Āryabhaṭa on the heliacal rise and set of canopus

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An interesting but controversial precept of Āryabhaṭa that has come down in the tradition is the rule given for viewing the heliacal rising and setting of Canopus. In later times, Lalla and Vaṭeśvara found it to be inaccurate. The present note examines the precept with reference to two locations, viz. 10°51′N where Palabhā is 2.3 and Ujjayinī, where Palabhā is 5; the former location being identified as the place of observation of Āryabhaṭa by the present author in his earlier works. It is found that the rule of Āryabhaṭa is quite precise for the location 10°51′N (Camravattam) in Kerala, where the meridian of Ujjayinī intercepts the west coast. Analysis also brings out the rationale of the Āryabhaṭa rule through regression analysis.

An interesting but controversial precept of Āryabhaṭa that has come down in the tradition is the rule given for viewing the heliacal rising and setting of Canopus, named after the great Sage Agastya in India. As has comedown to us through Khaṇḍakhādyaka, the rule is[1]:

राशिचतुष्केण यदा स्वाक्षांशयुतेन भवति तुल्यो*ऽ*कैः।
उदयो अगस्त्यस्य तदा चक्रार्धाच्छोधिते*ऽ*स्तमयः।।

'Agastya rises when the solar longitude is four rāśis plus the latitude ϕ and sets when sun equals six signs minus the same, i.e. Agastyodaya Sūrya = 120° + ϕ and Agastyāsta Sūrya = 180°– (120° + ϕ), which reduces to 60°– ϕ'.

Shukla has discussed the verse as appearing in the old *Śūryasiddhānta* compiled by Varāhamihira and suggests that probably Āryabhaṭa also held the same view. However, Shukla[2] has provided confirmation, quoting Mallikārjuna Sūri, about the fact that the precept really belonged to *Āryārdharātra-siddhānta*.

Additional facts we meet with in the above references are: (a) Verse occurring at the end of Chapter VI, *Pūrva-Khaṇḍakhādyaka* had been rejected by Bhaṭṭotpala on the ground that it does not yield correct results. (b) Lalla[3] has quoted the rule, according to Mallikārjuna Sūri, as is accepted by some pupils of Āryabhaṭa.

गृहद्वयेनाक्षविवर्जितेन समे स्वावस्तमुपैत्यगस्त्यः।
च्युतेन तेनैव समेन षड्भादुदेति
केचिज्जगुरेवमन्ये।११.२१।

'Some (pupils of Āryabhaṭa, according to Mallikārjuna Sūri) say that Agastya sets when the true longitude of the sun is two signs minus the local latitude. And it rises when the true longitude of sun gains six signs minus the same'.

Lalla is well known as one of the great pillars of the Āryabhaṭa school and in the light of Mallikārjuna Sūri's statement that Lalla is quoting the disciples of Āryabhaṭa, it stands confirmed that the original precept originated from Āryabhaṭa only.

A scenario such as the above and the name of Āryabhaṭa attached to it raises curiosity – what may be the rationale of this simple rule which others have found inaccurate, but have quoted it only because it could be traced to Āryabhaṭa? The present note is an attempt to examine the rationale of the rule and the circumstances of its possible origin.

Simple but inaccurate and controversial

It is apparent that the rule of Āryabhaṭa presents the altitude of Canopus in terms of the longitude of the sun. The veracity of the same can be therefore tested by examining the altitude of Canopus during the relevant course of the sun when its longitude (λs) is less than 60° – ϕ for Indian latitudes and more than 120° + ϕ in the same manner. For convenience, we shall choose two latitudes, viz. 10°51′N in Camravattam, Kerala, and 24°00′N, the traditional latitude of Ujjayinī to represent the northern latitudes.

Agastya sets when λs = 60 – ϕ

Figure 1 shows the altitude of Canopus vs longitude of sun (λs) at the time of sunset during λs = 25–60° of AD 519 to represent the times of Āryabhaṭa. The declination and right ascension of Canopus has been chosen for the same epoch. The altitudes of Canopus for setting at solar longitudes according to the rule of Āryabhaṭa are:

1. For ϕ = 24N, when λs = 60 – 24° = 36°, Canopus has an altitude of only 7° at sunset (18 : 20), when the centre of the solar disc crosses the horizon. Obviously Canopus is not visible at this time as the altitude is well below the Kalamsa[4] of

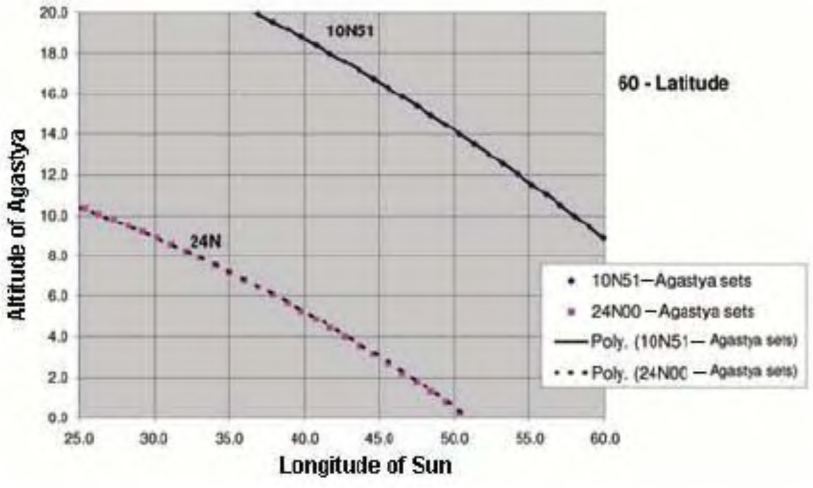

Figure 1. The setting of Agastya.

Table 1. Polynomial regression to understand the precept of Āryabhaṭa

Polynomial	Description of Agastya	λs for $A = 0$ (degrees)	Reference Āryabhaṭa	Difference λs Cols 3 – 4 (degrees)
$A = -0.0043*\lambda s\text{^}2 - 0.0646*\lambda s + 28.165$	Sets 10°51′N	73.8	$60° - \phi = 49°$	25
$A = -0.0052*\lambda s\text{^}2 - 0.0061*\lambda s + 13.743$	Sets 24°00′N	50.8	$60° - \phi = 36°$	15
$A = -0.0032*\lambda s\text{^}2 + 1.3401*\lambda s - 109.12$	Rises 10°51′N	110.7	$120° + \phi = 131°$	20
$A = -0.005*\lambda s\text{^}2 + 1.8536*\lambda s - 158.37$	Rises 24°0′N	133.5	$120° + \phi = 144°$	10.5

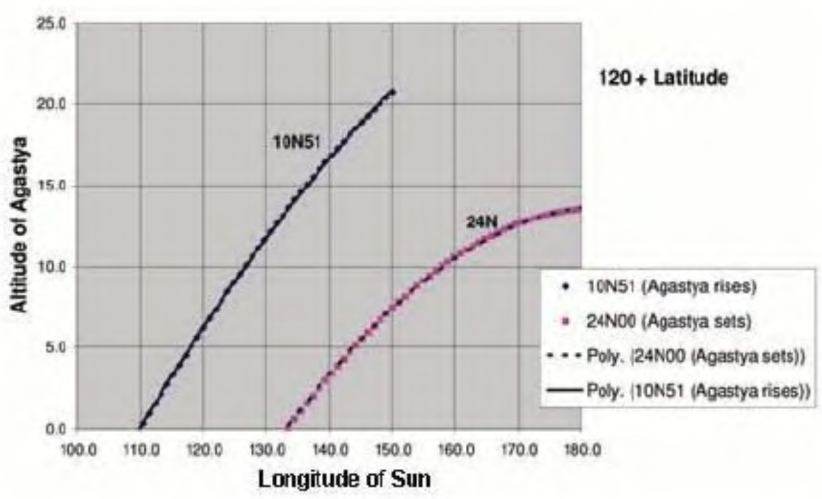

Figure 2. The rising of Agastya.

14–12° ascribed to Canopus in Indian astronomy. The date 25 April when $\lambda s = 36°$, therefore may not be the date of last visibility at the latitude of 24°N. Trend of altitude with λs may be understood from the plot.

2. For $\phi = 10°51'$N when $\lambda s = 49°$, Canopus has an altitude exceeding 14° at sunset when the middle of the orb of the sun is going down the horizon and matches perfectly with the Kālāmśa prescribed for visibility. The declining altitude of Canopus at $\lambda s = 49°$ may thus mark the last visibility at 10°51′N and hence the rule of Āryabhaṭa holds perfectly true at the Kerala latitudes.

Agastya rises when $\lambda s = 120 + \phi$

1. For $\phi = 24°00'$N, when $\lambda s = 120 + 24° = 144°$, Canopus has an altitude of 5° only as the centre of the solar orb rises at the horizon. The altitude of 5° is well below Kālāmśa of 14–12° and hence the star is not heliacally visible and the rule of Āryabhaṭa appears to be wrong.

2. For $\phi = 10°51'$N when $\lambda s = 120 + 10.85° = 131°$, Canopus has an altitude of 12° when the sun rises and thus the rule of Āryabhaṭa appears quite precise at the location for heliacal rising of Agastya.

Figure 2 shows the altitude of Agastya at sunrise varying with respect to the longitude of the sun for $\lambda s = 100$–$180°$ for the year AD 519.

The low altitude of Agastya at 24°00′N and north latitudes for λs at $60° - \phi$ and $120° + \phi$ suggest that the rule has its origin in the southern latitudes and the precise agreement of the rule of Āryabhaṭa at his location identified as 10°51′N, 75°45′E (Camravattam) renders further evidence for the place of observation of Āryabhaṭa.

It is clear that the $60° - \phi$ and $120° + \phi$ criteria could not have been of any use in predicting the heliacal visibility of Canopus in places like Ujjayinī and hence rejection of the rule by most astronomers of later times.

Analysis by polynomial regression

The above discussed altitude variation of Canopus can be studied using polynomial regression to understand the precept in detail. The altitude turns out to be a quadratic as may be expected from the bipolar nature of the heliacal rise and set phenomenon and the roots have to be chosen according to relevance to the sector $60° - \phi$ and $120° + \phi$ as is shown in Table 1.

It is apparent from the relative distances of λs at $A = 0$ and the $60° - \phi$ and $120° + \phi$ points that at high latitudes like $\phi = 24°$, the rule of Āryabhaṭa does not provide sufficient time for Agastya to gain the required Kālāmśa. For setting, the rule gives a 25-day span at 10°51′N, while the span is only 15 days at 24N. For heliacal rising the span is 20 days at 10°51′N, while it is only 11 days at 24N. Though the slope was proportional to λs, additional days were required at high latitudes for Agastya to gain the Kālāmśa of 12° or 14°, as accepted by different Indian astronomers.

Rules of other astronomers

In the light of the above analysis, it becomes apparent that the rules given by other Indian astronomers were all mere approximations and did not tally with the observations of the heliacal phenomenon. Discussed along with the movement of Saptarṣis in many treatises, the rising and setting of Agastya too were perhaps observed only to the extent of the Saptarṣis completing the circuit of heavens in 2700 years. As for example:

(a) The heliacal rise of Agastya at $\lambda s = 98 + 42*$Palabhā/5, obviously indicated that $\lambda s = 98 + 42 = 140°$ at Ujjayinī (22°30′N) where Palabhā equalled 5.

The heliacal setting was likewise given as $\lambda s = 6 - 42*$Palabhā/5, i.e. $\lambda s = 34°$ at 22°30′N. Both are derivatives of the Āryabhaṭa rule and arbitrary modifications were not supported by observations.

It is quite likely that the original rule of Āryabhaṭa was of the form:

Heliacal setting: $\lambda s_{(A=0)} - 42*$Palabhā/5 giving $\lambda s = 70 - 8.4*2.3 = 70 - 19.32 = 50°$ at 10°51′N, which may be approximated as $60° - \phi$ at low latitude of 10°51′N, where Palabhā = 2.3.

Heliacal rise: $\lambda s_{(A=0)} + 42*$Palabhā/5, i.e. $\lambda s = 110 + 8.4*2.3 = 130°$ at 10°51′N,

which is $120° + \phi$ and will hold true at low latitudes.

Similar rules had no superiority over the original precept of Āryabhaṭa based on observations at the latitude of 10°51′N and the meridian of Ujjayinī. Shukla[5] has shown that Sumati's rule was an adaptation of the Āryabhaṭa rule for the latitude of 27. Shukla[6] has also quoted *Bṛhatsamhitā* of Varāhamihira, wherein the Āryabhaṭa rule has been given.

(b) Manjula[7] gives the rule as λs = 97 + 8P and 77 – 8P, which is obviously simplification of the (42/5)*Palabhā rule.

Conclusion

The analysis given above for Āryabhaṭa's precept for the heliacal phenomenon illustrates that the rule is precise at the latitude of 10°51′N according to the Kālāmśa specified by Indian astronomical tradition for the observation of the heliacal phenomena of stars, especially Agastya or Canopus.

The hitherto unpopular rule of Āryabhaṭa for *Agastyodaya* and *astamaya* by its precision at the latitude of 10°51′N turns out to be another jewel in his crown and also in the case of Indian astronomy for which Āryabhaṭa heralded the age of scientific observations at Camravattam (10°51′N), where the west coast of Kerala intercepted the meridian of Ujjayinī.

The present note is a companion submission to the earlier ones on the eclipse observations of Āryabhaṭa at 10°51′N on 15 February 519 AD and at 8°24′N, near Kanyākumāri on 11 August 519 AD. The rationale for the equatorial circumference and the controversial precept on Arkāgrā has also shown that Āryabhaṭa observed the sky at the southern latitudes of 10°51′N and 8°24′N.

1. Shukla, K. S., *Vateśvara Siddhānta and Goḷa of Vateśvara*, INSA, New Delhi, 1985, Shukla has quoted the verse from *Khaṇḍakhādyaka* with the relevant details.
2. Shukla, K. S., Glimpses from *Āryabhaṭa Siddhānta*. *Indian J. Hist. Sci.*, 1977, **12**, 184.
3. Chatterjee, B. (ed.), *Śiṣyadhīvṛddhida tantra*, INSA, New Delhi, 1981, p. 167, verse 11.21.
4. *Vateśvara Siddhānta Part-II*, INSA, New Delhi, 1985, p. 599, Bhāskara-II has given 2 nāḍis as iṣṭakālanāḍīs of Agastya in *Siddhāntaśiromaṇī* I.11.12. Brahmagupta too had prescribed the Kālāmśa or time-degrees as 12. Vaṭeśvara adopts 14° as a general value and Mallikārjuna Sūri has given the same in his commentary on *Śiṣydhīvṛdhida Tantra*.
5. Shukla, K. S., *Vateśvara Siddhānta Part-II*, INSA, New Delhi, 1985, p. 605.
6. Sukla, K. S., *Vateśvara Siddhānta Part-II*, INSA, New Delhi, 1985, p. 604, Shukla quotes *Varāhamihira*.
7. Shukla, K. S., *A Critical Study of the Laghumānasa of Manjulā*, INSA, New Delhi, 1990, p. 181.

Dedication: This paper is dedicated to the memory of late Dr K. V. Sarma. Also, I remember with gratitude all the authors who have helped me understand the ancient works through their painstaking editions of translations with critical notes.

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