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Explaining the obvious: the place of mathematics in the sciences*

If there is anything obvious in the rapidly changing world of modern science, it is the essential role of mathematics. Even biologists, many of whom react negatively (if not violently) to the claim that mathematics is necessary, have to acknowledge the various direct and indirect ways in which mathematics influences the growth of their disciplines. However, the problem of obviousness is that what is obvious is soon taken for granted. Not only is the use of mathematics now taken for granted, at least in the physical sciences, it has for long become indispensable to these disciplines. The state of indispensability, as many people in power realize, is always a sure sign of future decline! Although we do not yet see the sign of any impending decline of mathematics on the horizon, there are already claims about new kinds of science, including the one by Wolfram which attempts to model the world not through traditional mathematics but with algorithmic rules.

It is also the case that when something becomes so obvious, we tend to forget what it is doing there in the first place. Mathematics is so entrenched in modern science that scientists have forgotten to query the relation between mathematics and science. Those eminent scientists who reflected on this relation, including Galileo, Newton, Einstein, Wigner and Feynman, were struck by the mysteriousness of the applicability of mathematics.

There is really nothing obvious about using mathematics to understand our world. The reasons for this are at least three: mathematics seems to be independent of our world, only a small fraction of the mathematical literature is used in science and the same mathematics can be used to describe worlds contradictory to ours. These characteristics actually embody a simple truth about mathematics that is worth reflecting upon – mathematics is actually indifferent to the nature of our world. How then can such an indifferent activity be so essential to science? Furthermore, the mysteriousness of mathematics is enhanced when we discover that certain mathematical structures best describe some physical concepts. So what we find is that there is really nothing obvious about *explaining* the efficacy of mathematics although its use is taken for granted.

Part of the problem lies in our ambiguous understanding of the nature of mathematics. Is mathematics a language? Or a set of logical statements? Is it always about truth? Why is symbolization so important to it? Is there a relation between mathematics and natural language, like English? What are allowed as mathematical entities and operators? What is the notion of proof in mathematics? And so on. It is indeed remarkable how much mathematics has accomplished in the sciences while it has been so little understood by scientists and mathematicians!

This collection of articles hopes to address some of these issues. Many of them are written in the spirit of trying to understand what we take to be obvious today, namely, the indispensability of mathematics in the sciences. The set of articles here, we hope, will help the reader to reflect on this most important issue from different perspectives.

Rajaraman (**page 360**) begins with analysing the notion of applicability of mathematics. He notes that mathematics is a human activity, created and sustained by a community of mathematicians. Mathematical truth is that which is collectively accepted by the community. What then guarantees that its truth is independent of human interests? What makes mathematical truths different from other 'truths' in activities such as music, which is also a community dependent activity? It is in this distinction that Rajaraman finds a clue for understanding the applicability of mathematics. The difference between the mathematical world and the physical world lies in the exactness of mathematical statements in contrast to the approximate, context-dependent descriptions of the world. Contrary to common claims that mathematics is independent of our physical world, Rajaraman suggests that mathematics developed as a response to the physical world but increasingly grew apart as its scope of abstraction increased.

Mukhi (**page 366**) develops the above theme and describes how modern physics increasingly depended on mathematical results. Although physicists in the latter part of the last century made various contributions to the mathematics which they used as part of their physics, such as in the areas of Lie groups, differential geometry, fibre bundles and so on, they were not of significant use to the mathematicians. He notes that the role of mathematics, in this period, was characterized by the fact that mathematical results that were used by the physicists had already been obtained by the mathematicians. However, in the 1980s this situation changed and physics began to make significant contributions that are of use to the mathematical community. He captures the spirit of this new trend by invoking the phrase 'The unreasonable effectiveness of physics in mathematics'! He illustrates his argument through examples of knot theory, Morse theory, moduli spaces of Riemann surfaces and mathematics used in string theory.

In the scientific community one often hears the comment that chemistry is reducible to physics. Two different approaches to this question are developed by Mukherjee and Desiraju. Mukherjee (**page 371**) notes that, historically, physics evolved more rapidly as an exact science in comparison to chemistry. This is because the underlying laws of binding and transformations of chemical substance have their basis in the quantum behaviour of the constituents of matter. However, the study of chemical properties has a degree of autonomy in the sense that the desirable goals of a chemist (control of emergent chemical behaviour, designing molecule with specific chemistry) are determined by questions of chemical nature. In this sense, chemistry is more complex than physics. Chemistry is thus compatible with physical laws but not reducible to them. Further, he points out that the emerging frontier areas of theoretical chemistry already use algebraic topology, homotopy theory, advanced computer methods including simulations, theories of non-linearity and complexity, and various other mathematical techniques. Mukherjee argues that these activities will only intensify in the near future and modern mathematics will become more indispensable to chemistry, thereby redefining the boundaries of molecular science.

Offering a different perspective, Desiraju (**page 374**) begins with the problematic reduction of chemistry to physics and argues that new developments in chemistry counter this reductive picture. What we have today is chemistry that embodies both reducible and irreducible character. The reductive argument is important since physics itself is

*I am grateful to S. K. Rangarajan for suggesting this particular title.

seen to be reducible in some sense to mathematics. Desiraju points out that while there were branches of chemistry amenable to this reductive mode into physics and therefore into mathematics, organic chemistry posed a fundamental problem to such reduction. The development of supramolecular chemistry posed a new question for the chemists, namely, whether biology could be reduced to chemistry. He argues against such a simple reductive picture because of the ideas of complexity inherent in both supramolecular chemistry and biology. Emergence is an idea related to complexity and he argues that emergence and reductionism are antithetical to each other. An important observation that follows from this is that rather than looking at a top-down approach through reduction one could look at a bottom-up approach through the idea of emergence.

The question of mathematization in the context of biology is somewhat controversial. There are many biologists who claim that mathematization of biology goes against the very essence of biology. There are others who point to the uses of various mathematical methods in biology. Both these views are represented in this collection. Ramaswamy (**page 381**), speaking for the use of mathematics in biology, extends Wigner's comment about the unreasonable effectiveness of mathematics in the natural sciences to that of the biological sciences. He considers various examples of the use of mathematical methods in biology such as stochastics like Markov models, game theory, graph theory, computational techniques and so on. In general, the rich field of complexity has a wide variety of mathematical ideas that can be used in biology. Ramaswamy also offers an interesting suggestion to explain the importance of mathematics for any knowledge system. This is based on the possibility that language is hard-wired in humans. In particular, if mathematics is so hard-wired, then it is possible that mathematical ability gives an evolutionary advantage to humans.

In contrast, Nanjundiah (**page 388**) questions the idea that mathematics is necessary for biology in the same way as it is for physics. He argues that the evolutionary aspect of life and living beings makes it very difficult to 'reduce' biology to physics and chemistry. The ideas of change and chance are at the core of evolution, because of which evolution is unpredictable. Further, evolution is described by serious discontinuities. These discontinuities make it impossible to describe or explain biological progress in a rational, sequential way where what happens later can be explained in terms of what happened earlier. Moreover, the idea of history is central to evolution and therefore becomes essential to biology. Thus, one of the serious challenges to mathematization of biology comes from the discontinuous, historical and the arbitrary nature of evolution.

The character of applied mathematics has undergone a great change in the sciences. The shift from analytical expressions to computational methods is one that portends significant changes in the future use of mathematics. Hari Dass (**page 394**) considers this issue of computation as part of the larger culture of mathematization. He notes that the great impact of computers is seen in numerical simulations, which are computer simulations of systems. This method can be applied to a wide range of problems such as turbulence, large scale weather forecast, behaviour of biomolecules and quark confinement. The importance of such simulations lies in their ability to study phenomena across vastly different scales. Hari Dass concludes by noting that these new computational methods transcend artificial disciplinary boundaries, thereby suggesting the possibility that in the future science will be dominantly computational in character and, because of this, it will seem less dispersed.

Madhavan (**page 397**) offers a different analysis of computation, from the perspective of computer science. Reflecting the close relation between mathematics and computer science, he argues that mathematics and computer science mutually draw upon one another. The act of computing brings computer science and mathematics together in the first instance. The foundation of theoretical computer science is based on algorithms and programs. Solving problems, which are an essential part of mathematics, are also what algorithms by their very nature are designed to do. Programs are concerned with the various aspects of language that embody algorithms. Madhavan also discusses the notion of proof both in computer science and mathematics, and analyses various similarities in this notion as it is used in both these disciplines. In this rich interaction, computer science contributes new ideas to mathematics, such as in the area of discrete mathematics. Such a 'feedback' from computer science to mathematics is similar to Mukhi's observation about the impact of physics on mathematics today.

Just as biology stands between the natural and social sciences, computer science stands midway between science and engineering. The use of mathematics in engineering is sometimes seen to be qualitatively different when compared to its use in the natural sciences. Chatterjee (**page 405**) isolates two important differences: one is the emphasis on approximation in contrast to the emphasis on truth as in physics, and the other is an epistemic responsibility that engineers have when making knowledge claims. Engineers are concerned about converting their theories into action and thus the kinds of questions they ask, what Chatterjee calls technological questions, are different in spirit from scientific or mathematical questions. He considers many examples of the use of mathematics in engineering which reflect these unique concerns of engineering.

Many of these contributions invoke the phrase 'unreasonable effectiveness of mathematics', a phrase used by Wigner in the title of his highly influential paper. Sarukkai (**page 415**) revisits this paper to understand exactly what Wigner meant when he described the use of mathematics in the natural sciences as being unreasonably effective. Sarukkai argues that the 'mysteriousness' of applicability is partly a reflection of the mysteriousness of mathematics itself. His basic contention is that to explain the applicability of mathematics is first to recognize that mathematics is applied to language and not to the world. To know why mathematics seems to be so effective is to understand the capacity of languages in general to describe the world. By considering examples of how mathematics is applied, he explains the effectiveness of mathematics by arguing that the use of mathematics is a strategy to proliferate narratives about the world. Among the many possible narratives that can be generated with the help of mathematics, there is an increased probability that we will indeed find some that match approximately with the world.

In this collection, we have, for the most part, managed to talk about mathematics without using the language of mathematics. Such an approach is already a small step in creating a space for a dialogue not just about mathematics, but about larger issues concerning the practice and claims of science, a dialogue which is sensitive to the philosophical, historical and sociological dimensions of mathematics and science.

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