Self-distortion of a short laser pulse in a plasma

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Self-distortion of a high-intensity laser pulse due to relativistic effects propagating in a high-density plasma is investigated. As the pulse energy increases, its front portion expands and tail portion get compressed. The effect of nonlinearity outweighs the effect of dispersion.

Keywords: Laser, plasma, relativistic effects, self-distortion.

LASER–PLASMA interaction is an area of significant scientific activities since the past several decades due to wide-ranging applications in materials processing[1], laser-induced fusion[2], harmonic generation[3], etc. Various nonlinear effects have been observed, viz. parametric instabilities[4], self-focusing[5], harmonic generation[6], phase modulation[7], etc. In a moderate-power, long-pulse laser–plasma interaction, the major source of nonlinearities is thermal[8] (collisional) and ponderomotive[9]. The development of short-pulse lasers has made possible the attainment of terawatt powers in a short duration[10]. This has opened up a new parameter regime for nonlinear effects with some novel applications in pair production[11], particle acceleration[12], etc. In this parameter regime the major source of nonlinearity arises due to relativistic effect, which also affect the laser pulse propagation dynamics, as the laser beam propagates[13]. The laser pulse expands both spatially and temporally as it propagates through the plasma due to diffraction and dispersion effects, and as a consequence, the pulse gets severely distorted. Esarey *et al.*[14] and Sprangle *et al.*[15] have given elegant reviews of self-focusing and guiding of short laser pulses in ionized gases and plasmas, in the paraxial ray approximation. Upadhyay *et al.*[16] have developed a formalism for asymmetric self-focusing of a laser pulse in the plasma. Sharma *et al.*[17] have studied the effects of all three nonlinearities, viz. collisional, ponderomotive, and relativistic on the propagation dynamics of the laser beam in a plasma. Faure *et al.*[18] have performed an experiment using 10 Hz, 20 TW, 820 nm, 600 mJ, 35 fs, 10^{19} W/cm^2 laser in under-dense plasma and studied the effects of pulse duration on self-focusing of ultra-short lasers. Their observations showed that a laser will get self-distorted if its pulse duration is short compared to the plasma period. Li and Crowell[19] have provided the formalism for shortening of a laser pulse with self-modulated phase at the focus of a lens. Shin *et al.*[20] have controlled the pulse dispersion of laser in an erbium-doped fibre amplifier by controlling input pulse width, pump power and background-to-pulse power ratio. Rostami and Matloub[21] have analytically investigated and simulated optical pulse distortion and propagation through quasi-periodic structures, generally and specifically for the Fibonacci class.

In this communication, we address the problem of nonlinear relativistic interaction of an ultrashort pulse with a cold plasma following the technique employed by Akhmanov *et al.*[22] to solve the nonlinear wave equation. The plasma is considered cold, where the electron quiver velocity in the pulse is larger than the electron thermal velocity. The pulse is assumed to be one-dimensional, circularly polarized and Gaussian, both spatially and temporally.

Consider the propagation of a laser through a plasma of electron density n_0. The electric field of the laser can be written as

$$\vec{E} = (\hat{x} + \mathrm{i}\hat{y})E_0 \exp[-\mathrm{i}(\omega t - kz)], \tag{1}$$

where ω is the angular frequency of the laser and k the wavenumber. On solving the equation of motion $m(\mathrm{d}\vec{v}/\mathrm{d}t) = -e\vec{E} - (e/c)\vec{v} \times \vec{B}$, the velocity of electron $\vec{v}$ is

$$\vec{v} = \frac{e\vec{E}}{m\mathrm{i}\omega}, \tag{2}$$

where $\vec{B}$ is the magnetic field of the laser, c the velocity of light in vacuum and $-e$ and m are the electronic charge and mass respectively.

The current density $\vec{J}$ is given by

$$\vec{J} = -n_0 e\vec{v}. \tag{3}$$

Using eqs (1)–(3) in the wave equation governing the propagation of laser beams in a plasma $\nabla^2\vec{E} - (1/c^2)(\partial^2\vec{E}/\partial t^2) = (4\pi/c^2)(\partial\vec{J}/\partial t)$ we obtain,

$$c^2k^2\vec{E} = (\omega^2 - \omega_\mathrm{p}^2)\vec{E}, \tag{4}$$

where $\omega_\mathrm{p} = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency.

We now consider the laser amplitude E_0 to be slowly varying as $E_0 = E_0(z, t)$. The consequence of this can be incorporated in the wave equation on replacing $\omega \to \mathrm{i}\partial/\partial t$ and $k \to -\mathrm{i}\partial/\partial z$ in eq. (4). Thus we obtain

$$2\mathrm{i}kc^2\frac{\partial\vec{E}_0}{\partial z} + 2\mathrm{i}\omega\frac{\partial\vec{E}_0}{\partial t} - \frac{\partial^2\vec{E}_0}{\partial t^2} + c^2\frac{\partial^2\vec{E}_0}{\partial z^2} - \Delta^2\vec{E}_0 = 0. \tag{5}$$

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In writing eq. (5) we have assumed

$$\omega_{\mathrm{p}}^2 = \omega_{\mathrm{p0}}^2 + \Delta^2, \tag{6}$$

in compliance with $m \cong m_0[1 + (1/2)(|v^2|/c^2)]$, where ω_{p0} is the unperturbed plasma frequency, Δ the perturbation given by $\Delta^2 = -(\omega_{\mathrm{p0}}^2/2)(|v^2|/c^2)$ and m_0 is the electron rest mass.

To the first-order perturbation we get

$$\frac{\partial \vec{E}_0}{\partial z} = -\left(\frac{\omega}{kc^2}\right)\frac{\partial \vec{E}_0}{\partial t},$$

$$\frac{\partial^2 \vec{E}_0}{\partial z^2} = \left(\frac{\omega^2}{k^2c^4}\right)\frac{\partial^2 \vec{E}_0}{\partial t^2}.$$

Using above equations in eq. (5) we get

$$\mathrm{i}\left(\frac{\partial}{\partial t} + v_{\mathrm{g}}\frac{\partial}{\partial z}\right)\vec{E}_0 + \frac{1}{2\omega}\frac{\partial^2 \vec{E}_0}{\partial z^2}(c^2 - v_{\mathrm{g}}^2) - \frac{\Delta^2}{2\omega}\vec{E}_0 = 0, \tag{7}$$

where v_{g} is the liner group velocity given by $v_{\mathrm{g}} = kc^2/\omega$.

Equation (7) is the wave envelope equation. In the group velocity frame, eq. (7) can be rewritten as

$$\mathrm{i}\frac{\partial \vec{E}_0}{\partial t'} + \left(\frac{c^2 - v_{\mathrm{g}}^2}{2\omega}\right)\frac{\partial^2 \vec{E}_0}{\partial z'^2} - \left(\frac{\Delta^2}{2\omega}\right)\vec{E}_0 = 0, \tag{8}$$

where $t' = t$ and $z' = z - v_{\mathrm{g}}t$. Following Akhmanov *et al.*[14], the solution of eq. (8) may be written as

$$\vec{E}_0 = \vec{A}_0(t', z')\exp[\mathrm{i}S(t', z')], \tag{9}$$

where S is the eikonal.

Using eq. (9) in eq. (8) and after some algebraic manipulations we obtain the coupled equations for wave amplitude $\vec{A}_0$ and eikonal S as

$$\frac{\partial A_0^2}{\partial t'} + \left(\frac{c^2 - v_{\mathrm{g}}^2}{\omega}\right)\frac{\partial S}{\partial z'}\frac{\partial A_0^2}{\partial z'} + \left(\frac{c^2 - v_{\mathrm{g}}^2}{\omega}\right)A_0^2\frac{\partial^2 S}{\partial z'^2} = 0, \tag{10}$$

and

$$\frac{\partial S}{\partial t'} - \frac{1}{A_0}\left(\frac{c^2 - v_{\mathrm{g}}^2}{2\omega}\right)\frac{\partial^2 A_0}{\partial z'^2} + \left(\frac{c^2 - v_{\mathrm{g}}^2}{2\omega}\right)\left(\frac{\partial S}{\partial z'}\right)^2 + \frac{\Delta^2}{2\omega} = 0. \tag{11}$$

We solve eqs (10) and (11) for a Gaussian pulse

$$|A|_{z=0}^2 = A_{00}^2\exp(-t^2/\tau'^2), \tag{12}$$

where τ' is the initial pulse width parameter. We express S and A_0^2 as

$$S = \beta(t')z'^2/2 + \phi(t'),$$

and

$$A_0^2 = \frac{A_{00}^2}{f(t')}\exp\left[-\frac{z'^2}{z_0^2\{f(t')\}^2}\right].$$

Using these values of S and A_0^2 in eqs (9) and (10), β can be evaluated as

$$\beta = \left(\frac{\omega}{c^2 - v_{\mathrm{g}}^2}\right)\frac{1}{f}\frac{\mathrm{d}f}{\mathrm{d}t'}. \tag{13}$$

Equations (11) and (13) may be used to obtain a differential equation for f as

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2} = \frac{1}{f^3} - \frac{\alpha}{f^2}, \tag{14}$$

where $\tau = t'\sqrt{K_1}$, $\alpha = K_2/K_1$, $K_1 = [(c^2 - v_{\mathrm{g}}^2)/\omega^2 z_0^4]^2$ and $K_2 = [(c^2 - v_{\mathrm{g}}^2)\omega_{\mathrm{p0}}^2|v|^2]/2\omega^2c^2z_0^2$.

Solving eq. (14) using the initial conditions at $\tau = 0$, $f = 1$ we get

$$\tau = \sqrt{\frac{cf^2 + 2\alpha f - 1}{c}} + \frac{\alpha}{c}\frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{cf - \alpha}{\sqrt{\alpha^2 + c}}\right), \tag{15}$$

where $c = 1 - 2\alpha$.

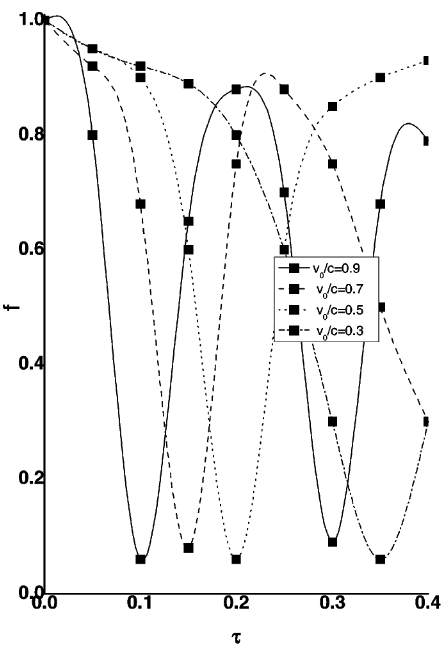

Figure 1. Variation of f as a function of τ at different values of v_0/c.

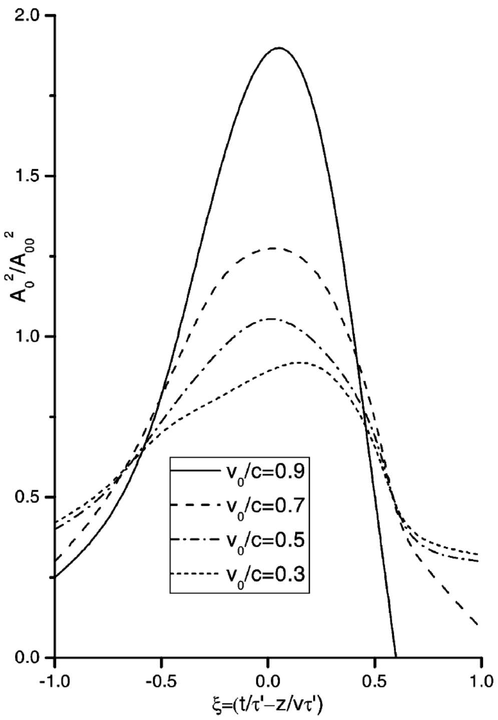

Figure 2. Normalized pulse amplitude A_0^2/A_{00}^2 as a function of $\xi = (t/\tau' - z/v\tau')$ at different values of v_0/c.

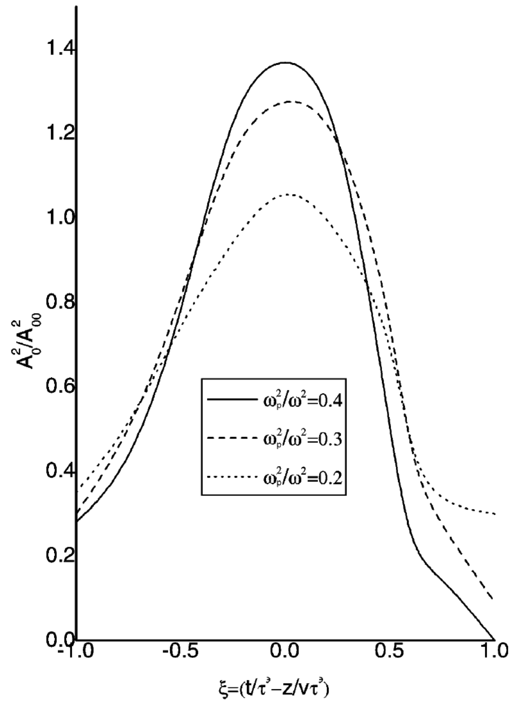

Figure 3. Normalized pulse amplitude A_0^2/A_{00}^2 as a function of $\xi = (t/\tau' - z/v\tau')$ at $v_0/c = 0.7$ for different values of ω_p^2/ω^2.

In Figure 1 we have shown the variation of beam width parameter f as a function of τ at various values of v/c for $\omega_p^2/\omega^2 = 0.3$, $v_g = 2.5 \times 10^{10}$ cm/s and $\tau' = 10^{-14}$ s. f is a monotonically decreasing function of τ and as the pulse strength increases, it takes a shorter duration for the pulse to get focused. Figure 2 shows the variation of amplitude square as a function of dimensionless parameter $\xi = t/\tau' - z/v_g\tau$. For $\lambda = 1$ μm, $\tau = 10^{-14}$ s, $v_0/c = 0.9c$ (corresponds to $I = 4 \times 10^{18}$ W/cm^2) at the distance $z = 5v_g\tau'$ = 0.0125 cm, the pulse is expected to be severely distorted, the peak amplitude is doubled, the front portion expanded and the tail portion compressed. This effect is enhanced as the pulse strength increases. Figure 3 shows the variation of amplitude square as a function of dimensionless parameter $\xi = t/\tau' - z/v_g\tau$ at $v_0/c = 0.7$ for various values of ω_p^2/ω^2. The distortion effect increases with plasma density.

In a dispersive medium, viz. plasma, the group velocity is a function of frequency and pulse strength. Consequently, the front portion of the pulse where amplitude is progressively increasing, acquires higher group velocity and moves faster than the tail portion, whereas amplitude decreases steadily. As a result, the front portion of the pulse is expanded and the tail portion is compressed. These effects become more prominent as the pulse energy increases. Here two processes compete with each other as dispersion broadens the pulse and nonlinearity compresses it. In this analysis nonlinearity is proportional to $|E|^2$, and contraction of the pulse due to nonlinearity outweighs the expansion due to dispersion. Distortion of a laser pulse could severely affect the processes like target heating in inertial confinement fusion scheme and laser induced ablation of material, resulting in nonuniform heating. Distortion of the laser could also adversely affect the excitation of the plasma wave in laser wakefield and beat-wave particle acceleration schemes. To avoid self distortion of a laser pulse, plasma channel created by a laser, a pre-pulse may be employed to guide the main laser pulse[23].

1. Stewart, R. and Isherwood, R., Rapid vaporization mechanism in enclosed laser ablation. *J. Phys. D: Appl. Phys.*, 2003, **36**, 303.
2. Canaud, B. *et al.*, High-gain direct-drive inertial confinement fusion for the laser Mégajoule: Recent progress. *Plasma Phys. Control. Fusion*, 2007, **49**, B601.
3. Schroeder, C. B., Esarey, E., Comier-Michel, E. and Leemans, W. P., High-harmonic generation in cavitated plasmas. *Phys. Plasmas*, 2008, **15**, 056704.
4. Pathak, V. B. and Tripathi, V. K., Supression of stimulated Raman scattering of a laser in a rippled density plasma. *Phys. Plasmas*, 2008, **15**, 022301.
5. Amrita and Sharma, A. K., Thermal self-focusing of a laser in collisional plasma: Beyond the near axis approximation. *Phys. Scr.*, 2006, **74**, 128.
6. Liu, C. S. and Tripathi, V. K., Third harmonic generation of short pulse laser in a plasma density ripple created by a machining beam. *Phys. Plasmas*, 2008, **15**, 023106.
7. Liu, C. S. and Parashar, J., Modulational instability of a self-guided laser in a magnetized plasma channel. *IEEE Trans. Plasma Sci.*, 2006, **34**, 2561.
8. Sodha, M. S. and Sharma, A., Mutual focusing/defocusing of Gaussian electromagnetic beams in collisional plasmas. *Phys. Plasmas*, 2006, **13**, 053105.

9. Sodha, M. S. and Faisal, M., Propagation of high power electromagnetic beams in overdense plasmas: Higher order paraxial theory. *Phys. Plasmas*, 2008, **15**, 033102; Dodin, I. Y. and Fisch, N. J., Particle manipulation with nonadiabatic ponderomotive forces. *Phys. Plasmas*, 2007, **14**, 055901.
10. Nilson, P. M. *et al.*, High-intensity laser-plasma interactions in the refluxing limit. *Phys. Plasmas*, 2008, **15**, 056308.
11. Khan, S. A. and Masood, W., Linear and nonlinear quantum ion-acoustic waves in dense magnetized electron positron ion plasmas. *Phys. Plasmas*, 2008, **15**, 062301.
12. Maksimchuk, A. *et al.*, Studies of laser wakefield structures and electron acceleration in underdense plasmas. *Phys. Plasmas*, 2008, **15**, 056703.
13. Qiao, B., Lai, C. H., Zhou, C. T., He, X. T., Wang, X. G. and Yu, M. Y., Nonlinear properties of relativistically intense laser in plasmas. *Phys. Plasmas*, 2007, **14**, 112301.
14. Esarey, E., Sprangle, P., Krall, J. and Ting, A., Self-focusing and guiding of short laser pulses in ionizing gases and plasmas. *IEEE J. Quant. Electron.*, 1997, **33**, 1879.
15. Sprangle, P., Esarey, E. and Krall, J., Self-guiding and stability of intense optical beams in gases undergoing ionization. *Phys. Rev. E*, 1996, **54**, 4211.
16. Upadhyay, A., Tripathi, V. K., Sharma, A. K. and Pant, H. C., Asymmetric self-focusing of a laser pulse in plasma. *J. Plasma Phys.*, 2002, **68**, 75; Upadhyay, A., Tripathi, V. K. and Pant, H. C., Pulse front sharpening of a laser beam in plasma. *Phys. Scr.*, 2001, **63**, 326.
17. Sharma, A., Prakash, G., Verma, M. P. and Sodha, M. S., Three regimes of intense laser beam propagation in plasmas. *Phys. Plasmas*, 2003, **10**, 4079.
18. Faure, J. *et al.*, Effects of pulse duration on self focusing of ultra-short lasers in underdense plasmas. *Phys. Plasmas*, 2002, **9**, 756.
19. Li, Y. and Crowell, R., Shortening of a laser pulse with a self modulated phase at the focus of a lens. *Opt. Lett.*, 2007, **32**, 93.
20. Shin, H. *et al.*, Reducing pulse distortion in fast light pulse propagation through an erbium doped fiber amplifier. *Opt. Lett.*, 2007, **32**, 906.
21. Rostami, A. and Matloub, S., Optical pulse distortion in Fibonacci-class multilayer stacks. *Turk. J. Phys.*, 2006, **30**, 21.
22. Akhmanov, S. A., Sukhorukov, A. P. and Khokhlov, R. V., Self focusing and self trapping of intense laser light beams in a nonlinear medium. *Sov. Phys. JETP*, 1966, **23**, 1025.
23. Durfee III, C. G. and Milchberg, H. M., Light pipe for high intensity laser pulses. *Phys. Rev. Lett.*, 1993, **71**, 2409.

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Characterization of clay bound organic matter using activation energy calculated by weight loss on ignition method

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Soil organic matter (SOM) through intimate associations with clay particles forms clay bound organic matter (CBO), which contributes mainly to long-term stability of SOM in most soils. The CBO fraction contributes mainly to more meaningful carbon sequestration due to their long-term stability and hence storage. The formation and stabilization of CBO is a complex process and studies on determination of the exact composition of this fraction are meagre. In this study, an attempt was made to characterize the CBO of a soil in terms of the other SOM fractions such as fulvic fraction (FF), humic fraction (HF) and fine soil litter. The weight loss on ignition (LOI) data of extracted SOM fractions and fine soil litter were used to characterize CBO. Rate constants (*k*) of the oxidation reactions during the LOI were calculated using percentage weight losses at different temperatures of the FF, HF and fine soil litter. Calculated activation energies (E_a) of the other SOM fractions at different temperatures were used in a multiple regression analysis to predict the E_a of the CBO fraction. The results showed that the activation energy of the CBO [E_a (CBO)] was positively related to that of the fine soil litter [E_a (fine soil litter)] and the FF [E_a (FF)]. The relationship indicates that the CBO of the soil is composed of a mixture of fine soil litter and the FF, as also proposed in the literature.

Keywords: Activation energy, clay bound organic matter, fine soil litter, fulvic fraction, humic fraction.

THE soil organic matter (SOM) is a physically and chemically heterogeneous mixture of organic compounds of plant, animal and microbial origin, and has components at different stages of decomposition. The type of land use and soil are important factors controlling SOM storage in the soils. Their storage reflects the relative importance of different mechanisms of SOM stabilization[1]. Several mechanisms have been proposed to contribute to the SOM stabilization in soils[2]. The SOM can be (i) biochemically stabilized through the formation of recalcitrant SOM compounds, (ii) protected by intimate association with silt and clay particles, and (iii) physically stabilized through aggregate formation.

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