# Resonant sum frequency generation in a semiconductor with a magnetic wiggler

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**Two laser beams propagating collinearly through a semiconductor in the presence of a wiggler magnetic field produce phase-matched sum frequency electromagnetic radiation. The magnetic wiggler provides the uncompensated momentum required to make the process a resonant one. The required wiggler wave number falls with plasma frequency and sum frequency.**

PROPAGATION of electromagnetic waves through plasmas and semiconductors is an area of significant research activity with applications in signal processing, optical switching, communication, inertial confinement fusion, material processing etc.[1–4]. Many interesting phenomena have been identified and investigated, notable among them are harmonic generation, parametric instabilities, phase conjugation, tunnel ionization, charged particle acceleration, wave mixing, etc.[5–10]. Amongst these, wave mixing has applications in coherent XUV generation, indirect inertial confinement fusion, diagnostics, harmonic generation and wavelength conversion. Federici[7] has given an excellent review of four-wave mixing and phase conjugation in plasmas. In sum frequency generation process, two photons of energy $\hbar\omega_{1,2}$ and momentum $\hbar\vec{k}_{1,2}$ each interact to generate a photon of sum frequency radiation of energy $\hbar\omega_+$ and momentum $\hbar\vec{k}_+$, where $\omega_{1,2},\vec{k}_{1,2}$ and $\omega_+$, $\vec{k}_+$ satisfy the linear dispersion relation for electromagnetic waves. The energy and momentum conservation in sum frequency generation process demands.

$$\omega_+ = \omega_1 + \omega_2,\ \vec{k}_+ = \vec{k}_1 + \vec{k}_1. \tag{1}$$

These conditions are quite restrictive. Plasmas and semiconductors are dispersive media (which means that index of refraction increases with frequency). Hence eq. (1) is not satisfied, i.e. under usual conditions, one cannot have efficient generation of sum frequencies. In plasmas, phase-matched high order difference frequency mixing has been achieved by varying the plasma pressure. Moore[11] has given theoretical and numerical analysis of resonant sum frequency generation in an external cavity by varying the input coupler reflectivity.

In this communication we propose a scheme for generating the resonant sum frequency in a semiconductor by employing a transverse wiggler magnetic field[12] to it. Placing bar magnets of alternate polarity over the semiconductor can produce the latter. The wiggler can be viewed as a photon $(0,\ k_0\hat{z})$ of zero energy and momentum $\hbar\vec{k}_0$ that can compensate for the unbalanced momentum between the generated and interacting photons,

$$\hbar\vec{k}_+ - \hbar(\vec{k}_1 + \vec{k}_2) = \hbar\vec{k}_0. \tag{2}$$

Consider the propagation of two laser beams,

$$\vec{E}_j = \hat{x}A_j\exp[-i(\omega_j t - k_j z)],$$

$$\vec{B}_j = \hat{y}(ck_j/\omega_j)A_j\exp[-i(\omega_j t - k_j z)], \tag{3}$$

$j = 1, 2$, in an $n$-type semiconductor (cf. Figure 1) in the presence of a wiggler magnetic field,

$$\vec{B}_{\mathrm{w}} = \hat{y}B_0 e^{ik_0 z}. \tag{4}$$

The incident and generated electromagnetic waves obey the linear dispersion relation,

$$k^2 \approx (\omega^2/c^2)[\varepsilon_L - \omega_p^2/\omega(\omega + i\nu)], \tag{5}$$

where $\varepsilon_L$ is the lattice permittivity, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, $c$ is the velocity of light in vacuum, $n_0$ is the carrier(electron) concentration, $-e$, $m$ and $\nu$ are the charge, mass and collision frequency of electron respectively. The wave vector $\vec{k}$ increases more than linearly with frequency $\omega$, hence $\vec{k}(\omega_+) > \vec{k}(\omega_1) + \vec{k}(\omega_2)$.

Taking all the $k$s parallel to each other and employing eqs (2) and (5), one obtains the wiggler wavenumber required for sum frequency generation as

$$k_{0+} = \frac{\omega_+}{c}\left(\varepsilon_L - \frac{\omega_p^2}{\omega_+(\omega_+ + i\nu)}\right)^{1/2} - \left\{\frac{\omega_1}{c}\left(\varepsilon_L - \frac{\omega_p^2}{\omega_1(\omega_1 + i\nu)}\right)^{1/2} + \frac{\omega^2}{c}\left(\varepsilon_L - \frac{\omega_p^2}{\omega_2(\omega_2 + i\nu)}\right)^{1/2}\right\}. \tag{6}$$

In Figure 2 we have shown the variation of normalized required wiggler wavenumber $ck_{0+}/\omega_p$ with normalized laser frequency $\omega_1/\omega_p$ and frequency ratio of the frequency mixing waves $\omega_1/\omega_2$. The required wiggler wavenumber decreases with the plasma frequency and the sum frequency.

On solving the equation of motion $m(\mathrm{d}\vec{v}/\mathrm{d}t) = -e\vec{E} - e(\vec{v} \times \vec{B})/c - m\nu\vec{v}$, we obtain the electron velocity due to two waves as

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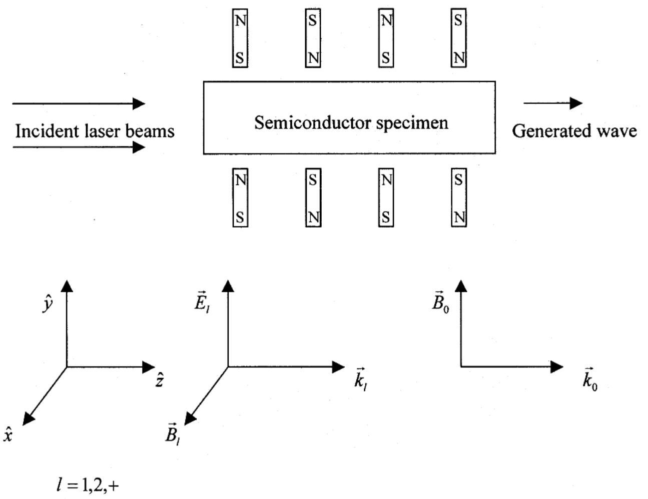


**Figure 1.** Schematic of the process.

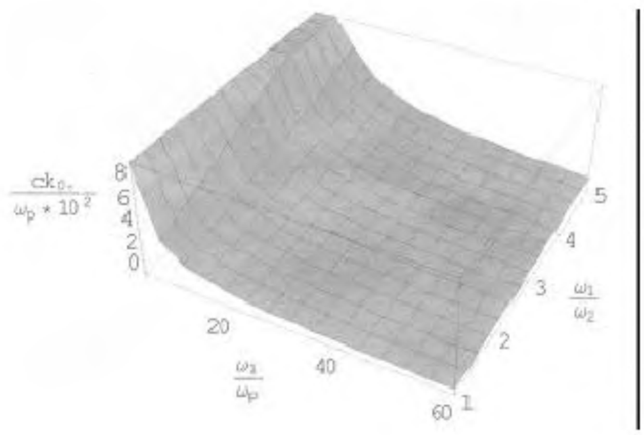


**Figure 2.** Variation of normalized wiggler wavenumber with laser frequency and frequency ratio of two incident waves.

$$\vec{v}_j = \frac{\hat{x}eA_j}{mi(\omega_j + iv)} \exp[-i(\omega_j t - k_j z)]. \tag{7}$$

$\vec{v}_1$ beat with $\vec{B}_2$ and $\vec{v}_2$ with $\vec{B}_1$ to exert a ponderomotive force $\vec{F}_{p+}$ on electrons at $(\omega_1 + \omega_2,\ \vec{k}_1 + \vec{k}_2)$.

$$\vec{F}_{p+} = -\frac{e}{2c}[\vec{v}_1 \times \vec{B}_2 + \vec{v}_2 \times \vec{B}_1],$$

producing

$$\vec{v}_+ = -\hat{z}\frac{e^2 A_1 A_2}{2m^2 \omega_+}\left[\frac{k_1}{\omega_+(\omega_2 + iv)} + \frac{k_2}{\omega_2(\omega_1 + iv)}\right], \tag{8}$$

$\vec{v}_+$ beat with $\vec{B}_w$ to produce transverse sum frequency ponderomotive force $\vec{F}_{p+}$, at $(\omega_+,\ \vec{k}_+ + \vec{k}_0)$ as

$$\vec{F}'_{p+} = -\frac{e}{2c}\vec{v}_+ \times \vec{B}_W.$$

Electron oscillatory velocity due to $\vec{F}'_{p+}$ is

$$\vec{v}'_+ = \hat{x}\frac{e}{2c}\frac{v_+ B_w}{mi(\omega_+ + iv)} e^{-i[\omega_+ t - (k_+ + k_0)z]}. \tag{9}$$

The self-consistent field $\vec{E}'_+ \exp[-i(\omega_+ t) - (k_+ + k_0)z]$ also produces electron oscillatory velocity as

$$\vec{v}'^{L}_+ = \frac{e\vec{E}'_+}{mi(\omega_+ + iv)}. \tag{10}$$

Using eqs (9) and (10), the nonlinear current density at $(\omega_+,\ \vec{k}_+ + \vec{k}_0)$ can be written as

$$\vec{J}'_+ = -n_0 e(\vec{v}'_+ + \vec{v}'^{L}_+)$$

$$= -\frac{n_0 e^2 \vec{E}'_+}{mi(\omega_+ + iv)} + \hat{x}\frac{e^4 A_1 A_2 n_0 B_w}{4m^3 i(\omega_+ + iv)\omega_+}$$

$$\left[\frac{k_2}{\omega_2(\omega_1 + iv)} + \frac{k_1}{\omega_1(\omega_2 + iv)}\right] e^{-i[\omega_+ t - (k_+\ k_0)z]}. \tag{11}$$

The sum frequency field is governed by the wave equation[13]

$$\nabla^2 \vec{E}'_+ = \frac{4\pi}{c^2}\frac{\partial \vec{J}'_+}{\partial t} + \frac{\varepsilon_L}{c^2}\frac{\partial^2 \vec{E}'_+}{\partial t^2}. \tag{12}$$

Using the expression for $\vec{J}'_+$ from eq. (11) in eq. (12), one gets,

$$\vec{E}'_+ = \hat{x}\frac{\omega_p^2 B_w e^2 A_1 A_2}{4cm^2(\omega_+ + iv}\left[\frac{k_2}{\omega_2(\omega_1 + iv)} + \frac{k_1}{\omega_1(\omega_2 + iv)}\right]$$

$$\frac{1}{[\omega_+^2 \varepsilon_L - (k_1 + k_0)^2 c^2 - \omega_p^2 \omega_+ /(\omega_+ + iv)]}. \tag{13}$$

We set the real part of the denominator corresponding to phase matching condition

$$\omega_+^2 \varepsilon_L - (k_+ + k_0)^2 c^2 - \omega_p^2 \approx 0. \tag{14}$$

Assuming $v \ll \omega_1, \omega_2$, eq. (13) reduces to

$$\left|\frac{E'_+}{A_1}\right| \approx \frac{e^2 A_2 B_w}{4cm^2 \omega_1 \omega_2 v}\left[k_+^2 + v^2\left(\frac{k_1}{\omega_2} + \frac{k_2}{\omega_1}\right)^2\right]^{1/2}. \tag{15}$$

**Table 1.** Variation of $|E_+'/A_1|$ with $\omega_1/\omega_2$ and $\lambda_w$

| $\omega_1/\omega_2$ | $\lambda_w$ (cm) | $|E_+'/A_1|$ |
|---|---|---|
| 1.5 | 1.47 | 0.12 |
| 2 | 1.07 | 0.17 |
| 2.5 | 0.89 | 0.23 |
| 3 | 0.76 | 0.29 |

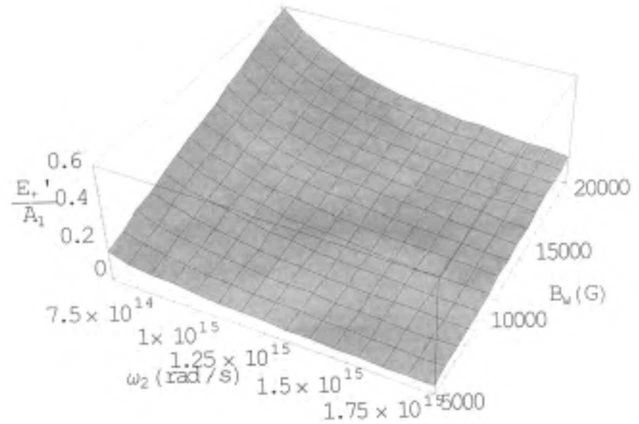


**Figure 3.** Variation of $|E_+'/A_1|$ with the frequency of the second laser $\omega_2$ and wiggler field strength $B_w$.

For a typical case of $n$-type germanium with electron density $10^{17}$ cm$^{-3}$, $\varepsilon_L = 14$, $v = 2 \times 10^{11}$ s$^{-1}$, $m = 0.3\ m_0$, $m_0 = 9.1 \times 10^{-28}$ g, second laser with power $2 \times 10^6$ W/cm$^2$, $\omega_1 = 1.8 \times 10^{15}$ rad/s (Nd:YAG laser), $B_w = 10$ kG, the variation of $|E_+'/A_1|$ with $\omega_2$ and $\lambda_w$ (wiggler period) is shown in the Table 1. The variation $|E_+'/A_1|$ with $\omega_2$ and $B_w$ is shown in Figure 3. The required wiggler wavenumber falls with plasma density and sum frequency. At higher values of plasma density the required wiggler period attains impractically smaller values. The scheme is useful in low-density plasmas and semiconductors where the amplitude of the density oscillations is not large. Employing a stronger wiggler magnetic field as well as a guide magnetic field can increase the efficiency. The estimated efficiency of the process is only an upper bound as laser propagation through plasmas and semiconductors is affected by a variety of processes, viz. insertion losses, parametric instabilities, diffraction, divergence and other nonlinear effects. In the present analysis the effect of wiggler imperfections has not been considered, which can play an important role in the generation of harmonics[14].

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# On solving Schrödinger equation for the ground state of a two-electron atom using genetic algorithm

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**A recipe is proposed for solving the radial Schrödinger equation (SE) for ground state of helium atom using genetic algorithm. A fitness landscape is generated and the problem of solving the radial SE is reduced to a search for the maximum on this landscape.**

THE use of genetic algorithms (GAs)[1,2] for solving the Schrödinger equation (SE) numerically, is of contemporary origin[3–9]. The primary motivation has been to look

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