

## Modelling of foF<sub>2</sub> using neural networks at an equatorial anomaly station

A. D. Sarma<sup>1,\*</sup> and T. Madhu<sup>2</sup>

<sup>1</sup>Research and Training Unit for Navigational Electronics, Osmania University, Hyderabad 500 007, India

<sup>2</sup>ECE Department, SRKR Engineering College, Chinna-Amiram, Bhimavaram 534 204, India

**The critical frequency of the F<sub>2</sub>-layer (foF<sub>2</sub>) is an important parameter in estimating the total electron content (TEC) of the ionosphere, which is necessary for predicting the ionospheric time delay in GPS applications. The foF<sub>2</sub> data from Ahmedabad, which is in the equatorial anomaly region, are modelled using a multilayer neural network trained with back-propagation algorithm. The IRI-2001 model together with this neural network model can be used for predicting/forecasting of the foF<sub>2</sub> parameter within the Indian subcontinent. The foF<sub>2</sub> values thus predicted can be used to estimate the critical TEC parameter.**

**Keywords:** Back-propagation, GPS, ionosphere, navigation, neural network.

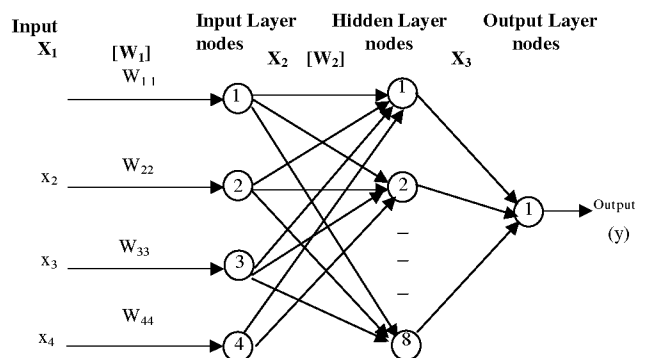
THE introduction of GPS has led to a major improvement in the worldwide navigation facilities. The positional accuracy of GPS is limited by the precision in measuring the atmospheric time delays. Precise ionospheric and tropospheric time delay estimation is needed for achieving better accuracy in position fixing, satellite navigation and geodesy. While the refractive index of vacuum is unity for electromagnetic waves (radio waves), the presence of free electrons in the terrestrial ionosphere decreases the refractive index significantly below unity and thus increases the phase velocity of propagation. Since group velocity and phase velocity are inversely related, the group velocity of radio waves (which constitute the GPS signal) decreases and its value is considerably reduced below the free space velocity of propagation. For example, 1 total electron content unit (TECU) of  $10^{16}$  el/m<sup>2</sup> at GPS L<sub>1</sub> frequency (1.575 GHz) causes a delay of 0.54 ns. As the troposphere is a non-dispersive medium for frequencies up to 15 GHz, standard models are available to accurately measure the time delay error. But as the ionosphere is an ever-changing medium, it is difficult to model it. Various electron density models such as Bent and International Reference Ionosphere (IRI-2001) have been developed to estimate the TEC. However, these models are not effective in low latitude and anomaly region TEC modelling. Further, the ionosphere over India is known for its large temporal and spatial variability. The dominant variability is diurnal due to the large variation in incident solar radiation. This is

true of all low latitude stations<sup>1</sup>. It is reported that the maximum ionization occurs at around 1500 h local time in India<sup>2</sup>. As there is no suitable and dedicated TEC model available for the Indian subcontinent (6–38°N), as a first step, it is proposed to model the foF<sub>2</sub> values using multilayered neural networks (MNNs). The neural network model can be used as a subroutine in IRI-2001 to increase the prediction accuracy of the model in the equatorial anomaly region. Earlier, Wintoft and Cander<sup>3</sup> used time-delay feed-forward neural networks with back-propagation to predict the hourly values of foF<sub>2</sub> at a single station. Also, Lamming and Cander<sup>4</sup> used the monthly median values of foF<sub>2</sub> along with month, local time and solar sunspot number (ssn) in predicting the foF<sub>2</sub> values at Poitiers station. However, no significant contribution is reported from the Indian researchers in modelling the ionospheric parameters using MNNs.

Hence, in this communication modelling of the Ahmedabad station foF<sub>2</sub> data over the solar cycle 20 (October 1964 to June 1976) is attempted using an MNN trained with back-propagation algorithm. As the Ahmedabad station (23.01°N, 72.60°E) is at the equatorial anomaly crest, day-to-day variations of the ionosphere are large, and modelling of the ionospheric parameters at this station assumes significance. This model can be extended to other stations also.

National Physical Laboratory (NPL), New Delhi collects ionospheric data from Delhi, Ahmedabad, Haringhata, Mumbai, Hyderabad, Tiruchirapalli, Kodaikanal and Thumba stations spread all over India. Here, the monthly median values of ionosonde foF<sub>2</sub> data of Ahmedabad station are used for modelling. The solar cycle comprised of 141 months. A few missing data values are replaced with those generated using the NPL-derived coefficients<sup>5</sup>. The data were arranged sequentially, month-wise, in four columns for use in the developed program. The four columns denote month, local time, SSN and ionosonde foF<sub>2</sub> data respectively.

Multilayered feed-forward neural networks became well known as a neural model after Rumelhart developed a learning algorithm called backwards error propagation or



**Figure 1.** Multilayer feed-forward neural network with input/output and one hidden layer.

\*For correspondence. (e-mail: ad\_sarma@yahoo.com)

**Table 1.** Comparison of foF<sub>2</sub> prediction error at Ahmedabad station for 1994 and 1997

No. of hidden nodes	March	May	September	November
1994	ssn = 34.1	ssn = 32.5	ssn = 26.6	ssn = 26.2
8	0.9773	0.6230	0.6611	0.9472
10	1.0004	0.6470	0.7134	0.9770
12	1.0156	0.6532	0.7202	0.9847
1997	ssn = 13.5	ssn = 18.3	ssn = 28.3	ssn = 35.0
8	0.8168	0.8269	0.8031	0.9944
10	0.8338	0.8161	0.7998	0.9994
12	0.8076	0.7874	0.8348	1.0260

simply back propagation. The MNN is a multi input and multi output nonlinear system having a layered structure, and its learning algorithm can be regarded as parameter estimation for such a nonlinear system. In the present work, the network consists of three layers, namely the input, hidden and output layers (Figure 1).

The input layer consists of four nodes and the output layer consists of one node. The number of nodes in the hidden layer can be varied and is approximately taken as  $(2n-1)$ , where  $n$  is the number of inputs. The network is trained by varying the number of hidden nodes with 8, 10 and 12. However, the network converged for an optimal solution with 8 hidden nodes. Inputs are normalized so that the maximum value of any input does not exceed unity. The input to the 1st, 2nd and 3rd nodes of the input layer are month, hour and ssn respectively. A bias value of 1 is given as input to the 4th node. These four inputs constitute the  $X_1$  matrix. The initial link weights  $W_1$  and  $W_2$  are selected from normal random values.  $W_1$  is a matrix of size  $8 \times 4$ , which gives the link weights between the input layer and the hidden layer.  $W_2$  is a matrix of size  $1 \times 9$ , which represents the link weights between the hidden layer and output layer. Inputs  $X_2$  (size  $8 \times 1$  matrix) to the hidden layer are given by

$$X_2 = [\text{sigmoid}(W_1 X_1)]. \tag{1}$$

The sigmoid function used is  $1/(1 + \exp(-x))$ .

Subsequently, another row of bias value is added to  $X_2$  to make it a  $9 \times 1$  matrix. Inputs to the hidden layer nodes are multiplied with the link weights in  $W_2$  using the sigmoid function to get  $X_3$ .  $X_3$  is the obtained output ( $y$ ) given by

$$X_3 = \text{sigmoid}(W_2 X_2). \tag{2}$$

For each iteration, the estimated output ( $y$ ) is compared with the desired output  $d(i)$ , and mean square error is calculated. After each iteration, the error terms for the output and hidden nodes are calculated. Based on the output error and using the momentum term, learning parameter and weights on the hidden and output nodes, new link weights  $W_1$  and  $W_2$  are calculated. This procedure is repeated iteratively, till minimum error is reached.

foF<sub>2</sub> data corresponding to the solar cycle 20 are used to train the three-layer MNN. While training the network, the learning rate parameter is varied from 0.8 to 0.01 along with the number of iterations. The outputs of the neural network program are two sets of weights  $W_1$  and  $W_2$ , which can be used to calculate the foF<sub>2</sub> value for any given hour and ssn. These weights are used to predict the foF<sub>2</sub>:

$$X_1 = [1/\text{month } 1/\text{hour } 1/\text{ssn } 1], \tag{3}$$

$$X_2 = [1/(1 + \exp(-(w_1 \cdot X_1))); 1], \tag{4}$$

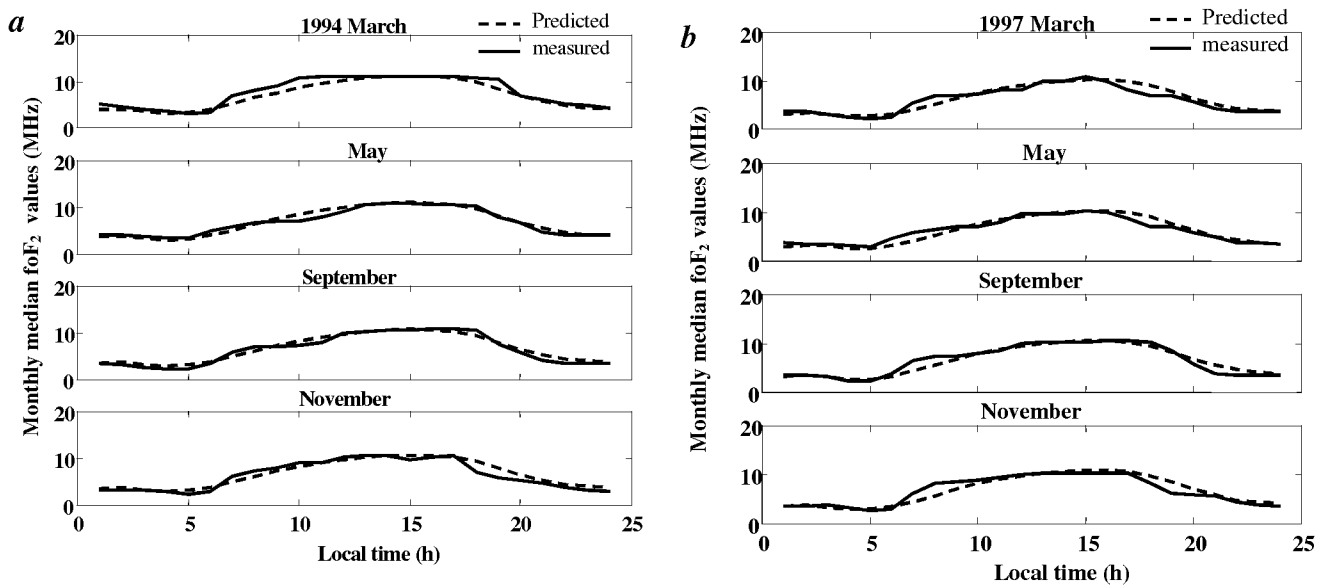
$$X_3 = 1/(1 + \exp(-(w_2 \cdot X_2))). \tag{5}$$

The predicted foF<sub>2</sub> can be obtained by multiplying  $X_3$  with the normalization factor (NF).

$$\text{Predicted foF}_2 = \text{NF} \times X_3. \tag{6}$$

Using the procedure described above, hourly foF<sub>2</sub> values are estimated for the years 1994 (solar cycle no. 22) and 1997 (solar cycle no. 23) by varying the number of hidden nodes with 8, 10 and 12. The corresponding rms error between the predicted and measured foF<sub>2</sub> values is presented in Table 1.

For 1994 data, the network with 8 hidden nodes is predicting foF<sub>2</sub> more accurately than the network with 10 or 12 hidden nodes. For 1997 data, the network with 8 hidden nodes is predicting the parameter better in November only, whereas the network with 12 hidden nodes is predicting better in March and May, while the network with 10 hidden nodes is predicting better for September data. Although, at times, the convergence and prediction capabilities of the network are better with increased number of hidden nodes, taking the complexity of the network into consideration, this aspect can be neglected. It is also evident from the results that the network is predicting the parameter foF<sub>2</sub> efficiently with 8 hidden nodes. The accuracy of the network can be improved by training the network with more data from several stations over 3–5 solar cycles. The actual and predicted foF<sub>2</sub> values are superimposed on one another and compared for the months of March, May,



**Figure 2.** Comparison of predicted and measured foF<sub>2</sub> data at Ahmedabad station with 8 hidden nodes for the year 1994 (a) and 1997 (b).

September and November corresponding to vernal equinox, summer, autumn equinox and winter respectively (Figure 2 a, b).

These plots show that the prediction error is less than 1 MHz. Therefore, this technique can be conveniently used for other stations also. It may be noted that median foF<sub>2</sub> values were used in this work. The idea of using the median values is to provide a reference base for an expected delay which should be corrected further based on the real time data that is continuously obtained.

Modelling of the monthly median ionosonde foF<sub>2</sub> data of Ahmedabad station is carried out using a three-layered neural network trained with back-propagation algorithm. Ahmedabad station is located close to the equatorial ionization anomaly, where the ionization density levels are unpredictable and also result in large day-to-day variability. The trained network with 8 hidden nodes is performing better than the network with 10 or more hidden nodes. The network is predicting foF<sub>2</sub> data efficiently in other solar cycles also. Further, the accuracy of the network in predicting foF<sub>2</sub> values can be improved by modelling the data over several solar cycles. Similar results over several stations together with IRI-2001, would provide better prediction of TEC accurately for GPS applications.

1. Aggarwal, S., Lakshmi, D. R. and Reddy, B. M., A simplified indexing of F-region geophysical noise at low latitudes. In *Solar Terrestrial Predictions (Proc.)* (ed. Donnelly, R. F.), US Department of Commerce, 1979, 1, pp. 134–139.
2. Dabas, R. S., Bharadwaj, R. M. and Lal, J. B., A study of total electron content and slab thickness variations at Kurukshetra. *Indian J. Radio Space Phys.*, 1979, vol. 8, pp. 15–19.

3. Wintoft, P. and Cander, L. R., Twenty-four hour predictions of foF<sub>2</sub> using time delay neural networks. *Radio Sci.*, 2000, **35**, 395–408.
4. Lamming, X. and Cander, L. R., Monthly median ionosphere frequencies prediction with neural networks. In Proceedings of the 2nd International Workshop on Artificial Intelligence Applications in Solar-Terrestrial Physics, ESA WPP-148 Proc., 1998, pp. 229–234.
5. Aggarwal, S. and Shastri, S., Atlas of ionospheric communication parameters over the Indian subcontinent, Radio Science Division, National Physical Laboratory, New Delhi, 1991.

**ACKNOWLEDGEMENTS.** Thanks are due to Dr B. M. Reddy, NGRI, Dr D. R. Lakshmi, NERTU, Osmania University and Mr B. S. V. Prasad, RCI, Hyderabad for valuable suggestions. Thanks are also due to Dr R. S. Dabas, NPL for providing data.

Received 24 March 2005; revised accepted 26 August 2005