Alleged mistake of Āryabhaṭa – Light onto his place of observation

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An attempt is made here to set correct the record of the 1500-year-old tradition of Āryabhaṭīyam. Āryabhaṭa's immortal work had been alleged of mistake by Brahmagupta in AD *629 and the modern researchers too did concur with him as they failed to see any rationale in the statement of Āryabhaṭa vide verse 31 of Gola. Effort has been made to establish that the statement of Āryabhaṭa, viz. 'sun in northern hemisphere enters the prime vertical when the agra is less than Rsine latitude' reflects the ingenuity of the astronomer who worked at the low latitudes of Kerala. At the low latitudes declination (δ) is very nearly equal to agra when it equals the latitude (ϕ) – limiting condition ($\delta = \phi$) for the prime vertical shadow of the gnomon. In view of the special situation as above at his latitude of 10°51'N, Āryabhaṭa chose to instruct the sun's entry to prime vertical in terms of the cut-off in agra equal to Rsine of the latitude of the place. Latitude of the place was the most basic parameter in the use of the gnomon and therefore given the errors of observation and measurement, it was more pragmatic to state the sun's entry into prime vertical in terms of Rsineϕ. Reference has been made to the evidence of spaṣṭa-bhūparidhi, viz. Āryabhaṭa's choice of 3299 as equatorial circumference to facilitate the spaṣṭa-bhūparidhi of 3240 or 9 Yojanas per degree at the latitude of 10°51'N where the coastline of Kerala intersected the meridian of Ujjayinī.*

Beginning with Brahmagupta, many scholars and astronomers have found fault with Āryabhaṭa for verse 31 of the *Goḷapāda* of *Āryabhaṭīyam*. Āryabhaṭa's precept is concerned with the condition for the existence of prime vertical altitude and the relevant verses are:

परमापक्रमजीवामिष्टज्यार्धाहतां ततो विभजेत्।
ज्या लम्बकेन लब्धार्काग्रा पूर्वापरे क्षितिजे ॥३०॥
सा विषुवज्ज्योना चेद् विषुवदुदग्लम्बकेन सङ्गुणिता।
विषुवज्ज्यया विभक्ता लब्धः पूर्वापरे शङ्कुः ॥३१॥

Verse 30 describes the computation of the 'agra' of Sūrya, i.e. the departure of the rising or setting sun from the east-west line.

'*R*sine of the maximum declination (ω) multiplied by *R*sine of the longitude (λ) and divided by the *R*sine of the co-latitude (90°– ϕ) gives the deviation of sun at the horizon' (Note 1).

$R\sin\lambda * R\sin\omega / R\cos\phi = \text{agra} = \alpha$, where ϕ is the latitude of the place and α shall be used to denote agra in the following discussion.

Verse 31 mentions: 'When that (agra) is less than the *R*sine of the latitude and the sun is in the northern hemisphere, multiply that by the *R*sine of the co-latitude and divide by the *R*sine of the latitude to obtain the *R*sine of the sun's altitude when the sun is on the prime vertical'.

i.e. $R\sin h = \alpha * R\cos\phi / R\sin\phi = \alpha / R\tan\phi$, in modern terms.

Criticism of Brahmagupta

All modern studies of *Ārybhaṭīyam* and researches on the works of Āryabhaṭa's followers like Bhaskara-I, Lalla, etc. have brought out the fact that Āryabhaṭa did commit the mistake which Brahmagupta had brought out in the following words[1]:

'The statement (of Āryabhaṭa) that the sun, in the northern hemisphere, enters prime vertical when the (sun's) agra is less than the *R*sine of the latitude is incorrect, because this happens when the *R*sine of the sun's declination satisfies this condition (and not the sun's agra).'

Shukla[1,2] has mentioned that the commentator Bhāskara-I (AD 629), committed the same error in his *Mahā-Bhāskarīya*, but had corrected himself in his *Laghu-Bhāskarīya*. (Shukla[1] has discussed the issue quoting important astronomers like Sūryadeva (b. AD 1191), Someśvara, etc.)

Āryabhaṭa, the great observer of the heavens

Āryabhaṭa, the great astronomer and mathematician who was far ahead of the astronomers of his times in observing that the earth rotated and the starry heavens were stationary, could not have made such a mistake as is being alleged since the 7th century CE, when Brahmagupta began his tirade against him. Expressed in modern terms, the prime vertical altitude h is:

$$\sin h = \sin\delta / \cos\phi,$$

where δ is the declination of the sun and ϕ the latitude. Declination δ has to be more than ϕ for the altitude h to be positive. This fact could have been obvious to a mathematician who is rightly believed to be the originator of the modern trigonometric functions. What then inspired him to make the wrong statement as is being alleged for 14 centuries? Table 1 furnishes the situation where $\delta = \phi$ for the declination of the sun up to $\omega = 24°$.

Notable aspects of the data shown in Table 1 are as follows:

Table 1. Agra for $\delta = \phi$

Sun (δ)	Latitude (ϕ)	$R\sin\phi$	$\sin\delta/\cos\phi$ = R sin agra	Agra in degrees	R sin agra – $R\sin\phi$	agra – δ
3	3	179.93	180.17	3.004	0.24	0.00
6	6	359.37	361.33	6.033	1.96	0.03
9	9	537.82	544.49	9.113	6.67	0.11
10.85	**10.85**	**647.16**	**658.90**	**11.050**	**11.74**	**0.20**
12	12	714.8	730.72	12.272	15.92	0.27
15	15	889.82	921.15	15.542	31.33	0.54
18	18	1062.4	1117.00	18.961	54.60	0.96
21	21	1232.1	1319.64	22.573	87.57	1.57
24	24	1398.4	1530.60	26.438	132.24	2.44

(a) At lower latitudes ($R\sin$ agra – $R\sin\phi$) and (agra – δ) tend to be lower.

(b) For the latitudes of Kerala between 8.5° and 12.5°, the difference between δ and agra is only quarter of a degree – the minimum error observed in ancient astronomical measurements of longitude.

(c) Indian astronomy had been employing at the beginning of the 6th century CE, $\omega = 24°$ against the true value of obliquity $\omega = 23°38'$ and this alone could have introduced errors in conversion of coordinates. For example, in the λ to δ conversion the error is 1.33–1.4%.

(d) Therefore, at lower latitudes as in Kerala, observation and measurement of agra and its conversion to declination was too delicate a task to be accomplished as to annul the error that we see in verse 31. Prime vertical shadow of the gnomon is stated in terms of the agra that could be observed and measured by the astronomer. Anyone computing the declination of sun from agra in the latitude of Kerala, like say 10°51′N (identified as that of Āryabhaṭa's place)[3] need not have necessarily obtained a declination greater than the agra. It is this practical situation that is reflected in verse 31 stated above.

(e) *Rsin ϕ the cut-off value of agra at low latitudes:* Table 2 presents the hypothetical situation of a 2% error, i.e. overestimation in the declination values. What made Āryabhaṭa prescribe the rule in terms of Rsine of latitude, i.e. the boundary condition where $\delta = \phi =$ agra at lower latitudes is evident and thus nothing barred him from framing the rule in terms of agra that is observed and measured.

(f) It can be understood that the sun's agra had to be less than $R\sin\phi$ when there was over estimation of a mere 2% in declination computed from agra.

(g) Latitude ϕ being a basic parameter known, the observational precepts had to be in terms of such parameters and therefore Āryabhaṭa's instruction that the agra must be less than the $R\sin\phi$.

(h) Methods[4,5] of finding the sun's agra by observation using a neck-high circular platform and further derivations have been mentioned in the works of Bhāskara-I and other disciples of Āryabhaṭa, and it is towards such purpose that the verse 31 is also intended.

(i) In fact, Āryabhaṭa showed ingenuity by making $R\sin\phi$ the cut-off value of agra for the prime vertical shadow of the gnomon, taking advantage of his position at the low latitude of 10°51′N in Kerala.

(j) Verse 31 is thus reflective of his realization of the constraints at low latitudes to have a more accurate determination of declination. Table 1 shows that declination was nearly equal to agra in degrees at low latitudes and therefore the instruction in terms of $R\sin\phi$ was an observational thumb rule and cut-off value used in gnomonics of the region.

(k) Rather than theory, the ancient astronomers were more interested in practical applications, so that important celestial events could be predicted and their exercise began with the gnomon.

Oversight in expressing a universal astronomical precept

As described at the outset, the location-specific rule had been under severe criticism since the time of Brahmagupta (AD 628). It has to be admitted that there was some oversight by Āryabhaṭa. The (f) description of the phenomenon should have been applicable to all latitudes to avoid confusion with other astronomers in different places. As an irony it may be said that the oversight has today become an aid, as supporting evidence in fixing the place of his observation.

Evidence of the Spaṣta-bhūparidhi

I had earlier demonstrated that the astronomers who have given original values of the earth's diameter or circumference vis-a-vis their measure of the unit 'Yojana' have done so to take advantage of an integer number of Yojanas or half-integers per degree with the spaṣṭa-bhūparidhi of their location (Table 3).

Āryabhaṭa's choice of 3299 Yojanas as the equatorial circumference was to obtain a spaṣṭa-bhūparidhi of 3240 Yojanas at his place of observation 10°51′N, 75°45′E, where the coastline of Kerala intercepted the Laṅkā–Ujjayinī yāmyottara-rekha or the Hindu prime meridian. His choice of the earth's diameter as 1575 Yojanas in the Ārdharātrika system[6] and circumference as 4948 Yojanas also point towards the same choice of place at 10°51′N. Vaṭeśvara's choice of 3311.24 Yojanas is remarkable to make

Table 2. Impact of 2% over estimation

Sun (δ N)	δ – 2%	Latitude (ϕ)	R sin ϕ	R sin agra
1	0.98	10.85	647.16	59.87
2	1.96	10.85	647.16	119.72
3	2.94	10.85	647.16	179.53
4	3.92	10.85	647.16	239.30
5	4.90	10.85	647.16	298.99
6	5.88	10.85	647.16	358.59
7	6.86	10.85	647.16	418.10
8	7.84	10.85	647.16	477.47
9	8.82	10.85	647.16	536.71
10	9.80	10.85	647.16	595.79
10.85	10.63	10.85	647.16	645.88

Table 3. Evidence of earth's circumference

Astronomer	$2\pi r$ at 0°N (C)	Yojanas per degree Y at ϕ	$\cos\phi = Y*360/C$	Latitude (ϕ)	Place of choice/ native
Eratosthenes	4320	12	0.86	31.00	Alexandria
	5040	14	1.00	0.00	Equator
Āryabhaṭa	3299	9	0.98	10.85	Ponnāni
	4948	13.5	0.98	10.82	Ponnāni
Brahmagupta	5000	13	0.94	20.61	Bhilmala
Bhāskara-II	4967	13	0.94	19.57	Bid.
	3927	10	0.92	23.55	Ujjayinī
Vateśvara	3311.24	8.5	0.92	22.50	Ujjayinī
Varāhamihira	3200	8	0.90	25.84	Kusumapura
Manjula	3600	8	0.90	25.84	Prakāśa
	4800	12	0.90	25.84	25°36′N

the spaṣta-bhūparidhi at 1/16th of the circumference (Ujjayinī according to Āryabhaṭa[7]) to have 8.5 Yojanas per degree. Varāhamihira too had accepted Āryabhaṭa's value of 3299 Yojanas to place Ujjayinī of Āryabhaṭa at 200 Yojanas and Avanti at 213.33 Yojanas and then had changed the value to 3200 with the choice of reference location as Kusumapura.

Though Āryabhaṭa in his brief treatise is silent about the spaṣṭa-bhūparidhi, the idea is well attested in the tradition of Āryabhaṭīya as may be noted from *Śiṣyadhīvṛdhida tantra* of Laḷḷācārya (Lalla has also discussed the correction of planets computed for the meridian of Ujjayinī to the local meridians)[8]:

खखामरा योजनवेष्टनं भुवो नभः
शराभ्रक्षितयो*f*स्य विस्तृतिः
दिवाकरघ्नं पलकर्णभाजितं स्फुटं
महीगोलकवेष्टनं भवेत् ।१.४३।

The equatorial circumference of the earth is 3300 Yojanas. Its diameter is 1050 Yojanas. Equatorial value multiplied by 12 and divided by the hypotenuse of the equinoctial shadow (palakarṇa) of a place gives the true circumference of the earth at that place.

As 12/palakarṇa = cos ϕ, where ϕ is the latitude of the place, the rule is

Spaṣṭa bhūparidhi =
Bhūmadhyaparidhi * cosϕ.

This for the equatorial value of Āryabhaṭa, viz. 3299 Yojanas gave spaṣṭa bhūparidhi as 3240 Yojanas at ϕ = 10°51′N. Though Āryabhaṭa has not given the rule explicitly in his brief treatise, the rule is apparent in verse 11 of the Goḷapāda, where Meru the abode of Gods at the pole 90°N is described:

मेरुर्योजनमात्रं प्रभाकरो हिमवता परिक्षिप्तः
नन्दनवनस्य मध्ये रत्नमयः सर्वतो वृत्तः ।११।

Commentators have taken the verse to mean the height of the Meru, whereas Āryabhaṭa has meant the circumference and so the interpretations have been confusing. Taking the verse as referring to circumference at 1′ short of the pole at 90°N, i.e. 89°59′, we get the right meaning of the verse as 3299 * cos 89°59′ = 0.96 ≈ 1 Yojana. It is just about 1 and so he expressed *Meruryojana mātram*, which is correct rather than *Meruryojana matrah*, which translate as Meru is exactly one yojana in height (Shukla and Sharma have discussed the verse as referring to the height of the Meru mountain.)[9]. It is unlikely that Āryabhaṭa meant 1 Yojana height as that of a mountain, when the earth's circumference is 3299 Yojanas and 1 Yojana is just 6′ of arc, not even quarter of the diameter of the solar or lunar disc.

It is therefore evident that Āryabhaṭa's choice of an odd number like 3299 was to have the spaṣṭa-bhūparidhi at 10°51′N as 3240 = 360*9, giving 9 Yojanas for each degree of terrestrial longitude at his place where the Ujjayini meridian had intercepted the west coast of Kerala.

Further, *Āryabhaṭīyam* offers no clue in terms of the Yojanas east or west of his place[8,10] from the meridian of Ujjayinī. This is possible only under a situation where the astronomer had been living at a place where he could use the Ujjayinī meridian as the prime meridian.

Evidence of the mistake of the Ujjayinī latitude at 22°30′N

Another of Āryabhaṭa's alleged mistake, viz. latitude of Ujjayinī as 22°30′N, having the equinoctial shadow 5:12, also suggests that at the time of writing the *Āryabhaṭīyam*, Āryabhaṭa was not familiar with North India and the popular Ujjayinī marked by the Mahākāleśvar temple at the latitude 24°N and where the equinoctial shadow would have been in excess by 20 Vyaṅgulas[11].

Āryabhaṭīya *Gola*-verse 14 specifies the Indian or Hindu prime meridian in the words:

स्थलजलमध्याल्लंका भूकक्ष्याया भवेच्चतुर्भागे।
उज्जयिनी लंकायाः तच्चतुरंशे समोत्तरतः ।१४।

'From the centre of the land and water, at a distance of one-quarter of the earth's circumference lies Laṅkā; and from Laṅkā at a distance of one-fourth thereof, exactly northwards, lies Ujjayinī.'

The verse spells out that on the prime meridian, Ujjayinī is located at one-sixteenth of the earth's circumference north of Laṅkā and thus the latitude of Ujjayinī turns out to be 360°/16 = 22°30′N. Shukla and Sharma[12] have given a discussion on this aspect in their critical edition of *Aryabhatiyam*. To quote:

'... This makes the latitude of Ujjayinī equal to 22°30′N. This is in agreement with the teachings of the earlier followers of Āryabhaṭa, such as Bhāskara-I (AD 629), Deva (AD 689) and Lalla and the interpretations of the commentators Someśvara, Sūryadeva (b. AD 1191) and Parameśvara (AD 1431). Even the celebrated Bhāskara-II (AD 1150) has chosen to adopt it.

But Brahmagupta (AD 628) differed from this view. He takes Ujjayinī at a distance of one-fifteenth of the earth's circumference from Laṅkā and the likewise the latitude of Ujjayinī as equal to 24°N. Some of the commentators of Āryabhaṭīya who favoured Brahmagupta's view changed the reading taccaturamśe into pancadaśamśe. The commentator Sūryadeva, who first interprets the original reading taccaturamśe, later remarks: 'Ujjayinī laṅkāyāḥ pancadaśāmśe samottarataḥ' (i.e. Ujjayinī is at a distance of one-fifteenth of the earth's circumference to the exact north of Laṅkā) is the proper reading because Brahmagupta writes: Laṅkottaratofvanti bhūparidheḥ pancadaśabhāge ...'.

This long quotation brings out the salient features of the conflict which may be enumerated as follows:

(i) Āryabhaṭa gave the latitude of Ujjayinī as 360°/16 north of Laṅka and it had acceptance only among his followers.

(ii) Brahmagupta and a host of others like Varāhamihira[13] did not agree with Āryabhaṭa and this had given rise to an alternate school of thought and tradition.

(iii) Bhāskara-II apparently agreed with Āryabhaṭa. However, some followers of Āryabhaṭa like Sūryadeva could not find any rationale underlying Āryabhaṭa's notion and they did tacitly accept Brahmagupta as correct.

(iv) Apart from what Shukla and Sharma have discussed, we can see that the *Sūryasiddhānta* also did not agree with Āryabhaṭa in the matter.

(v) Shukla has quoted Nīlkaṇṭha who has tried to explain the conflict by crediting Āryabhaṭa's reference of 22°30′N to a different Janapada at that latitude. But this is not correct as any reference to Ujjayinī in ancient texts obviously hinted at the location of the Mahākāleśvar temple whose latitude according to modern determination is 23°13′N.

Thus it becomes apparent from the above that:

(1) Āryabhaṭa was correct in his statement that the sun in the northern hemisphere enters the prime vertical when the agra is less than the cut-off,

viz. *R*sine of the latitude and Āryabhaṭa chose agra as the reference as it is nearly equal to declination when the sun enters the prime vertical at lower latitudes like that of Kerala.

(2) The 1400-year-old criticism on Āryabhaṭa which began with the tirade unleashed by Brahmagupta is irrelevant and unwarranted given the location of Āryabhaṭa as Kerala.

(3) In giving expression to the phenomenon universally observed, oversight is apparent. But the precept rightly understood in terms of the location renders supporting evidence to the fact that Āryabhaṭa had his observations in the latitudes of Kerala.

(4) Reference is made to the evidence of spaṣṭa-bhūparidhi. Āryabhaṭa's choice of 3299 Yojanas as equatorial circumference is shown to be inspired by the 3240 Yojanas at 10°51′N, the location of Āryabhaṭa where the Ujjayinī meridian intercepted the coastline of Kerala (10°51′N, 75°45′E), viz. Camravattam, near Ponnāni, an ancient port and Arab trade centre.

Note 1. Symbols used: (1) *R*sine refers to 3438* modern sine and similarly with other trigonometric functions. (2) Symbol multiplication (*) and division (/) and other mathematical operations have been given as is being widely used now in 'Excel' worksheets.

1. Shukla, K. S., *Āryabhaṭīya*, Indian National Science Academy, New Delhi, 1976, p. 143.
2. Shukla, K. S., *Mahābhāskarīyam*, Lucknow University, Lucknow, 1960, p. 86.
3. Hari, C. K., Critical evidence to fix the native place of Āryabhaṭa – I. *Curr. Sci.*, 2007, **93**, 1177–1186.
4. Shukla, K. S., *Mahābhāskarīyam*, Lucknow University, Lucknow, 1960, p. 94.
5. Chatterjee, B., *Śiṣyadhīvṛdhida tantra*, Indian National Science Academy (INSA), New Delhi, 1981, vol. 2, pp. 92–93.
6. *Mahābhāskarīyam* (ed. Sastry, T. S. K.), Madras Government Oriental Series, No. CXXX, Govt Oriental Manuscripts Library, Madras, 1957, p. XLVI.
7. *Ibid*, Gola: Verse-13, p. 123.
8. Chatterjee, B., *Śiṣyadhīvṛdhida tantra*, INSA, New Delhi, 1981, vol. 2, pp. 28–33.
9. Shukla, K. S., *Āryabhaṭīya*, Indian National Science Academy, New Delhi, 1976, p. 121.
10. Bhāskara-I has devoted a chapter to discuss the 'Deśāntara-Yojanas' in *Mahābhāskarīyam*. See Shukla[2]. Āryabhaṭa himself speaks in verse 13 of *Goḷa* about the four mythical cities marking the four quarters of the earth's circumference from Laṅkā between which obviously 15 nāḍikās becomes the interval of planetary motion that must be adjusted. However, Āryabhaṭa gives no rule for practical application to his local place. This is quite unnatural given the fact that Bhāskara-I repeatedly takes the name of Āryabhaṭa and the tradition of his disciples while enunciating the different aspects of Deśāntara in chapter 2 of *Mahābhāskarīyam*. Āryabhaṭa himself, having introduced two systems of reckoning ahargaṇa from Laṅkā differing by 15 nāḍikās, needed to have related his observations to the local meridian and Ujjayinī meridian for reconciling the two systems without computational conflicts. Āryabhaṭa's adherence to the Ujjayinī meridian is evident from the fact that he reduced the year length in such a way as to avoid any conflict at his epoch of Kali 3623 when the two systems coincided. Even if it was improper for a Siddhānta to make reference to a specific location, any reference given by Āryabhaṭa to his disciples on the local meridian would have come down to us through the works of his disciples. But we meet with no such reference to the meridian of Kusumapura in the history of Indian astronomy. As shown in the discussion on spaṣṭa-bhūparidhi, Āryabhaṭa's choice of two values for the equatorial circumference had as reference the local latitude 10°51′N and was located on the prime meridian of Ujjayinī.
11. Immense evidence is available to realize that the Indian astronomers were conversant with the use of gnomons and the equinoctial shadow of places. Ānandapura situated on the tropic of Cancer was known to be of the equinoctial shadow of 5.33:12 as known in tradition. Govindaswāmi, Śaṅkaranārāyaṇa, Sūryadeva, Parameśvara, etc. also have made well-known reference to their places by mentioning the equinoctial shadows and the deśāntara Yojanas.
12. Shukla, K. S. and Sharma, K. V., *Aryabhatiya*, Indian National Science Academy, New Delhi, pp. 123–126.
13. *Ibid*, Varāhamihira's opinion is discussed on p. 125.

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