tension to complex geometries, the scope of the method has been extended greatly, making it the method of choice to solve linear boundary value problems on complex geometries.

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Is there a need for variable density option in making combined mass correction to gravity data acquired over high relief?

Rambhatla G. Sastry*, Neeta Chaudhary and Md. Israil

Department of Earth Sciences, Indian Institute of Technology, Roorkee 247 667, India

Traditional gravity data processing involves terrain and Bouguer corrections with uniform density. However, inclusion of variable densities honouring surface geology in such corrections is a long-felt need. This matter assumes fundamental importance for gravity data acquired over high relief. So, improved Bouguer and terrain correction scheme is proposed and its utility is demonstrated on a gravity profile along Mahe–Sumdo–Tso Morari of Ladakh Himalaya.

A maximum difference of about 50–70 mGal in the final Bouguer anomaly is observed between data processed through normal procedure with uniform Bouguer density (= 2.67 g/cm^3) and those proposed with variable density. This underlines the importance of the proposed scheme.

THE purpose of making corrections to gravity data is to arrive at Bouguer anomaly free of non-geologic effects that are unavoidable components of the basic measurement. In-depth analysis of gravity data corrections is clearly missing in basic English language textbooks[1]. In recent years, La Fehr[2] and Talwani[3] have dealt with the problem of making Bullard *B* correction. The method proposed by Banerjee[4] for gravity data acquired over high relief uses a digital terrain model and constant Bouguer density.

Northwestern Himalaya is characterized by high elevations and severely tectonized zones with steep dips, which pose challenging problems for geophysical data processing in general and gravity data, in particular. A new improved gravity data reduction method is proposed, involving variable density information. The effectiveness of the proposed procedure is illustrated on a gravity profile along Mahe–Sumdo–Tso Morari of Ladakh Himalaya.

The conventional land Bouguer gravity, Δg_B based on single constant density for combined mass correction is given by

$$\Delta g_B = g_{obs} - LC \pm FAC \mp BC + TC, \tag{1}$$

where g_{obs} refers to observed gravity reading, LC is latitude correction, FAC is free-air correction, BC is Bouguer correction and TC is terrain correction.

To incorporate the terrain correction term with variable density option in eq. (1), let us adopt Hammer's template for the terrain around the gravity station covered by N circles of radii R_i ($i = 1, n$ for the inner zone and $i = n + 1, N$ for the outer zone), with $m(i)$ compartments the inner zone ($j = 1, m(i)$) and $M(i)$ compartments in the outer zone ($j = 1, M(i)$). Then TC in eq. (1) can be expressed as

$$TC = \sum_{i=1}^{n}\sum_{j=1}^{m(i)} f_{ij}\rho_{ij} + \sum_{i=n+1}^{N}\sum_{j=1}^{M(i)} f_{ij}\sigma, \tag{2}$$

where ρ_{ij} and σ are the variable and constant Bouguer densities of the inner and outer zones (Figure 1) respectively, and following Grushinsky and Sazhina[5], one can have

$$f_{ij} = \frac{1}{C}(2\pi G((R_{i+1} - R_i) + \mathrm{sqrt}(R_i^2 + h_{ij}^2) - \mathrm{sqrt}(R_{i+1}^2 + h_{ij}^2))), \tag{3}$$

where

$$C = \begin{cases} m(i), i = 1, n, & \text{for the inner zone} \\ M(i), i = n+1, N, & \text{for the outer zone.} \end{cases} \tag{4}$$

h_{ij} are elevation differences of the ijth curvilinear prism of topography with respect to station elevation d. G is the universal gravitational constant. It is to be noted that the topographic masses around a gravity station delimited by n zones of Hammer's chart in the first summation are for a variable density inner zone and the remaining (N–n) zones in the second summation constitute the outer zone with a constant density σ respectively.

So far, we have made necessary modifications in terrain correction term for the case of variable density option. However, this situation also demands a modification of Bouguer correction term.

BC in the absence of variable densities is given by

$$BC = 2\pi G\sigma d. \tag{5}$$

*For correspondence. (e-mail: rgss1fes@iitr.ernet.in)

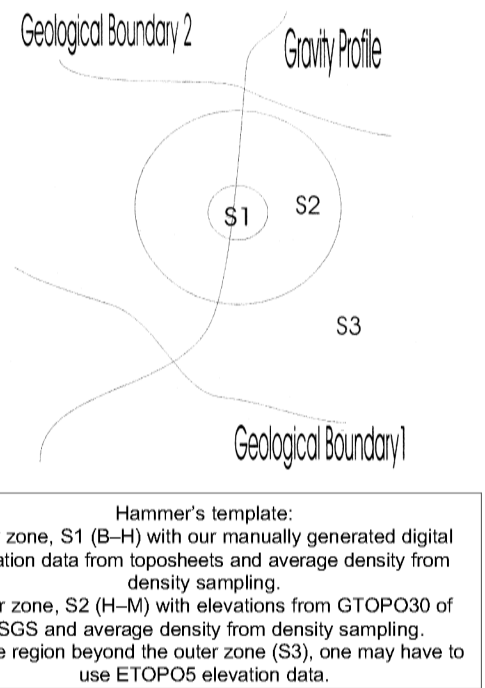

Figure 1. Schematic diagram in plan view explaining the adopted scheme of the two-zone Hammer's method. S1 and S2 are respectively, variable density inner and outer zones and S3 is the outermost zone of constant density (= 2.67 g/cm^3) beyond the M zone (≈ 21.9 km) of Hammer's template. The outermost zone of density S3 is not taken into account in the reported gravity examples as its use demands digital elevation data from ETOPO5 of USGS.

However, for incorporation of variable densities in the traditional BC, the Bouguer slab of thickness d can be divided into two regions, variable density (ρ_{ij}) inner zone and constant density (σ) outer zone.

The estimation of modified Bouguer correction can be achieved in two stages: (i) Initially by carving out a circular region around the gravity station delimited by R_i and removing that mass to have the gravity effect of constant density (σ) outer zone and (ii) later by filling that circular region by mass of variable density constituting the inner zone.

Considering the first stage, the constant density outer zone gravity effect, $\mathrm{BC_{outer}}$ in the Bouguer slab is given by:

$$\mathrm{BC_{outer}} = 2\pi G\sigma d - \sum_{i=1}^{n}\sum_{j=1}^{m(i)} F_{ij}\sigma, \tag{6}$$

where

$$F_{ij} = \frac{1}{C}(2\pi G((R_{i+1}-R_i)+\mathrm{sqrt}(R_i^2+d^2)-\mathrm{sqrt}(R_{i+1}^2+d^2)))$$

and C is given by eq. (4).

In the second stage, filling in the carved-out region with mass of variable density in the inner zone (eq. (6)) leads to the following equation for total Bouguer effect:

$$\mathrm{BC} = 2\pi g\sigma d - \sum_{i=1}^{n}\sum_{j=1}^{m(i)} F_{ij}\sigma + \sum_{i=1}^{n}\sum_{j=1}^{m(i)} F_{ij}\rho_{ij}. \tag{7}$$

In view of eqs (1), (2) and (7), we can have the following expression for Bouguer anomaly:

$$\Delta g_{\mathrm{B}} = g_{\mathrm{obs}} - \mathrm{LC} \pm \mathrm{FAC} \mp \left(2\delta + G\sigma d\sum_{i=1}^{n}\sum_{j=1}^{m(i)} F_{ij}(\rho_{ij}-\sigma)\right) + \left(\sum_{i=1}^{n}\sum_{j=1}^{m(i)} f_{ij}\rho_{ij} + \sum_{i=n+1}^{N}\sum_{j=1}^{M(i)} f_{ij}\sigma\right). \tag{8}$$

The last term in eq. (8) corresponds to TC of eq. (2). Equation (8) can be implemented using Hammer's terrain correction scheme. It may be noted that eq. (8) is in most general form for implementing both Bouguer and terrain corrections for variable densities. Depending on the geological situation at hand, some of the densities in the inner zone could even be constant and the processing scheme offered in eq. (8) is flexible in accommodating them. Further, the choice of datum for elevations is user-defined and in no way could the proposed scheme be held responsible for the resulting Bouguer anomaly.

A regional profile from Ladakh Himalaya along Mahe–Sumdo–Tso Morari (Figure 2) is processed with the help of eq. (8) and GRAVMASTER software of GEOTOOLS. Density measurements of collected representative samples from different geological formations along the profile (Figure 3) formed the input data for eq. (8). The reference Bouguer datum is mean sea level (msl).

The processing of Mahe–Sumdo–Tso Morari profile (Figure 4) was undertaken with two options, viz. (a) conventional constant Bouguer density, D = 2.67 g/cm^3 (for both Bouguer and terrain corrections) and (b) variable density for both Bouguer and terrain corrections (Figure 3).

Considering the geological map (Figure 2), the available density measurements for key lithologies, the digital elevation data from manually digitized toposheets (1 : 50,000) and the global elevation data, it was decided to have an inner zone extending up to a radius of 2.6 km (up to H zone of Hammer's template; Table 1) and an outer zone extending from 2.6 to 21.9 km (from H to M zones of Hammer's template; Table 1).

The outer zone (H–M zones) elevations are taken from GTOPO30 (USGS digital elevation data) available in GEOTOOLS software, while toposheets provided elevations for the inner zone. Beyond 21.9 km (M zone), one has to use a global digital elevation model (ETOPO5) constructed by USGS. It has to be noted that GRAVMASTER of GEOTOOLS is intervened and in association with MS Excel the proposed scheme is implemented.

Accordingly, Figure 1 illustrates the deployed scheme of making corrections to gravity data with n = 7 and N = 12 with corresponding zone-wise compartments (Table 1)

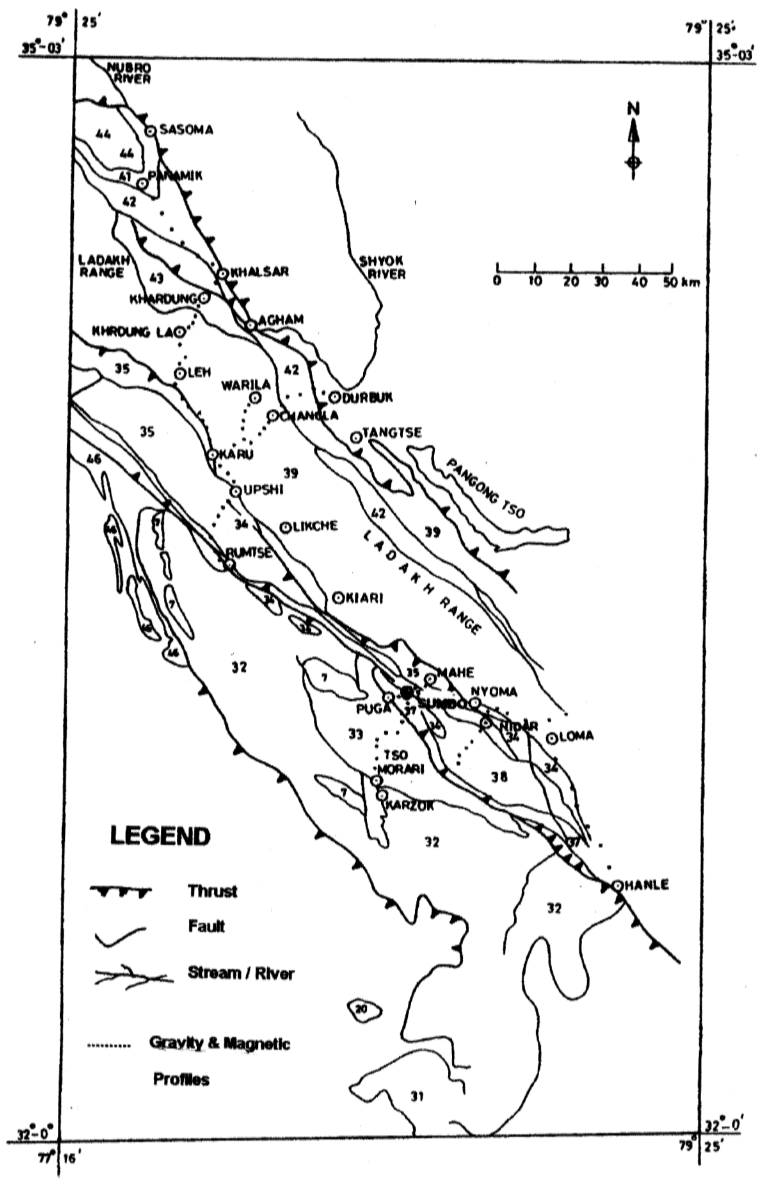

Figure 2. Geological map of Ladakh Himalaya showing position locations of our gravity and magnetic profiles. Dotted lines are profiles for acquiring gravity and magnetic measurements (Adapted from Thakur[8]; Thakur and Misra[9] and Thakur and Rawat[10].) Formations identified by numerals 34 and 37 under Indus and Shyok Suture Zones are in same geological group.

Tethys Himalayan Zone

Formation code	Formation/lithology
31	Undifferentiated Palaeozoic
7	Granite, 500 Ma (Jispa)
Tso Morari Crystallines	
7	Granite, 500 Ma (Polokong, Ruptsu)
32	Taglang la, Low-grade Mansarovar metasediments
35	Puga, Gurla, Mandhata

Indus Suture Zone

Indus Zone	Shyok Suture Zone
34 Kargil, Lyon, Kailas	40 Diong Formation
35 Indus Formation	34 Kole Molasse
36 Dras volcanics, Nindam	41 Saltoro Molasse
37 Shergol, Zildat ophiolitic Melange	42 Hundri Formation
38 Nidar ophiolite	43 Khardung volcanics
39 Ladakh Plutonic Complex	37 Ophiolitic Melange
	44 Shyok Formation
45 Spongtong Klippe	
46 Lamayuru	

Table 1. Hammer's gravity terrain correction

Zone (i =)	B (i =1)	C (i = 2)	D (i = 3)	E (i = 4)	F (i = 5)	G (i = 6)	H (i = n = 7)	I (i = n + 1 = 8)	J (i = 9)	K (i = 10)	L (i = 11)	M (i = N = 12)
Compartment	$m(1)$ = 4	$m(2)$ = 6	$m(3)$ = 6	$m(4)$ = 8	$m(5)$ = 8	$m(6)$ = 12	$m(7)$ = 12	$M(8)$ = 12	$M(9)$ = 16	$M(10)$ = 16	$M(11)$ = 16	$M(12)$ = 16
Inner radius (m)	2	16.6	53.24	170	390	894.9	1529.49	2614.57	4468.98	6652.56	9902.95	14741.65
Outer radius (m)	16.6	53.24	170	390	894.9	1529.49	2614.57	4468.98	6652.56	9902.95	14741.65	21944.38

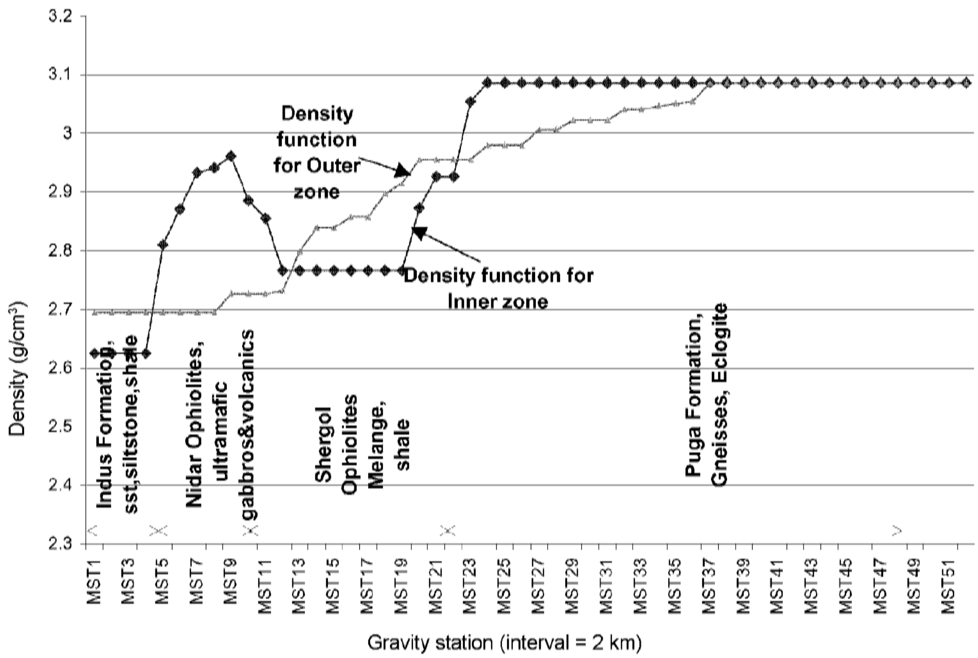

Figure 3. Density values for inner and outer zones for different gravity stations along Mahe–Sumdo–Tso Morari profile.

for the inner and outer zones respectively, with the help of eqs (2) and (8). Based on density sampling values spread over the entire inner zone, mean density (for all compartments confined to the inner zone) is computed; a similar computation is done for the outer zone also.

Figure 4 depicts the processed gravity profiles with and without variable density option. The gravity values are referred to a local datum. The Bouguer plate density is chosen to be that of the respective outer zone.

The effect of choice of variable density is clearly evident on the Mahe–Sumdo–Tso Morari profile, where a maximum of 70 mGal is noticed in the ultimate Bouguer anomaly. The implication of such differences could have wide ramifications in the interpretation. The Bouguer anomaly processed with variable density option clearly reflects (Figure 4) the signature of Nidar ophiolites.

The proposed variable density scheme based on density measurements on rock samples of geological formations encountered along profiles, is restricted to two zones in the cited profile (Figure 4). The prime reasons for such a choice are due to the existing geological maps in Ladakh Himalaya and steeply dipping geological strata. The appreciable difference in the processed anomaly using the proposed scheme needs to be noted, even though it has been implemented in its simple form. The choice of Bouguer density is solely guided by local geology rather than default Bouguer density corresponding to average crustal density. Further, choice of datum for elevations is user-dependent. The unit model opted here (eqs (6) and (8)) is a traditional vertical curvilinear prism, which suits the reported profile needs. However, it could be replaced by a 3D body of arbitrary shape (polyhedral bodies) to accommodate the structural geological inputs of the study region. Accordingly, gravity forward modelling can be used for polyhedral bodies[6].

Presently, the proposed method is explained on popular Hammer's terrain correction template and related charts. However, its replacement by alternate schemes[7] may alter the method of computing the combined mass correction, but not the basic equations mentioned here.

The proposed scheme is in a way similar to the 'geological correction' which geophysicists apply occasionally. It

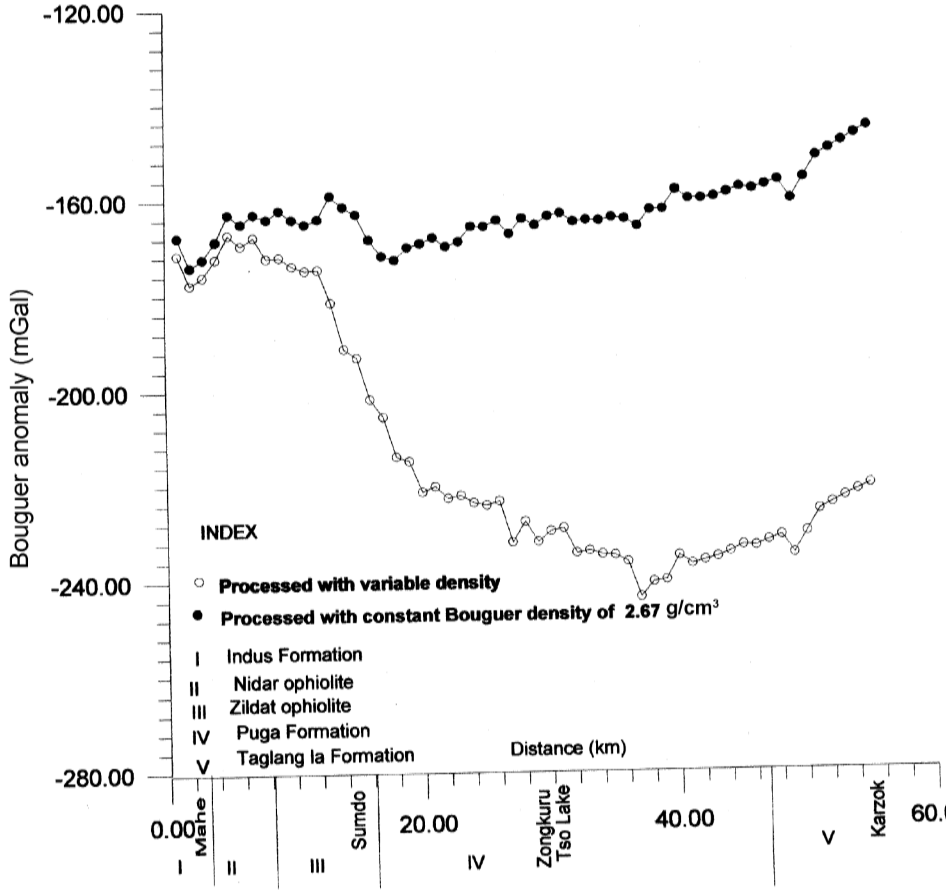

Figure 4. Bouguer anomaly plot along Mahe–Sumdo–Tso Morari profile with and without variable density option.

is usually a good idea to employ mean density in Bouguer and terrain corrections and defer use of variable densities until interpretation of Bouguer gravity anomalies. However, in our case, the profile passes through highly tectonized country involving variable density formations and the adopted procedure clearly highlights the anomalous ophiolite zone (Figure 4). It may be noted that here the Bouguer datum is set as msl and the resulting discrepancy in Bouguer anomaly clearly highlights the need for variable densities. However, continuance of surface lithologies up to msl is a hypothesis to predict the maximum amount of error in Bouguer anomaly computation. The nature of discrepancy in computed anomaly (phase) remains the same except for amplitude, if one delimits the datum to any higher level.

The present attempt is a small step in the right direction and one needs to evolve tailor-made gravity data processing to answer geological problems at hand, rather than conventional universal text-book approaches.

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