# How to convert subtraction into addition

The January 2004 edition of the *Scientific American* carried a fascinating article titled 'Curious History of the First Pocket Calculator', recounting the story of a hand-held mechanical calculator perfected before the dawn of the solid-state electronics age[1]. With a crank on the top, it closely resembled the pepper grinder on your dining table, handy and as small, but decorated with number readouts (Figure 1). Its inventor, Curt Hertzstark (1902–88), was of half-Jewish German descent and one does not have to stretch one's imagination too much to see the sort of dangers he was exposed to as a young man under the Nazi regime in the 1930s and the World War II years. What probably saved him was his usefulness as a precision instrument maker/repairer, a skill he acquired, even as a boy, in his father's fine mechanics workshop.

Thinking about the design of his calculator, Hertzstark decided it should operate on a cylindrical principle, not on a flat bed, using gears, cams and steps cut into or projecting from a rotatable drum and auxiliary shafts. The working numbers, which can be entered by moving sliders along the body of the calculator, would be visible at the upper ends of the slots. The results of a calculation could be read out in a register constituted by numbers visible in little windows around the periphery of the top rim. Hertzstark could easily visualize how his little gadget might work for carrying out additions and multiplication. However, he could not immediately see how the operation of turning a crank could bring the desired numbers into their operating positions for carrying out subtractions – certainly not by turning the crank backwards, the cams would prevent that! In Hertzstark's own words[1]: 'Adding two digits may create a carry condition *after* the operation, but subtraction requires a borrow *beforehand*. A single arithmetic step drum couldn't properly look ahead to see what might be coming'. But the method came to him in a flash: 'Traveling in a train through the Black Forest, I sat alone in a compartment. Looking out the window, I thought, "Good grief! One can get the result of a subtraction by simply adding the complement of a number".'

As Stoll[1] describes it, the first step in the process was taking the nines complement of a number by 'subtracting each digit from nine'. By adding a number to the complement of another number, you can simulate subtraction. For example, to calculate 788,139 minus 4890, first find the nines complement of 004,890, i.e. 995,109. Now add 788,139 and 995,109 to get 1,783,248. Remove the highest-order digit to arrive at 783,248. Finally, add one to find the answer: 783,249. Sweet – the same technique is used in computers today.

'Hertzstark's calculator would retain the single rotating step drum, but would have two sets of teeth: one set dedicated to addition, the other to subtraction. Lifting the crank three millimeters would engage the subtraction teeth to perform nines-complement addition. Subtraction would be as easy as addition. Repeated addition and subtraction would handle multiplication and division. And since the results register could be rotated in relation to the input sliders, several shortcuts would speed these operations. For instance, to multiply by 31,415, you don't spin the crank 30,000-plus times; the movable carriage cuts this to 14 turns: five turns for the 5, once for the 10, four times for the 400, and so on'. (Reference 1 has a detailed description of how the calculator operated, with beautiful illustrations of cutaway views of its inner workings.)

The capabilities of the Curta Calculator, as it came to be known, were advertised in glowing terms in the *Scientific American*: 'Weighs only 8 oz. THE CURTA IS A PRECISION CALCULATING MACHINE FOR ALL ARITHMETICAL OPERATIONS. Curta adds, subtracts, multiplies, divides, [takes] square and cube roots, [does] continuous multiplication, negative multiplication, standard deviations and all statistical calculations, squares and higher powers, coordinates and associated land survey formulae, and every other computation arising in science and commerce. Available on a trial basis. Price $ 125.00. Write for literature'.

Reading Stoll's article[1] brought back a faint memory of my having come across some kind of a 'nines-complement method' in an article or book. Wasn't it *Vedic Mathematics* by Bharathi Krishna Tirtha?[2] I wondered if the method discovered by Hertzstark, in the context of making a mechanical contrivance work for subtraction, had been anticipated by those who developed methods, in a non-mechanical context of course, of what has come

**Figure 1.** Curta calculator (source: ref. 1).

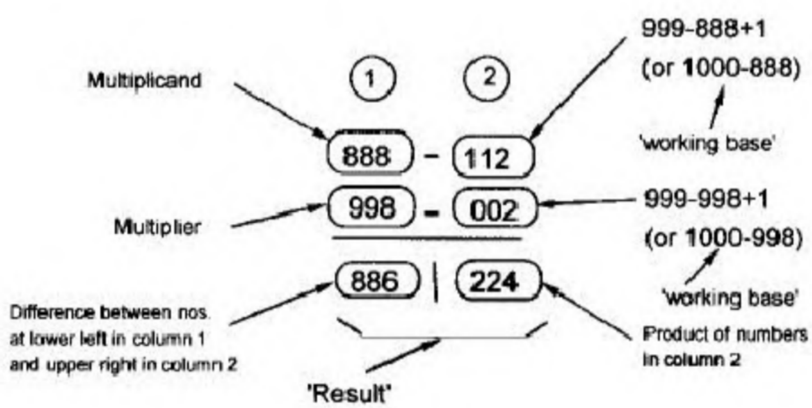


**Figure 2.** Example of multiplication using the 'nines complements plus one' method.

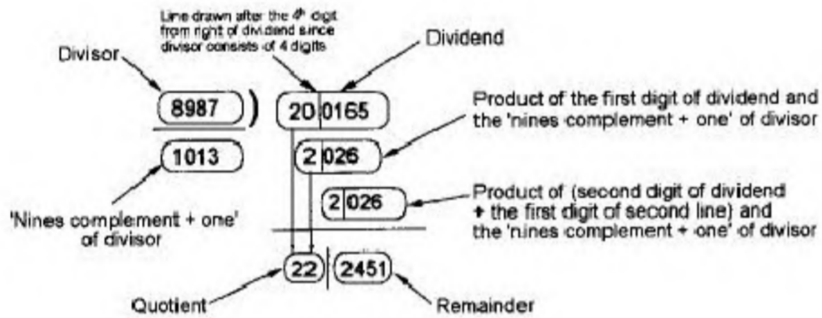


**Figure 3.** Illustration of division using the 'nines complements plus one' method.

to be known as 'Vedic mathematics.' I believe that Hertzstark's method is not 'derivable' in some formal sense, but is subject to what is called inductive 'proof' – just as the methods delineated in 'Vedic mathematics' are – if it works for $n$ and is found to work for $n + 1$, it works!

Yes, 'Vedic mathematics' had used something like the 'nines-complement method' in various places[2], but not quite in Hertzstark's manner for subtraction. Illustrated with a simple example in Figure 2, is the short-cut Vedic method of multiplication based on the *sutra*: *nikhilam navatascaramam dasatah* (everything from nine and the last from ten, that is, the last digit is to be subtracted from ten).

More generalized procedures are needed to meet contingencies that may arise even in the normal application of the method. For instance, either the multiplicand or the multiplier may not be near an integer power of 10. (This is not the case with the example of Figure 2, where the 'working base,' 1000 is close to both multiplicand and multiplier.) Such procedures, based on the very principles of the Vedic method, are fully described by Bharathi Krishna Tirtha[2].

A similarly simple example of division, based on the same *sutra*, is given in Figure 3. It is interesting to note that, except when taking the nines complements, no subtraction is needed in the division process and only single-digit multiplication is involved.

The originators/propagators of the Vedic procedures have not explicitly stated[2] how a subtraction can be carried out employing the 'nines-complement method' (or a modification thereof, whereby the last digit is subtracted from 10 and not 9). Would it, in effect, have been the same as that of Hertzstark's? If so, Hertzstark can only be regarded as having independently discovered a 'Vedic' method, assuming, of course, that the latter has priority. However, there seems no way of getting to know the date(s) of the origins of that method. We remark that, unlike Hertzstark, the originators of the 'Vedic' method went on to develop procedures for multiplication and division without any impetus from trying to make a mechanical contrivance work on the principles that they discovered.

It is too late in the day to ask whether a mechanical (or even an electronic) device would become 'simplified' when principled to operate using the Vedic procedures of multiplication and division. No one in this Age of Doped Silicon would want to try to construct a purely mechanical device just to answer that question!

1. Stoll, C., *Sci. Am.*, 2004, **290**, 82.
2. Bharathi Krishna Tirtha, *Vedic Mathematics*, Motilal Banarsidas, Delhi, 1992 revised edn (first edition 1965).

S. N. BALASUBRAHMANYAM

*'Sarasakshetra',*
*Machohalli,*
*Bangalore 560 091, India*
*e-mail: sub@orgchem.iisc.ernet.in*

# Flaw in statistical method

The article by Bhattacharyya *et al.*[1] on arsenic-polluted groundwater in West Bengal has serious flaws in the statistical methods used for analysis of data based on which some conclusions with far-reaching impact have been drawn. For example, 'the regression of percentage arsenic removal . . . against arsenic concentration produces a best-fit line ($R^2 = 0.001$) that parallels the abscissa'. The authors should know that expecting a best-fit line with an $R^2$ value of 0.001 for a relationship which is a loose scatter, is not correct. This relationship is statistically non-significant even at the 5% level of probability and therefore does not warrant any conclusions to be drawn from it, definitely not the kind the authors have made. An $R^2$ value of 0.06 and an $r$ value of 0.250 are necessary (for a sampling size of 62) for the relationship to be called even moderately strong and significant (i.e. at 5% level of probability).

Based on these loose relationships (Figure 3 for clay-candle filter and Figure 5 for membrane filter), the authors conclude that 60% of the arsenic is removed by filtration using the clay-candle filter, irrespective of initial arsenic content. This, if true, has immense benefits in tackling this grave problem which affects thousands of ordinary people in West Bengal, where arsenic in groundwater is a serious problem. The data and the relationships shown in the article do not warrant such a conclusion.

1. Bhattacharyya, D. *et al.*, *Curr. Sci.*, 2004, **86**, 1206–1209.

V. R. SASHIDHAR

*Department of Crop Physiology,*
*University of Agricultural Sciences,*
*GKVK Campus,*
*Bangalore 560 065, India*
*e-mail: vrsashi@yahoo.com*

*Response:*

The authors appreciate the comments of Shashidhar and find that there is absolutely no difference of opinion so far as