Self-distortion of a short laser pulse in a plasma

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Self-distortion of a high-intensity laser pulse due to relativistic effects propagating in a high-density plasma is investigated. As the pulse energy increases, its front portion expands and tail portion get compressed. The effect of nonlinearity outweighs the effect of dispersion.

Keywords: Laser, plasma, relativistic effects, self-distortion.

LASER–PLASMA interaction is an area of significant scientific activities since the past several decades due to wide-ranging applications in materials processing, laser-induced fusion, harmonic generation, etc. Various nonlinear effects have been observed, viz. parametric instabilities, self-focusing, harmonic generation, phase modulation, etc. In a moderate-power, long-pulse laser–plasma interaction, the major source of nonlinearities is thermal (collisional) and ponderomotive effects. The development of short-pulse lasers has made possible the attainment of terawatt powers in a short duration. This has opened up a new parameter regime for nonlinear effects with some novel applications in pair production, particle acceleration, etc. In this parameter regime the major source of nonlinearity arises due to relativistic effect, which also affect the laser pulse propagation dynamics, as the laser beam propagates. The laser pulse expands both spatially and temporally as it propagates through the plasma due to diffraction and dispersion effects, and as a consequence, the pulse gets severely distorted. Esrey et al. and Sprangle et al. have given elegant reviews of self-focusing and guiding of short laser pulses in ionized gases and plasmas, in the paraxial ray approximation. Upadhay et al. have developed a formalism for asymmetric self-focusing of a laser pulse in the plasma. Sharma et al. have studied the effects of all three nonlinearities, viz. collisional, ponderomotive, and relativistic on the propagation dynamics of the laser beam in a plasma. Faure et al. have performed an experiment using 10 Hz, 20 TW, 820 nm, 600 mJ, 35 fs, 10¹⁹ W/cm² laser in under-dense plasma and studied the effects of pulse duration on self-focusing of ultra-short lasers. Their observations showed that a laser will get self-distorted if its pulse duration is short compared to the plasma period. Li and Crowell have provided the formalism for shortening of a laser pulse with self-modulated phase at the focus of a lens. Shin et al. have controlled the pulse dispersion of laser in an erbium-doped fibre amplifier by controlling input pulse width, pump power and background-to-pulse power ratio. Rostami and Matlbou have analytically investigated and simulated optical pulse distortion and propagation through quasi-periodic structures, generally and specifically for the Fibonacci class.

In this communication, we address the problem of nonlinear relativistic interaction of an ultrashort pulse with a cold plasma following the technique employed by Akhmanov et al. to solve the nonlinear wave equation. The plasma is considered cold, where the electron quiver velocity in the pulse is larger than the electron thermal velocity. The pulse is assumed to be one-dimensional, circularly polarized and Gaussian, both spatially and temporally.

Consider the propagation of a laser through a plasma of electron density n0. The electric field of the laser can be written as

$$\vec{E} = (\hat{x} + i\hat{y})E_0 \exp[-i(\omega t - kz)],$$

where $\omega$ is the angular frequency of the laser and k the wavenumber. On solving the equation of motion $m(d\vec{v}/dt) = -e\vec{E} - (e/c)\vec{v} \times \vec{B}$, the velocity of electron $\vec{v}$ is

$$\vec{v} = \frac{e\vec{E}}{m\omega},$$

where $\vec{B}$ is the magnetic field of the laser, e the velocity of light in vacuum and $-e$ and m are the electronic charge and mass respectively.

The current density $\vec{J}$ is given by

$$\vec{J} = -n_e e\vec{v}.$$

Using eqs (1)–(3) in the wave equation governing the propagation of laser beams in a plasma $\nabla^2 \vec{E} - (\omega c^2)\vec{E}/\omega^2 = (4\pi/e^2)(\partial^2 \vec{E} / \partial t^2)$ we obtain,

$$c^2 k^2 \vec{E} = (\omega^2 - \omega^2_b)\vec{E},$$

where $\omega_b = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency.

We now consider the laser amplitude $E_0$ to be slowly varying as $E_0 = E_0(z, t)$. The consequence of this can be incorporated in the wave equation on replacing $\omega \rightarrow i\omega / \omega^2$ and $k \rightarrow -i\omega / \omega^2$ in eq. (4). Thus we obtain

$$2i\omega c^2 \frac{\partial \vec{E}_0}{\partial z} + 2i\omega \frac{\partial \vec{E}_0}{\partial t} = -\frac{\partial^2 \vec{E}_0}{\partial t^2} + \frac{\partial^2 \vec{E}_0}{\partial z^2} + c^2 \frac{\partial^2 \vec{E}_0}{\partial z^2} - \Delta^2 \vec{E}_0 = 0.$$

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In writing eq. (5) we have assumed
\[ \omega_p^2 = \omega_{p0}^2 + \Delta^2, \]

in compliance with \( m \equiv m_0[1 + (1/2)[\nu^2/c^2]] \), where \( \omega_{p0} \) is the unperturbed plasma frequency, \( \Delta \) the perturbation given by \( \Delta^2 = (\omega_{p0}^2/2)[\nu^2/c^2] \) and \( m_0 \) is the electron rest mass.

To the first-order perturbation we get
\[ \frac{\partial E_0}{\partial z} = -\left( \frac{\omega}{k c^2} \right) \frac{\partial E_0}{\partial t}, \]
\[ \frac{\partial^2 E_0}{\partial z^2} = \left( \frac{\omega^2}{k^2 c^4} \right) \frac{\partial^2 E_0}{\partial t^2}. \]

Using above equations in eq. (5) we get
\[ \frac{d}{dt} \left( \frac{\partial}{\partial z} + v_p \frac{\partial}{\partial z} \right) E_0 + \frac{1}{2\omega} \frac{\partial^2 E_0}{\partial z^2} \left( c^2 - v_p^2 \right) - \frac{\Delta^2}{2\omega} E_0 = 0, \]

where \( v_p \) is the linear group velocity given by \( v_p = k c^2/\omega \).

Equation (7) is the wave envelope equation. In the group velocity frame, eq. (7) can be rewritten as
\[ \frac{\partial^2 \tilde{E}_0}{\partial \tau^2} + \left( \frac{c^2 - v_p^2}{2\omega} \right) \frac{\partial^2 \tilde{E}_0}{\partial \tau^2} - \frac{\Delta^2}{2\omega} \tilde{E}_0 = 0, \]

where \( \tilde{E}_0 = \tilde{A}_0(\tau', z') \exp[iS(\tau', z')] \),

where \( \tau' = \tau - z/v_p t \). Following Akhmanov \textit{et al.} \cite{14}, the solution of eq. (8) may be written as
\[ \tilde{E}_0 = \tilde{A}_0(\tau', z') \exp[\alpha S(\tau', z')], \]

where \( \alpha \) and \( S \) are the eikonal.

Using eq. (9) in eq. (8) and after some algebraic manipulations we obtain the coupled equations for wave amplitude \( \tilde{A}_0 \) and eikonal \( \tilde{S} \)
\[ \frac{\partial \tilde{A}_0^2}{\partial \tau} + \left( \frac{c^2 - v_p^2}{\omega} \right) \frac{\partial \tilde{A}_0^2}{\partial \tau} + \left( \frac{c^2 - v_p^2}{2\omega} \right) \tilde{S}^2 + \frac{\Delta^2}{2\omega} \tilde{A}_0^2 = 0, \]

and
\[ \frac{\partial \tilde{S}}{\partial \tau} - \frac{1}{\tilde{A}_0} \left( \frac{c^2 - v_p^2}{2\omega} \right) \frac{\partial^2 \tilde{A}_0}{\partial \tau^2} + \left( \frac{c^2 - v_p^2}{2\omega} \right) \frac{\partial \tilde{S}}{\partial \tau} + \frac{\Delta^2}{2\omega} \tilde{S}^2 = 0. \]

We solve eqs (10) and (11) for a Gaussian pulse
\[ A(t) = A_0 \exp(-t^2/\tau^2), \]
where \( \tau \) is the initial pulse width parameter. We express \( S \) and \( A_0^2 \) as
\[ S = \beta(t') \xi^2/2 + \phi(t'), \]
and
\[ A_0^2 = \frac{A_0^2}{f(t')} \exp \left[ -\frac{z^2}{z_0^2 f(t')} \right]. \]

Using these values of \( S \) and \( A_0^2 \) in eqs (9) and (10), \( \beta \) can be evaluated as
\[ \beta = \frac{\omega}{c^2 - v_p^2} \frac{1}{f(t')} \frac{df}{dt'}, \]

Equations (11) and (13) may be used to obtain a differential equation for \( f \) as
\[ \frac{d^2 f}{dt^2} = \frac{1}{f^3} - \frac{\alpha}{f^2}, \]

where \( \tau = \sqrt{k_1/K_1}, \alpha = K_2/K_1, K_2 = [(c^2 - v_p^2)/\omega^2 z_0^4] \) and \( K_1 = [(c^2 - v_p^2)/\omega^2] v_p^2 \sqrt{2\omega^2 c^2 z_0^2} \).

Solving eq. (14) using the initial conditions at \( \tau = 0, f = 1 \) we get
\[ \tau = \sqrt{c f^2 + 2\alpha f - 1} + \frac{1}{\sqrt{c}} \sin^{-1} \left( \frac{c f - \alpha}{\sqrt{\alpha^2 + c}} \right), \]

where \( c = 1 - 2\alpha \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Variation of \( f \) as a function of \( \tau \) at different values of \( v_p/c \).}
\end{figure}

\textbf{Figure 1.} Variations of \( f \) as a function of \( \tau \) at different values of \( v_p/c \).
In Figure 1 we have shown the variation of beam width parameter $f$ as a function of $\tau$ at various values of $v/c$ for $\alpha^2/\alpha^2 = 0.3$, $v_0 = 2.5 \times 10^{10}$ cm/s and $\tau' = 10^{14}$ s.

$f$ is a monotonically decreasing function of $\tau$ and as the pulse strength increases, it takes a shorter duration for the pulse to get focused. Figure 2 shows the variation of amplitude square as a function of dimensionless parameter $\xi = \eta^2/\nu_0^2 \tau$. For $\lambda = 1$ $\mu$m, $\tau = 10^{14}$ s, $v_0/c = 0.9$ (corresponds to $f = 4 \times 10^{18}$ W/cm$^2$) at the distance $x = 5 \nu_0 \tau' = 0.0125$ cm, the pulse is expected to be severely distorted, the peak amplitude is doubled, the front portion expanded and the tail portion compressed. This effect is enhanced as the pulse strength increases. Figure 3 shows the variation of amplitude square as a function of dimensionless parameter $\xi = \eta^2/\nu_0^2 \tau$ at $v_0/c = 0.7$ for various values of $\alpha^2/\alpha^2$. The distortion effect increases with plasma density.

In a dispersive medium, viz. plasma, the group velocity is a function of frequency and pulse strength. Consequently, the front portion of the pulse where amplitude is progressively increasing, acquires higher group velocity and moves faster than the tail portion, whereas amplitude decreases steadily. As a result, the front portion of the pulse is expanded and the tail portion is compressed. These effects become more prominent as the pulse energy increases. Here two processes compete with each other as dispersion broadens the pulse and nonlinearity compresses it. In this analysis nonlinearity is proportional to $|E|^2$, and contraction of the pulse due to nonlinearity outweighs the expansion due to dispersion. Distortion of a laser pulse could severely affect the processes like target heating in inertial confinement fusion scheme and laser induced ablation of material, resulting in nonuniform heating. Distortion of the laser could also adversely affect the excitation of the plasma wave in laser wakefield and beatwave particle acceleration schemes. To avoid self distortion of a laser pulse, plasma channel created by a laser, a pre-pulse may be employed to guide the main laser pulse.

Characterization of clay bound organic matter using activation energy calculated by weight loss on ignition method

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Soil organic matter (SOM) through intimate associations with clay particles forms clay bound organic matter (CBM), which contributes mainly to long-term stability of SOM in most soils. The CBM fraction contributes mainly to more meaningful carbon sequestration due to their long-term stability and hence storage. The formation and stabilization of CBM is a complex process and studies on determination of the exact composition of this fraction are meagre. In this study, an attempt was made to characterize the CBM of a soil in terms of the other SOM fractions such as fulvic fraction (FF), humic fraction (HF) and fine soil litter. The weight loss on ignition (LOI) data of extracted SOM fractions and fine soil litter were used to characterize CBM. Rate constants (k) of the oxidation reactions during the LOI were calculated using percentage weight losses at different temperatures of the FF, HF and fine soil litter. Calculated activation energies (Ea) of the other SOM fractions at different temperatures were used in a multiple regression analysis to predict the Ea of the CBM fraction. The results showed that the activation energy of the CBM [Ea (CBM)] was positively related to that of the fine soil litter [Ea (fine soil litter)] and the FF [Ea (FF)]. The relationship indicates that the CBM of the soil is composed of a mixture of fine soil litter and the FF, as also proposed in the literature.

Keywords: Activation energy, clay bound organic matter, fine soil litter, fulvic fraction, humic fraction.

The soil organic matter (SOM) is a physically and chemically heterogeneous mixture of organic compounds of plant, animal and microbial origin, and has components at different stages of decomposition. The type of land use and soil are important factors controlling SOM storage in the soils. Their storage reflects the relative importance of different mechanisms of SOM stabilization². Several mechanisms have been proposed to contribute to the SOM stabilization in soils⁵. The SOM can be (i) biochemically stabilized through the formation of recalcitrant SOM compounds, (ii) protected by intimate association with silt and clay particles, and (iii) physically stabilized through aggregate formation.

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