The role of Alfven drag in the planetary cratering process

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While cratering on the rocky planetary surfaces is well known, details of the propagation of the impacting bodies through the atmospheres of planets and satellites are not well studied. The impacting bodies, which are mainly chondritic rocks, are likely to have finite electric conductivity. Hence, if they are orbiting in the magnetospheres of the planets and satellites, Alfven drags are normally expected. Based on the earlier study of Alfven drag effects in the Jovian system, in this investigation, the damping times for rocks of different sizes in the earth's orbit due to Alfven drag are estimated. It is shown to vary from about 1.915 × 10^8 yrs for rock sizes of 50 m, go through a minimum of about 3.8315 × 10^7 yrs for rock sizes of 250 m and increase thereafter to about 7.6630 × 10^7 yrs for rock sizes of 500 m. Thus it is clear that rocks of sizes ranging from 175 to 275 m, responsible for small craters of sizes less than a few kilometres like the Baringer, Arizona, USA and Lonar, Maharashtra, India, can be attributed to rocks orbiting in the earth's magnetosphere over a million years or so, though the impact events themselves may be less than hundred thousand years.

Keywords: Alfven drag, cratering, magnetosphere, planetary surfaces, rocks.

THOUGH there is daily input of more than 100 tonnes of extraterrestrial dust on the earth[1], the arrival of rocks which can cause a visible crater is not frequent. The dust, which is mainly meteors, originates in the cometary debris left after their passage through the solar system in the direction of various constellations, e.g. the Geminid meteor shower. These arrive gently and settle on the earth's surface without leaving any visible traces, except for their brilliant passage through the atmosphere on their journey to the earth's surface. Craters ranging from very small sizes to those of even up to 100 km are known on the rocky planets and satellites. Table 1 shows 30 known craters of sizes up to 2 km, their locations, age and other details. While the big craters of sizes 100 km or more are believed to be formed by asteroidal or cometary impacts, small craters result from the impacts of rocks of small sizes like meteorites. It is not known whether the impacting bodies result directly from the break-up of big rocks ejected from the asteroidal belt or comets at the time of formation of the craters, or the rocks from other outer sources which spend a long time orbiting the planets before impact. But it is known from the early investigations[2] that orbital energy of a body having a minimum size and made up of materials with minimum electrical conductivity will be lost due to Alfven wave dissipation. Anselmo and Farinella[3] have applied this mechanism to the Jovian satellite systems and rings. They have shown that this Alfven drag effect could have influenced in a significant way, the orbital evolution of small natural moons orbiting inside or in the proximity of Jovian rings. They specially noted this mechanism to be responsible for the absence of rocks of sizes ranging from 100 m to 1 km in the Jovian ring. In the present investigation, this mechanism is applied to rocks of different sizes orbiting the earth's magnetosphere. The damping times, namely the time taken for the Alfven wave dissipation to drag the body to the earth's surface, varies from 1.9157 × 10^8 yrs for rocks of size 50 m, decreases to a minimum of 3.8315 × 10^7 yrs for rocks of size 250 m and then increases to 7.6630 × 10^7 yrs for rocks of size 450 m. A rough qualitative explanation for the behaviour of the damping times to go through a minimum is possibly the competition between the generation of the Alfven waves and the time spent by the rock in the Alfven wave environment. Though the presence of big-sized rocks will have been known from other investigations, it is difficult to detect rocks of sizes less than 100 m in distant orbits. Thus it is possible that small rocks of sizes 175–275 m, responsible for craters of sizes less than a few kilometers like the Baringer, Arizona, USA[4,5] and Lonar, Maharashtra[6], India, may have spent about a million years orbiting in the magnetosphere of the earth before their impact.

As is well known[3], Alfven waves are magnetohydrodynamic disturbances of frequency ω much less than the ion cyclotron frequency $\Omega_i = eB/M_i c$, where B is the magnetic field strength in gauss, M_i the ion mass, and e and c are the electronic charge and the velocity of light respectively. Since a body of size D moving with a velocity v (v is the orbital velocity relative to magnetic field velocity $v = v_{orb} - v_{mag.fi.}$, where v_{orb} is the orbital velocity of the object and $v_{mag.fi.}$ is velocity of the magnetic field at that radius) radiates at frequencies of the order of v/D, generation of Alfven waves will be important for $v/D < \Omega_i$. Substituting the values for e and c and expressing the ion mass in terms of proton mass M_p yields the minimum size of the body for the Alfven waves to be important as

$$D > 10^{-4} (v/B)(M_i/M_p).$$

In the case of interest for us, in the earth's orbit with a radial distance 30,000 km, for a velocity of 3646.2 m/s and $B = 0.2963 \times 10^{-2}$ G, at that radial distance assuming $M_i/M_p = 1$, the minimum size turns out to be 50 m; for a radial distance 9000 km, for a velocity of 6653.8 m/s, and

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Table 1. List of 30 impact craters on the earth with diameter less than or up to 2 km (last updated 2 November 2004)

Crater	Location	Latitude	Longitude	Diameter (km)	Age (Ma)	Exposed	Drilled
Haviland	Kansas, USA	37°35′N	99°10′W	0.015	<0.001	Y	N
Dalgaranga	Western Australia	27°38′S	117°17′E	0.024	~0.27	Y	N
Slkhote Alin	Russia	46°7′N	134°40′E	0.027	0.000055	Y	N
Campo Del Clelo	Argentina	27°38′S	61°42′W	0.05	<0.004	Y	N
Sabolev	Russia	46°18′N	137°52′E	0.053	<0.001	Y	N
Veevers	Western Australia	22°58′S	125°22′E	0.08	<1.0	Y	N
Ilumetsa	Estonia	57°58′N	27°25′E	0.08	>0.002	Y	N
Morasko	Poland	52°29′N	16°54′E	0.1	<0.01	Y	N
Kaalijarv	Estonia	58°24′N	22°40′E	0.11	0.004 ± 0.001	Y	N
Wabar	Saudi Arabia	21°30′N	50°28′E	0.116	0.00014	Y	N
Henbury	Northern Territory, Australia	24°34′S	133°8′E	0.157	0.0042 ± 0.0019	Y	N
Odessa	Texas, USA	31°45′N	102°29′W	0.168	<0.05	Y	Y
Boxhole	Northern Territory, Australia	22°37′S	135°12′E	0.17	0.0540 ± 0.0015	Y	N
Macha	Russia	60°6′N	117°35′E	0.3	<0.007	Y	N
Aouelloul	Mauritania	20°15′N	12°41′W	0.39	3.0 ± 0.3	Y	N
Amguid	Algeria	26°5′N	4°23′E	0.45	<0.1	Y	N
Monturaqui	Chile	23°56′S	68°17′W	0.46	<1.0	Y	N
Kalkkop	South Africa	32°43′S	24°34′E	0.64	<1.8	Y	Y
Wolfe Creak	Western Australia	19°10′S	127°48′E	0.875	<0.3	Y	N
Tswaing	South Africa	25°24′S	28°5′E	1.13	0.220 ± 0.052	Y	Y
Barrianger	Arizona	35°2′N	111°1′W	1.186	0.049 ± 0.003	Y	Y
Tabunkharaobo	Mangolia	44°6′N	109°36′E	1.3	150 ± 20	Y	N
Saarijarvi	Finland	65°17′N	28°23′E	1.5	>600		Y
Karikoselka	Finland	62°13′N	25°15′E	1.5	<1.88	Y	
Liverpool	Northern Australia	12°24′S	134°3′E	1.6	150 ± 70	Y	N
Talemzane	Algeria	33°19′N	4°2′E	1.75	<3	N	Y
Lonar	India	19°58′N	76°31′E	1.83	0.052 ± 0.006	Y	Y
Tenoumer	Mauritania	22°55′N	10°24′W	1.9	0.0214 ± 0.0091	Y	N
B.P. Structure	Libya	25°19′N	24°20′E	2.0	<120	Y	N
Tvaren	Sweden	58°46′N	17°25′E	2.0	~455	N	Y

$B = 0.1103$ G the minimum size turns out to be 05 m. Hence the small dusty particles are not affected by this mechanism and only macroscopic bodies of larger than a decametre size are to be considered.

The amount of energy P dissipated is given in terms of the Alfven velocity v_A given by

$$v_A = c\,[1 + 4\pi\rho c^2/B^2]^{-1/2}$$

as

$$P = B^2v^2D^2/(2\pi v_A).$$

But the above dissipation is valid only in the linear regime, namely $v < v_A$, and has to be multiplied by a factor $(D/L)^2$, if the dimension of the body D is less than the length L (ionic inertial length) corresponding to the plasma frequency ω_p given by $\omega_p = (4\pi e^2 n_e/m_e)^{1/2}$ as $L = c/\omega_p$. Further, the above dissipation is valid only if the electrical conductivity of the body is greater than a critical value $\sigma_{cr} \equiv c^2/(2\pi D v_A)$. In the case of chondritic bodies, this condition is easily fulfilled, as the experimental measurements of Schwerer *et al.*[7] have shown. Thus, it is possible to estimate the orbital damping time $\tau = E_{orb}/P$, where E_{orb} is the body's orbital energy.

$$\tau = (\pi^2\rho_s D v_{orb}^2 v_A)/(6v^2B^2),$$

where ρ_s is the density of the orbiting body. For a body of density 2 g per cubic cm we discuss here two cases, of initial orbital radii 30,000 and 9000 km with orbital speeds $V_{orb} = 3616.2$ and 6653.8 m/s respectively, and the damping times for objects of different sizes are calculated (Tables 2 and 3). The damping times given in Tables 2 and 3 show that there exists a minimum value as the rock size increases. While there is no obvious physical explanation for this behaviour, one can qualitatively see that the drag will be most effective when the Alfven waves generation is efficient, as well as when the rock spends enough time in the Alfven wave environment. The Alfven waves generation is more with increasing kinetic energy, but chances of the rock spending enough time in the Alfven wave environment will decrease with increasing kinetic energy. Hence it is possible to understand the minimum size as the one where these two conditions match. This is at present qualitatively the most promising explanation for the behaviour of the damping time. A more detailed analysis of this is under progress.

It is clear from Tables 2 and 3 that orbital damping by Alfven drag is important for the decay of orbital energy within a reasonable period in the earth's history. The mechanical energy decay due to friction and other causes is less than the Alfven drag decay estimated above.

Table 2. Orbital damping time τ for primordial rocks/fragments of asteroids produced due to collisions among them and thrown towards the earth/parts of cometary nuclei or from different sources mentioned in the text of different sizes D (in m), average density $\rho_s = 2$ g per cubic cm with initial orbital radius of 30,000 km (this radial distance is within stationary orbit), magnetic field B of earth at that distance being 0.2963×10^{-2} G, v_A (velocity of Alfven wave at that distance) = 20.7426 m/s, v_{orb} (orbital velocity) = 3646.2 m/s, $v_{mag.fi.}$ (magnetic field velocity) = 2182 m/s, v (relative velocity, $v = v_{orb} - v_{mag.fi.}$ = 1464.2 m/s, ω_p (plasma frequency) = 1.2×10^6 Hz, L (ion inertial length) = c/ω_p = 250 m

Size of the object D (in m)	Orbital damping time τ in units of 10^7 yrs
50	19.157
100	9.5758
150	6.3348
200	4.7894
250	3.8315
300	4.5978
350	5.3641
400	6.1304
450	7.6630

Table 3. Orbital damping time τ for primordial rocks/fragments of asteroids produced due to collisions among them and thrown towards earth/parts of cometary nuclei or from different sources mentioned in the text of different sizes D (in m), average density $\rho_s = 2$ g per cubic cm with initial orbital radius of 9000 km, B of earth at that distance being 0.1103 G, v_A = 761.615 m/s, v_{orb} = 6653.8 m/s, $v_{mag.fi.}$ = 655 m/s, v = 5998.8 m/s, $\omega_p = 1.2 \times 10^6$ Hz, $L = c/\omega_p$ = 250 m

Size of the object D (in m)	Orbital damping time τ in units of 10^7 yrs
05	101.62
10	50.812
20	25.406
40	12.7031
50	10.1625
100	5.0812
150	3.3604
200	2.5406
250	2.0320
300	2.4390
400	3.252
500	4.065

Rocky planetary surfaces show small or big craters. While big craters are known to form by fragments of big asteroid, it is unknown as to how small craters of size less than 2 km are formed. To understand this, we invoke here Alfven drag in the magnetospheres of planets (if they have them to appreciable extent) to show that it can spiral in slowly, small asteroids in the size range 5–300 m, ultimately making them hit the rocky planetary surfaces forming craters of size about or less than 2 km.

The question remains as to how and where these small bodies, which are responsible for creating on the rocky planetary surfaces, small craters of size 2 km and less, originate. In January 1991, the 0.9 m Space Watch Telescope made the first observation[8] of an asteroid outside the earth's atmosphere, but in the neighbourhood of the earth–moon system. Since then, more than 40 earth-approaching asteroids (defined as objects with perihelion of less then 1.3 AU) have been discovered, including 13 smaller than 50 m. Using these data, Rabinowitz *et al.*[9] have shown that there is an excess of earth-approaching asteroids with diameters less than 50 m relative to the population inferred from the distribution of larger objects. Rabinowitz *et al.*[9] argued that these smaller objects – characterized by low eccentricity, widely ranging inclinations and unusual spectral properties – form a previously undetected asteroid belt concentrated near the earth. The recent discovery of additional small, earth-approaching asteroids supports this conclusion.

Small craters of size about 2 km across, may be created for some asteroids from the near-earth asteroid belt mentioned above. Due to planetary perturbations, some small asteroids of sizes ranging from 5 to 300 m from this near-earth asteroid belt lose their orbits and come towards the earth, wherein they are captured by the earth's gravity. Then the Alfven drag takes over to spiral them in slowly, ultimately making them hit the earth forming small craters of size about 2 km.

Lunar ejecta or an undetected population at Lagrangian points on the earth's orbit (analogous to the Trojan groups on Jupiter's orbit) may be other sources for small bodies in the size range 5–300 m, which due to planetary perturbations lose their orbits and come near the earth. They too are captured by the earth's gravity and then taken over by Alfven drag to spiral in slowly, ultimately hitting the earth forming small craters of size about 2 km across. Rawal[10–13] proposed that analogous to Oort clouds of comets circumscribing the solar system, there exist Oort clouds of comets circumscribing each of the satellite systems. Due to planetary perturbations, they enter the gravitational field of the earth and then Alfven drag takes over to spiral them in slowly, ultimately hitting the earth forming small craters of size about 2 km.

Meteorites and rocks of sizes capable of creating a visible crater on the earth's surface can arise due to the ejection of collision products from the asteroidal belt or various other sources discussed above, and may either directly reach the earth's surface or can be caught within a stationary orbit depending on the ejection velocity. Bigger rocks would have been detected by ground-based observations. But rocks of smaller sizes may be still in orbit and are difficult to observe and detect, as discussed above. The calculations outlined here precisely refer to these orbiting rocks. It is shown here that rocks of sizes 175–275 m reach the earth's surface within a few hundreds of thousands of years of the initial orbital capture. Thus the craters which may be recent in the earth's history, namely less than tens of thousands of years, can be caused by rocks residing within the stationary orbit for hundreds of thousands of years. This is relevant especially for the Lonar crater, which is estimated to be 50,000 years old. The same will be true for the Barringer Crater.

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Optimal ordering policies of inventory model for deteriorating items having generalized Pareto lifetime

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In this communication, we develop and analyse an inventory model with the assumption that the lifetime of the commodity is random and follows a generalized Pareto distribution. It is also assumed that the demand is a function of stock and the money value is subject to inflation. Using the differential equations, the instantaneous state of inventory is derived. With suitable cost consideration, the total cost function is obtained. Minimizing total cost function, the optimal ordering quantity and cycle length are obtained. This model is useful in practical situations arising at places like the food and vegetable markets, oil industry and photochemical industry.

Keywords: Generalized Pareto distribution, inflation, perishable models, stock-dependent demand.

RECENTLY, much emphasis has been given in developing inventory models for deteriorating items with random lifetime. Several workers have reviewed the inventory models for deteriorating items[1–3]. In the study of inventory models for deteriorating items, the lifetime of the commodity plays a dominant role. Several researchers have studied the inventory models with exponential lifetime Tadikamalla[4] developed inventory models with gamma distribution for deterioration. Inventory model with Weibull distribution for the lifetime of a commodity has also been studied[5–7]. Nirupama Devi[8] has studied the inventory model with the assumption that the lifetime of a commodity follows a two-component Weibull distribution. No serious attempt has been made to develop and analyse inventory models with generalized Pareto distribution, except the work of Srinivasa Rao *et al.*[9], who studied the models with the assumption that the demand is a function of selling price or is time-dependent. They assumed that the money value is fixed and remains constant (without inflation). However, in deteriorating items like food and vegetables, photographic films and electronics, when a price increase is anticipated, then a large amount of items may be purchased, but the money value may change during the planning period with an inflation rate and the demand is stock-dependent. Several researchers have examined the inflationary effect on an inventory policy. Buzacott[10] developed an approach of modelling inflation

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