approach using the value of 1000 danda to the goruta provided a coherent understanding and analysis of the data.

At this juncture, we briefly digress to note that the garhapatya dhanus, which was also used for measurement according to the KA, contains 108 angulams\(^1,12\). This has not been considered separately but it is easy to note that this is a simple double multiple of the F-hasta. We will, therefore, not add the garhapatya dhanus (\(\approx 190.4\) cm) as a separate multiple, but note here that it offers a coherent explanation of the dimensions of several Harappan Civilization settlement sites\(^2,8\) and the dimensions of the Delhi Iron Pillar\(^10\). Another ‘double’ measure that appears to have been popular through the ages is the measure of twice the P-hasta (Table 1). The earth’s circumference is determined to be 39,096.4 km, with angulam taken as 1.763 cm and yojanam as defined by the K-hasta. The deviation from the exact circumference is –2.50%. Again, we notice that this gives an excellent match with the known value of circumference, which again goes to prove the accuracy of the calculations of the Indian astronomers.

The above analysis suggests that the astronomers who estimated the earth’s circumference as approximately 3300 yojayams have utilized the measure based on the 42-angulam hasta (i.e. K-hasta), while those who estimated it to be 5000 yojanams have used the measure based on the 28-angulam hasta (i.e. C-hasta). The angulam remained the same at 1.763 cm.

The implicit assumption in the above argument is that the early Indian astronomers were able to measure the earth’s circumference fairly precisely, since the actual value of the earth’s circumference was used to work out the value of the yojanam that was in use. Recent statistical analyses of Aryabhata’s tables of planetary longitudes\(^15\)–\(^17\) have clearly established the existence of a rich tradition of accurate observational astronomy in the Siddhantic period. Therefore, a precise measure of the earth’s circumference would therefore have been well within the abilities of the sants of the time.

The confusion from astronomical texts is primarily caused because one does not know the precise length of the basic unit, based on which the measures are described. An example here will illustrate this point. In verse 7 of section I (Gitiyapada) of Aryabhatiya, the diameter of the earth is mentioned as 1050 yojanams\(^18\). The circumference is to be obtained using the value 62,832/20,000 = 3,1416 for the ratio between the circumference and the diameter, as defined in verse 10 of Ganitapada. The circumference of the earth is, therefore, obtained as 3298.68 yojanams (which is quite close to the value of 3300 yojanams used in this article and has been explained based on the use of the K-hasta). The only clue to the measure of the yojanam that Aryabhatiya gives (in the same verse 7 of the Gitiyapada) is that one yojanam is equal to 8000 measures of nara, the height of an average person. Using the value of yojanam as 11.847 km, according to the present article, the value of the nara can be estimated as 1.48 m. The problem has to be analysed further by searching out clues for the length of the nara defined
in Aryabhata. This is beyond the scope of the present article.

At this juncture, it is important to note that the exact details available in the Sanskrit astronomical texts are not being specifically referred to. This is for the simple reason that differences will be found in the reasoning and in the measures that are specified. As pointed out earlier, literary evidences apart from the KA will not be critically analysed here. Much of the existing confusion is apparently due to different interpretations based on various textual sources. Therefore, the constant angulam (proven by material evidences) has been used to present a coherent argument by which the two measures of 3300 and 5000 yojanams for the earth’s circumference can be reconciled satisfactorily. This is the novelty of the present work and there is no need to interpret the present results in terms of quotes from different historical texts.

The above proof has cleared the confusion regarding the two values for the estimate of the earth’s circumference in traditional Indian astronomical texts. It is now clear that the two estimates resulted due to the kind of hasta that was used in defining the larger measurement units of the yojanam. There are several other major conclusions which are immediately obvious based on the above analysis. The first major conclusion is that the angulam remained the same. The second major conclusion is that the value of the angulam of 1.763 cm can explain the circumference of the earth with accuracy. This is indeed significant because this value of the angulam was derived from archaeological evidences without any a priori assumptions. Another conclusion is that the goruta defined as 1000 dandas provided logical interpretation of data. More importantly, the present article has provided a new independent way of understanding historical aspects of metrology that is not dependent on manuscript readings.

A significant outcome of the present study is that the dimensions noted in structures of the Indian subcontinent must be in multiples of the five different kinds of basic hastas, namely 42.312, 49.364, 56.416, 74.046 and 95.202 cm. Analysis of dimensions of earliest human engineering caves (rock cut) at Barabar and Nagarjun hills dated to the time of Ashoka (272–232 BC) has verified the use of 42.312 cm standard, while analysis of dimensions of the Delhi Iron Pillar dated to the time of Chandragupta II Vikramaditya (AD 375–414) has verified the use of the standard of 95.202 cm.

Incidentally, the same multiples can also be used in the analysis of Islamic structures of the subcontinent. This specific issue will not be discussed here in detail. However, the above statement can be proved by the appearance of the same angulam and hasta units of Arthasastra in the Ain-i-Akbari. This will be of significance because, apart from establishing the continuity of the angulam over a longer period of Indian history, this will also validate the approach presented above to understand the earth’s circumference.

Circumference of the earth in Ain-i-Akbari

It is of interest to understand how the traditional linear measures of the Indian sub-continent were viewed and stated in historical Islamic literature. The most critical and thorough analysis of Mughal linear and area measures is by Habib. Attention here will be specifically focused on the Ain-i-Akbari, because the specific connection between the traditional unit of measurement and the unit of measure standardized during Akbar’s period has been stated, and this is with particular reference to the circumference of the earth. Surprisingly, Habib has completely neglected this important piece of evidence while establishing the relationship between various measurement units in use during the Mughal period. The analysis by Habib, however, clearly establishes the widespread use of Akbar’s measurement units, especially the area units, up to the pre-modern period in Indian history, till the adoption of the British units of measurement.

The Ain-i-Akbari is a significant document because, like the Arthasastra it gives a fairly detailed account of the state of affairs, especially in matters related to administration, of the Mughal empire in its heydays under the rule of Akbar (AD 1555–1605). Of equal significance is the continuation of Akbar’s measures till the adoption of the British units. There is a period of almost 1900 years separating the Arthasastra (~300 BC) and the Ain-i-Akbari (AD ~1600). It will be of interest to decipher the Arthasastra hasta measures in the Ain, to further understand the constancy of the traditional angulam unit of 1.763 cm.

The system of measures is mentioned in Ain and the relevant portions will be consulted to gain further insights. The circumference of the earth and the linear measures are also discussed in the book.

The Ain specifically mentions that there were two values of the circumference of the earth and the number depended on ‘old’ school and ‘new’ school of geographers (astronomers).

The Ain mentions that according to the ‘old’ school, the earth’s circumference was 8000 farsakh, with each farsakh being equal to 3 koz. Further, Ain specifies that each koz contains 4000 gaz of 24 ‘old digits’ or 3000 gaz of 32 ‘old digits’. It is clear that the multiplication of 4000 by 24 will yield the same number as the multiplication of 3000 by 32, which the Ain also notes. We shall therefore stick to the former (i.e. 4000 gaz of 24 ‘old digits’) for further analysis. The Ain mentions the term ‘digit’ (specifically, angushth) for the division units, but we have specifically called it the ‘old digit’ to differentiate it from the ‘new digit’ of the new school.

Therefore, according to the Ain, the ‘old’ school of astronomers obtained the value of the circumference as 8000 ×
3 x 4000 x 24 ‘old digits’. Utilizing the known value of the circumference of the earth (40,075 km), the old digit can be calculated to be 1.739 cm. It is immediately obvious that this is remarkably close to the value of the traditional angulam of the Harappan Civilization. The match must be considered excellent, with an error of only about 1.38% from the Harappan Civilization angulam of 1.763 cm. Since the old digit in the above formula for the circumference of the earth is the traditional Indian angulam, then the factor 24 x old digit is precisely the P-hasta. Therefore, the Ain describes the circumference of the earth according to the old school in terms of the P-hasta of the Arthasastra. We can also recalculate the circumference as given in the Ain using the traditional angulam (i.e. old digit) taken as 1.763 cm. With this approach, the circumference will now be equal to 40619.52 km, which is away from the actual value by +1.34%.

The Ain also quotes that the earth’s circumference according to the ‘new’ school is 6800 farsakh. This ‘new’ school came about apparently because Akbar noticed that there was considerable confusion in the use of length measures in his empire and in order to systematize the measurement unit, he standardized it. Going by the same argument as before, the circumference will now be given by 6800 x 3 x 4000 x 24 x new digit. Assuming that the earth’s circumference is 40,075 km, we can estimate this new digit to be 2.046 cm.

The physical significance of the relation between new digit and old digit can be understood. We can write the relationship for the earth’s circumference as 6800 x 3 x 4000 x 24 x (old digit x 28/24), since the new digit is related to the old digit by the multiplication factor 28/24, namely (28/24) x 1.763 cm = 2.057 cm (which is off by a low error margin of 0.54% from the value estimated from the circumference of the earth of 6800 farsakh, namely 2.046 cm). This leads us to the conclusion that the new school determination utilizes the measure of 28 angulam C-hasta of the Arthasastra, but divides it into 24 equal parts, giving an apparent notion that the basic unit of measure had increased, whereas the basic measure (the angulam) was very much fixed at 1.763 cm. More important, this analysis proves the continuity of length measures from the time of Kautilya to Akbar, with the angulam remaining fixed at 1.763 cm.

As another example, the Ain-i-Akbari also states that an ‘Akbar Shahi gaz of 46 fingers was used...’. It would be interesting to understand what this 46 finger (apparently new digit) measure is. The 46 new digits will give us 46 x 2.057 cm = 94.622 cm. It is clear from Table 1 that this is the F-hasta of Arthasastra measuring 95.202 cm. The error is ~1.06%.

The direct connection between the traditional angulam and the basic unit of measure used during Akbar’s period has been established here. Its connection with the other units used during the Mughal Period can be derived, since the units of Akbar’s period are related to those under other Mughal monarchs. This has been done and the results are available elsewhere.

It should be possible to understand the dimensions of significant Islamic engineering structures of the Indian subcontinent using the measures defined in the Arthasastra. One of the well-known engineered structures of the Mughal Period is the Taj Mahal, located in the Taj Mahal Complex at Agra. It is one of the wonders of the modern world. Examination of the plan of the Taj Mahal Complex reveals that it was planned based on ordering of grids. It is indeed surprising that the first detailed scholastic examination of how the various dimensions of the Taj Complex might fit into a pattern was undertaken only in 1989. Using several accounts of the 17th century description of the Taj Mahal Complex in terms of the gaz or zira (Mughal linear measures), two different grid patterns have been proposed to account for the planned dimensions of the Complex. Begley and Desai concluded that a 400-gaz, simple, fixed grid and its sub-divisions were used. In contrast, Barraud utilized a generated grid system to explain the layout of the complex. He suggested that the Taj Mahal Complex was planned as a tripartite rectangle of 374-gaz squares. Both these analyses, unfortunately, are not accurate and the various discrepancies have been simply listed as being due to errors in the contemporary descriptions, rounding-off errors, inaccuracies of reporting from third persons and errors in workmanship. The apparently peculiar numbers for the dimensions (when expressed in terms of gaz or zira) were simply explained as being part of geometric understanding, whereas it is impossible to understand the use of illogical numbers when the dimensions are expressed as gaz units.

It should be possible to apply the results of the analysis presented in this article to the dimensions of the Taj Mahal Complex. Detailed analysis has revealed that a coherent pattern emerges when units described in the Arthasastra are used. This is not the theme of the present article, but a simple example here will shed some insights.

The most detailed measurements of the Taj Mahal Complex were made by Hodgson and his assistants in 1825. Hodgson was the then Surveyor General of India. The main purpose of his study was to determine the precise length of the linear measure gaz that was in popular use at that time all over North India. The emerging British power had to fix land records so that income can be generated by taxing the lands under their control. After a detailed analysis of the measurements obtained by his team, Hodgson stated that the square marble platform (on which the Taj Mahal mausoleum has been erected in a geometrically well-defined systematic plan) was the most convenient for measurement and simplest for explanation. The length of each side determined by Hodgson was 9587.79 cm. The direct connection of this dimension with 100 times F-hasta units of the KA is evident from
Table 1. This 100 F-hasta measurement can also be represented as 50 dhanus, with each dhanus measuring 108 angulams. Therefore, it can be seen that the length of each side of the marble platform on which the Taj Mahal rests can be reconciled as 50 dhanus (i.e., 50 x 108 x 1.763 cm = 9520.20 cm). This estimate is off from the measured value of the length of marble platform by -0.71%, low margin of error. In a similar manner, the entire dimensions of the Taj Mahal Complex and the Taj Mahal can be analyzed in terms of Arthasastra units, using a constant angulam of 1.763 cm. This implies continuity in the civil-engineering traditions of the Indian subcontinent, especially in matters related to metrology.

Conclusion

This study has shed further insights into the constancy of the basic unit of measurement the angulam through the ages of the Indian subcontinent. Utilizing the concept of a constant angulam of 1.763 cm and the metrological table defined in the KA, the conclusion regarding the prevalence of two different measures for the circumference of the earth in Sanskrit astronomical texts has been resolved. The apparent difference is due to the type of hasta unit used for defining larger measures like yojanam. Additionally, the relationship between the traditional angulam and hasta measures of the Arthasastra and the measures in the Ain-i-Akhbar has been explained by considering the same issue of circumference of the earth.

The connection of the angulam of the Harappan Civilization (~2000 BC) is therefore established with length measures of the Mughal Period (AD ~1600). This implies an unbroken tradition in the use of the angulam of 1.763 cm over a period of more than 3600 years in the Indian subcontinent.

Finally, the possible measurement units which can be used to analyse the dimensions of most engineered structures of the subcontinent through the ages are proposed to be 42.312, 49.364, 56.416, 74.046 and 95.202 cm, based on the five basic hasta units mentioned in the Arthasastra.


ACKNOWLEDGEMENTS. I thank Michel Danino for his critical comments and observations on the manuscript in particular, and on matters related to metrology of the Indian subcontinent through the ages, in general. I acknowledge valuable inputs received from M.D. Srinivas on Arvabhatiya and Najaf Haidar on Ain-i-Akbari.

Received 25 August 2008; revised accepted 26 December 2008

552 CURRENT SCIENCE, VOL. 96, NO. 4, 25 FEBRUARY 2009