Dissipation*†

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The interaction of a physical system with its environment generally leads to dissipation. For example, a large particle immersed in a liquid experiences frictional drag and velocity fluctuations, both resulting from repeated impact of the fluid particles. In this article we will discuss dissipation in the presence of cooperative effects in strongly interacting systems which exhibit physical phenomena like pattern formation, phase-ordering, domain growth kinetics, etc. We will also discuss quantum dissipation and its influence on tunnelling, Brownian motion and crossover from coherence to decoherence.

\[
\frac{dv(t)}{dt} = g \left( 1 - \frac{\rho_L}{\rho_S} \right) - \gamma v(t),
\]

where the first term on the right-hand-side is the buoyancy-corrected acceleration due to gravity \( g \), and \( \rho_L \) and \( \rho_S \) are the densities of the liquid and the solid ball, respectively. It is the second term, however, that is responsible for slowing down the ball, ultimately making it move with a terminal velocity \( v_T \), obtained by setting the left-hand-side to zero:

\[
v_T = \frac{g}{\gamma} \left( 1 - \frac{\rho_L}{\rho_S} \right)
\]

Our school experiment consists of measuring \( v_T \), thereby deducing also the viscosity \( \eta \) of the liquid making use of the Stokes–Einstein relation:

\[
\gamma = \frac{6 \pi \eta \alpha}{a}
\]

where \( a \) is the radius of the ball.

A similar equation such as eq. (1) is encountered in our discussion of Ohm’s law, though barring a few imaginative teachers, we are hardly educated on this remarkable connection between the flow of electric current and the flow of a foreign body through a liquid. The corresponding equation for current \( I(t) \) is

\[
\frac{dI(t)}{dt} = \frac{\varepsilon}{L} \left( R - \frac{R}{L} I(t) \right),
\]

where \( \varepsilon \) is the electro-motive force (EMF) applied to the wire which possesses an inductance \( L \) and resistance \( R \).

The fairly innocuous looking eqs (1) and (4) have a few astounding properties, the significance of which is unravelled only much later, as late as in our M Sc days. These are:

(i) The dissipative term (i.e. the last one in eq. (1)) is not derivable from the gradient of a potential, as would be the case for a ‘conservative’ system.

(ii) The dissipative term, proportional to \( v(t) \) (or \( I(t) \)), is linear in \( v(t) \) (or \( I(t) \)); why?

(iii) If we change the time \( t \) to \( -t \), \( v(t) \) (or \( I(t) \)) changes sign. Therefore, the dissipative term takes cognizance of the arrow of time – the equation of motion breaks time-reversal invariance. This is quite remarkable, and almost goes unnoticed as a novel phenomenon, as we are busy grappling with Lagrange’s or Hamilton’s equations in

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classical mechanics, or even Schrödinger’s or Heisenberg’s equations in quantum mechanics, all of which respect time-reversal invariance. A consequence of the dissipative force not being derivable from a mechanical potential is that the work done over a closed path is not zero! We are told that energy is dissipated. But, where does this energy go or come from? The answers to these tantalizing questions are to be searched in non-equilibrium statistical mechanics, a subject which seldom figures in our university curriculum.

**Brownian motion and Langevin equation**

The first person to systematically analyse the issues presented above was none other than Einstein\(^1\). He was intrigued by an observation due to a Scottish botanist Brown\(^2\), whose fascinating discovery of what physicists call Brownian motion, has been widely misunderstood\(^3\). Brown, a man of medicine and a great explorer, found that Clarkian pollens, suspended in a liquid like water, undergo zig-zag, random motion under a microscope, though it has been believed in some quarters that Brown’s attention was focused on the movement of pollen grains themselves as the very essence of life! It is to Brown’s credit that he was not swayed by others, and having seen the phenomenon in living plant specimens, repeated the experiment in ‘dead’ pollen grains preserved for about a year in an alcoholic solution\(^4\). Incidentally, the other red herring that Brown’s microscope was not adequate for observing such a minute phenomenon, has also been disposed of in careful recreation of his experiments with the aid of the original microscope\(^5\). Einstein pointed out that Brownian motion occurs due to thermal fluctuations which are present even when the liquid is in equilibrium, i.e. its temperature \(T\) and other external conditions are fixed. He further showed that the fluctuations arise from the colliding particles of the liquid, thereby synergizing dissipation with fluctuation\(^6\).

A nice pedagogical model to demonstrate Einstein’s ideas from statistical mechanics was put forward by Zwanzig\(^6\). He considered a spherical ball of mass \(M\) (like our earlier discussed Stokes–Einstein ball) moving in an arbitrary potential \(U(\vec{Q})\), and hence a force \(-\vec{V}U\), but getting impeded by a whole lot of smaller balls (imagined to be the fluid-particles of Brown), each connected by a spring of frequency \(w\) to the centre of the mass \(M\) (Figure 1). We should emphasize that Zwanzig’s starting point is Hamilton’s equations of motion, perfectly time-reversal invariant, arising from a Hamiltonian for the conservative system depicted in Figure 1. From these equations, Zwanzig ‘integrated out’ the equations of motion of the smaller balls of mass \(m_j\), \((j = 1, \ldots, N; N\) being suitably large). He arrived at a mathematically exact equation of motion of the larger mass \(M\) in terms of the initial values (i.e. at \(t = 0\)) of the coordinates \(\vec{q}_j\) and momenta \(\vec{p}_j\) of the smaller balls. This equation is an integro-differential equation for the momentum \(\vec{P}(t)\) of \(M\) and reads:

\[
\frac{d\vec{P}}{dt} = -\nabla U(\vec{Q}) - \frac{1}{M} \int_0^t dt' \vec{P}(t') \xi(t-t') + \vec{\Theta}(t),
\]

where

\[
\xi(t) = \sum_j \frac{C_j^2}{m_j w_j^2} \cos(w_j t),
\]

\[
\vec{\Theta}(t) = \sum_j \left\{ C_j \left[ \ddot{q}_j(0) - \frac{C_j \dot{q}_j(0)}{m_j w_j^2} \right] \cos(w_j t) + \frac{C_j \ddot{p}_j(0)}{m_j w_j} \sin(w_j t) \right\},
\]

\(C_j\) being an appropriate coupling constant between the \(j\)th spring and the mass \(M\).

It should be noted that eq. (5) contains an integral in the positive domain of time as the equations of motion of the smaller balls have been integrated in the forward direction in time. Thus, we have tacitly recognized the arrow of time. Up to eqs (5)–(7), only rules of classical mechanics have been applied. The next important step of Zwanzig, embodied in a deep concept, is to interpret the small balls \(m_j\) to constitute a (large, by taking \(N \to \infty\)) statistical mechanical system or a heat bath, in the sense of Gibbs and Boltzmann, the founding fathers of modern statistical mechanics. Thus we construct a **correlation function** for the variable \(\vec{\Theta}(t)\) by taking a statistical (i.e. thermal) average of the product of two different compo-

![Figure 1](image-url)
ments of $\Theta(t)$, denoted by angular brackets, and employ the *equipartition theorem* that relates the fluctuations in position and momentum variables to the temperature $T$ of the bath. The result is:

$$\langle \Theta_j(t) \Theta_{j'}(t') \rangle = kT \delta_{jj'} \xi(t - t'),$$

(8)

where $\mu$, $\nu$ denote cartesian axes $X$, $Y$ and $Z$. Indeed, a further step of going to the continuum limit (by replacing the summation over $j$ in eq. (6) by an integral over the frequencies $\omega_j$), and suitably modelling the density of frequency states, the kernel $\xi(t - t')$ can be replaced by $(\gamma M)$ times the delta function $\delta(t - t')$ (ref. 6). In that limit, known as Ohmic dissipation, the second term on the right-hand-side of eq. (5) looks exactly like the dissipative force of eq. (1). Furthermore, eq. (1) is now upgraded to the status of a stochastic differential equation, popularly known as the Langevin equation, in which $\Theta(t)$ plays the role of *noise*.

It is appropriate now to recapitulate the various steps of our treatment and assimilate in our mind the physical and microscopic basis of Brownian motion. Each pollen grain that Brown saw under the microscope is like the large mass $M$ in Figure 1. It suffers a frictional force or dissipation because of interaction with the surrounding liquid particles. The liquid is like a heat bath – a statistical system in thermodynamic equilibrium at temperature $T$, though the grain itself is not in equilibrium! The noise force that the grain encounters has its origin in the degrees of freedom of the liquid (cf. eq. (7)), and the same degrees of freedom cause the friction $\xi(t)$. This internal consistency of statistical mechanics is encapsulated in eq. (8) that relates noise fluctuations to dissipation, yielding the celebrated Fluctuation–Dissipation (FD) Theorem. We also have *inter alia* found the answer to the question posed at the beginning of this article – friction causes dissipation in a subsystem, the pollen-grains in this instance, and energy is lost to or gained from the heat bath with which the subsystem is in continual interaction. A similar interpretation is valid in the case of Ohmic flow of electric current. Current is due to motion of the subsystem of electrons, whereas resistance arises from the interaction (or scattering) of the electrons from the heat bath of lattice vibrations (or phonons), material defects, etc.

**Kinetic spin models**

In the above, the analysis of Brownian motion and the concomitant Langevin equation is carried out in the context of the continuum, i.e. the position and momentum variables are assumed to take an infinitely large set of values. Very often however, the subsystem defined above, is comprised of entities that are endowed with discrete degrees of freedom. The situation is commonly encountered in the important technique of nuclear magnetic resonance (NMR). In NMR, we have to deal with a bunch of magnetic spins, such as the proton spins in the case of magnetic resonance imaging (MRI), which are subjected to internal and externally applied magnetic fields. These spins are like arrows pointing either *up* or *down*, or a collection of two-level atoms in the context of a laser. Dissipation occurs due to coupling with a heat bath which, in the magnetic case, is usually made up of phonons that cause *spin–lattice relaxation*, or the electromagnetic fields of a cavity in the case of a laser. What we earlier called friction crops up now in the guise of rate parameters such as spin–lattice relaxation rate or spin–spin relaxation rate, and in the example of lasers as Einstein’s $A$ and $B$ coefficients.

If we focus our attention on a single spin for which the up projection is associated with a value $+1$ and the down projection is associated with a value $-1$, the heat bath-induced fluctuations can be schematically illustrated as in Figure 2. The spin-value $m$ as a function of time $t$ is a two-level jump process (in contrast to the velocity of a Brownian particle, being a continuous stochastic process), variously referred to as a telegraph process or a dichotomic Markov process. If the up and down projections occur with equal probability, i.e. $m$ retains the value $+1$ or $-1$ with equal duration of time on average, such as in a paramagnet in the absence of a magnetic field, the time-averaged value of the spin (denoted by an overhead bar), and proportional to the magnetization of the subsystem, obeys the equation:

$$\frac{d}{dt} \bar{m}(t) = -\gamma \bar{m}(t),$$

(9)

where with reference to Figure 2, $\gamma^{-1}$ is the mean time-difference between successive instants $t_1, t_2, t_3, \ldots$. Keeping in view the invisible heat bath that causes $m$ to jump between $+1$ and $-1$, $\gamma$ is called the spin–lattice relaxation rate.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Stochastic evolution of a spin in contact with a heat bath. The spin-value $m$ switches from $\pm 1$ to $\mp 1$ at randomly distributed times $t_1, t_2, t_3, \ldots$.
On the other hand, if an aligning magnetic field $H$ is present (somewhat akin to the gravitational field in the case of Brownian motion), which makes the spin stay in the up projection longer than in the down projection, the equation takes the form:

$$\frac{d}{dt} \bar{m}(t) = -\lambda \bar{m}(t) + \lambda \tanh \left( \frac{\alpha H}{KT} \right),$$

(10)

where $\alpha$ is the gyromagnetic ratio yielding what is called the Zeeman energy $(\alpha H)$ due to the magnetic field $H$. Unlike eq. (9) which makes the magnetization decay to zero exponentially with time at the rate $\lambda$, eq. (10) leads to an asymptotically stationary value:

$$\lim_{t \to \infty} \bar{m}(t) = \tanh \left( \frac{\alpha H}{KT} \right),$$

(11)

the expected Zeeman relation.

Two points are now in order. The overhead bar in eqs (9)–(11), as mentioned earlier, refers to a time-average of the spin as its trajectory evolves in accordance with Figure 2. It was the genius of Gibbs who interpreted this average in the context of a collection of a large number of noninteracting spins but independently interacting with a heat bath, as an ensemble-average based on what is called the ergodicity hypothesis. Thus the overhead bar can be replaced by the angular brackets introduced in the context of eq. (8). With this interpretation we come to the second point, viz. that both eqs (11) and the earlier discussed eq. (2) refer to steady-state or terminal values, albeit of two distinct physical quantities. But, more fundamentally, the discerning reader would note that while eq. (2) contains the friction $\gamma$, the relaxation rate $\lambda$ has disappeared from eq. (11)!

This underscores the basic difference between a transport property such as the velocity of a Brownian particle and what is called a thermodynamic property such as the magnetization.

As mentioned earlier, equations such as eq. (10) are at the heart of NMR or electron paramagnetic resonance. What is also relevant in that context is the analysis of what is known as the spin auto-correlation function:

$$C(t) = \langle \bar{m}(0) \bar{m}(t) \rangle_{eq}.$$  

(12)

The subscript ‘eq’ stands for equilibrium which is to emphasize the point that while eq. (1) or eq. (10) is relevant when the subsystem is out of equilibrium with the heat bath, $C(t)$ describes time-dependent correlations when the subsystem is in equilibrium with the heat bath – spontaneous spin-fluctuations or velocity-fluctuations can never be switched off! It is also remarkable that $C(t)$ and $\bar{m}(t)$ obey the same differential equation – a consequence of the so-called Regression Theorem$^5$:

$$\frac{d}{dt} C(t) = -\lambda [C(t) - C(t = \infty)],$$

(13)

where $C(t = \infty)$ indicates the steady-state value of $C(t)$ reached at $t = \infty$. Given that the correlation function $C(t)$ measures the overlap of a given spin with its own orientation at two distinct times $0$ and $t$, and that this memory is expected to be wiped out due to statistical fluctuations when the time-difference $t$ is sufficiently large, we expect that

$$C(t = \infty) = \langle \bar{m}(0) \rangle_{eq}^2 = (\bar{m}(t = \infty))^2,$$

(14)

where $\bar{m}(t = \infty)$ is obtained from the right-hand-side of eq. (11). Equation (14) is based on a property called mixing of statistical states, from which follows ergodicity alluded to earlier$^6$. Finally, eq. (13) for the correlation function leads to the definition of the correlation time $\tau_c$, an underlying property of all spectroscopy experiments\textsuperscript{11}:

$$\tau_c = \frac{\int_0^\infty dt [C(t) - C(t = \infty)]}{\int_0^\infty dt [C(t) - C(t = \infty)]}.$$  

(15)

Interestingly, $\tau_c$ equals $\lambda^{-1}$, but this equality is a special (linear) property of eq. (13). Thus we have come a full circle – correlation time, which connotes to a measure of coherence in a subsystem, is inversely related to the relaxation rate, a measure of dissipation: stronger the dissipation lesser the coherence. This issue acquires a new and intriguing flavour for quantum systems, discussed later.

**Pattern formation and domain kinetics**

The subject of condensed matter physics is replete with systems which display cooperativity due to strong interactions between the constituent elements of the subsystem. Often the interactions lead to ‘order’ making possible a phase transition, from say a paramagnet to a ferromagnet. On the other hand, dissipation due to coupling with the heat bath is responsible for bringing the subsystem to equilibrium, as argued already in the case of a Brownian particle (cf. eq. (1)) or a single spin (cf. eq. (9)). It is therefore a matter of great interest to study the effects of dissipation on a subsystem which has a tendency to phase-order. The simplest model of such phase-ordering kinetics is provided by the Ising Hamiltonian which describes the interaction between spins $m_i, m_j, \ldots$ placed on the sites of a lattice denoted by $i, j, \ldots$, etc.

$$H = -\frac{1}{2} \sum_j J_{ij} m_i m_j,$$

(16)

$J_{ij}$ being the coupling constants. The prime over the summation implies that $i \neq j$ terms (corresponding to self-interaction) are omitted.

Amongst the myriads of application of the Ising model the two that we want to focus on are the ferromagnet, as a natural extension of the single-spin case discussed earlier, and the equivalent problem of phase separation in a binary alloy (or liquid mixture). In the case of a binary system the spin variable $m_i$ is mapped into the occupancy of an $A$ or a $B$ type of atom at the lattice site $i$. 

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Footnotes:

1. Regmc Theorem.
3. Eigendecomposition.
4. Equilibrium.
5. Relaxation rate.
6. Correlation time.
7. Cooperativity.
8. Phase transition.
11. Spectroscopy experiments.
with the magnetic field $H$ playing the role of $(\mu_A - \mu_B)$, the difference in chemical potentials. The phase diagram of a ferromagnet in the $(T, H)$-plane is shown in Figure 3. The point $(T_c, H_c = 0)$ is a second-order critical point, commonly called the Curie point demarcating the ferromagnetic and paramagnetic phases. It may be noted that a magnet possesses time-reversal symmetry, i.e. if we run the time $t$ in the negative direction, each $m_i$ is flipped in sign leaving the Hamiltonian $H$ in eq. (16) invariant. This symmetry can be broken by the application of an external magnetic field $H$. Consequently, the line $(T < T_c, H_c = 0)$ is a line of first-order phase transition – crossing that line makes the magnetization (thermodynamically conjugate to $H$) jump discontinuously. Such a time-reversal symmetry is of course absent for a binary alloy or a two-component liquid mixture, the phase diagram of which in the $(C_A, T/T_c)$-plane is shown in Figure 4. The point at which the concentration $C_A$ of $A$-type atoms is one-half and $T/T_c = 1$, is a second-order critical point. Below the coexistence curve (denoted by a solid line), the system separates into $A$-rich and $B$-rich regions. Their relative proportions are determined by the lever rule. In the region between the spinodal (denoted by a dashed line) and the coexistence curve, an initially homogeneous system is locally stable.

Apart from the time-reversal symmetry, there is another crucial difference between a ferromagnet and a binary mixture when we consider heat bath-driven kinetics. While in a ferromagnet the kinetics proceeds by individual spin-flips (as indicated in Figure 2), causing the magnetization to change, the corresponding kinetics for a binary mixture must imply an interchange of $A$ and $B$ atoms at two different lattice sites; in the spin-language this interchange is a ‘flip-flop’ process. Consequently, the magnet is said to be driven by ‘non-conserved’ kinetics, whereas the binary mixture is characterized by ‘conserved’ kinetics, as the net concentration of $A$ and $B$ atoms remains invariant. Therefore, while the phase transition characteristics are identical in the two cases, both being governed by the same Ising models, the dissipative effects leading to domain kinetics are quite different, as discussed below.

In order to study phase-ordering we need to define an appropriate order parameter following Landau and Lifshitz. The order parameter is a macroscopic (thermodynamic) entity which is zero in the disordered phase and nonzero in the ordered phase. For the case at hand, the suitable order parameter is the average magnetization (or its equivalent in the case of a binary mixture) and is denoted by $\Psi(\bar{r}_i)$:

$$\Psi(\bar{r}_i) = \langle m_i \rangle,$$

(17)

where $\bar{r}_i$ is the position at which the $i$th spin is located and the angular brackets as usual refer to the statistical average. The basic framework for nonconserved kinetics is derived from a model due to Glauber, who put forward a time-dependent equation for the probability of a certain spin-configuration of the Ising spins. He assumed that an invisible heat bath causes spontaneous spin-flips at random instants of time. From this one can derive an equation for the order parameter (cf. eq. (17)), which is after all the first moment of the underlying probability. This equation is nonlinear and highly intricate, leading to a hierarchy of equations connecting all higher moments. A convenient technique to terminate this hierarchy is provided by the so-called mean field approximation. This approximation is useful in the theory of phase transitions.

Figure 3. Phase diagram of a ferromagnet in the $(T, H)$-plane.

Figure 4. Phase diagram of a binary mixture in the $(C_A, T/T_c)$-plane.
yielding the Landau expansion of the order parameter\textsuperscript{12}. Further, it is customary to give a coarse-grained description, i.e. go from a discrete lattice to a continuum. The resulting equation for the order parameter is known as the time-dependent Ginzburg–Landau (TDGL) equation, written as\textsuperscript{15}

\[
\frac{\partial \Psi(\vec{r}, t)}{\partial t} = \Gamma [a(T_c - T) \Psi - b \Psi^3 + H + K \nabla^2 \Psi], \tag{18}
\]

where \( \Gamma \) parametrizes dissipation, and \( a, b, K (> 0) \) are other parameters related to the coupling constants \( J_0 \) (cf. eq. (16)). The corresponding equation for conserved kinetics, appropriate for a binary mixture, is derived from the so-called Kawasaki model distinct from Glauber's in which one assumes simultaneous flip-flops of spins at different sites, as indicated earlier\textsuperscript{16}. A further mean field approximation and coarse-graining of the Kawasaki model yields the following Cahn–Hilliard–Cook (CHC) equation for phase separation for conserved kinetics\textsuperscript{17,18}:

\[
\frac{\partial \Psi(\vec{r}, t)}{\partial t} = \vec{\nabla} \cdot \left[ D \vec{\nabla} \left( -a(T_c - T) \Psi - b \Psi^3 - K \nabla^2 \Psi \right) \right], \tag{19}
\]

Here \( D \) is the diffusion coefficient that takes into account dissipation.

We now present numerical simulations of eqs (18) and (19) using an Euler-discretization mesh size: \( \Delta t = 0.01 \) and \( \Delta x = 1.0 \) in time and space, and a lattice size \( 256 \times 256 \). In Figure 5 we show the evolution of a ferromagnet from a disordered initial condition for \( T < T_c, H = 0 \) based on eq. (18). Regions with positive magnetization are marked in black and regions with negative magnetization are unmarked. A characteristic feature of nonconserved kinetics is that the domain length-scale obeys the Lifshitz–Allen–Cahn law\textsuperscript{19}, \( L(t) \sim t^{1/3} \) for dimension \( d \geq 2 \). The evolution of the CHC eq. (19) with \( T < T_c \), from a disordered initial condition, is shown in Figure 6. Regions with positive \( \Psi \) (say, \( A \)-rich) are marked in black, and regions with negative \( \Psi \) (\( B \)-rich) are unmarked. Although the snapshots in Figures 5 and 6 look similar, the discerning feature is the domain size which grows as \( L(t) \sim t^{1/3} \) for \( d \geq 2 \), where the characteristic one-third power law for the conserved case was derived by Lifshitz and Slyozov\textsuperscript{20}. Summarizing, we have shown that a class of models whose phase equilibria are identical, governed by the Ising model, trod different routes to equilibrium due to the presence of dissipative kinetics which may be conserved or nonconserved. Dissipation is responsible for rich pattern formation in these systems which is characteristic for observed complex behaviour of many physical and biological systems.

**Quantum dissipation**

Until now our discussion of dissipation has been restricted to classical systems. What happens when both the subsystem and the heat bath are governed by quantum laws? Is there something like quantum friction? Quite remarkably, the answer to these questions can be sought within the same model depicted in Figure 1. Instead of using Hamilton's equations of motion, we have to now employ

![Figure 5](image_url) Evolution of a ferromagnet from a disordered initial condition for \( T < T_c, H = 0 \), based on the TDGL equation.

![Figure 6](image_url) Evolution of the dimensionless CHC model (with \( T < T_c \)) from a disordered initial condition.
Heisenberg’s equations and arrive at the same eqs (5)–(7) as earlier, except that all the coordinates and momenta have to be interpreted as quantum operators with prescribed quantum commutation relations. In addition, the noise \( \Theta(t) \) is also a quantum operator whose spectral properties are however different from the classical FT theorem (cf. eq. (8)). They are characterized by the following symmetric and antisymmetric correlation functions:

\[
\langle \Theta_\mu(t), \Theta_\nu(t') \rangle = \delta_{\mu\nu} - \frac{2}{\pi} \int_0^\infty dw \text{Re} \left[ \frac{\xi(w+i0^+)}{w} \right] \coth \left( \frac{Bw}{2} \right) \cos[w(t-t')], \tag{20}
\]

\[
\langle [\Theta_\mu(t), \Theta_\nu(t')] \rangle = \delta_{\mu\nu} - \frac{2}{\pi} \int_0^\infty dw \text{Re} \left[ \frac{\xi(w+i0^+)}{w} \right] \sin[w(t-t')]. \tag{21}
\]

Here, we have defined the Laplace transform of the friction kernel \( \xi(t) \) as

\[
\xi(s) = \int_0^\infty e^{-st} \xi(t) dt. \tag{22}
\]

With this proviso, eq. (5) for the momentum operator of the mass \( M \) (cf. Figure 1) can be referred to as a \textit{quantum Langevin equation} and the underlying dynamics is called quantum Brownian motion.

One interesting application of quantum Brownian motion is to dissipative tunnelling. Imagine for simplicity, motion in one dimension, for which the underlying potential \( U(Q) \) in eq. (5) is a symmetric double-well. Quantum mechanically, the mass \( M \) is initially localized to be in one of the wells, keeps going back and forth between the two wells as time proceeds. This phenomenon is classically prohibited and is called quantum tunnelling. The motion is coherent, like that of a quantum clock. However, the presence of heat bath-induced dissipation leads to decoherence. This coherence-to-decoherence transition is best exemplified in the case of \textit{Ohmic} dissipation for which the friction kernel \( \xi(t) \) can be replaced by a constant \( \xi_0 \). A fascinating manifestation of quantum dissipation is a \textit{broken-symmetry} transition when the particle does not tunnel at all. In this, the wave function associated with the position of the particle, instead of having a symmetric two-peaked structure localized at the two wells, becomes a single peak around one of the wells beyond a critical value of \( \xi_0 \). The phenomenon is analogous to the \textit{watched-pot effect}, where the two minima of the double-well depict the \textit{boiling} and \textit{non-boiling} states of the pot – localization means the pot does not boil at all if it is continually watched. A dramatic application of the localization concept is found in \textit{C-axis} transport in layered superconductors. The transport across the \textit{C-axis} between adjacent CuO planes in a high \( T_c \) superconductor such as Y BaCuO\textsubscript{2} can be viewed to be a quantum-tunnelling process. While this process is coherent, quantum Brownian motion of the tunnelling electron within the \( ab \)- (i.e. CuO) plane renders it incoherent. The quantum Brownian motion is indeed the consequence of repeated inelastic scatterings that the tunnelling electron has to undergo, as a result of interaction with other electrons, phonons, defects, etc. Thus tunnelling is impeded, much like in the case of the \textit{quantum Zeno effect}. The consequence suppression of transport along the \( C \)-axis and the accompanying (incoherent) diffusion of the electron is believed to be responsible for the observed anisotropy in resistivity \( \rho \), which is found to be about two orders of magnitude higher than \( \rho_{ab} \) in the \textit{normal} state of layered superconductors. Remarkably, superconducting transport is not affected by quantum dissipation as that process involves tunnelling of a Cooper pair of electrons, and not a single electron. It remains coherent because the conjugate member of the pair traverses a time-reversed path of the other member, and hence any effect of decoherence due to inelastic in-plane scattering cancels out.

Another important application of quantum dissipation is in elucidating the question: does Landau diamagnetism survive dissipation? Landau diamagnetism is a property of a collection of charged particles in a box and the response of their orbital motion to an applied magnetic field \( \vec{B} \). This problem can be studied by adding the \textit{Lorentz force}

\[
\frac{e}{MC} \times \vec{P} \times \vec{B},
\]

to the right-hand-side of the quantum Langevin eq. (5). It was a great triumph of quantum mechanics when Landau showed that the discreteness of energy levels (and the consequent degeneracy of each level) results in a non-zero diamagnetic susceptibility. We may view diamagnetism as a coherent property – it arises from coherent cyclotron motion of the electron in a plane normal to the magnetic field. Now, the question is: what happens if the coherent precessional motion of the electron is disturbed due to dissipative effects arising from interactions with other degrees of freedom? The results of the calculation, based on eq. (5), for the dimensionless magnetization \( Y \) are plotted in Figure 7 as a function of the scaled resistance \( X \), which is the ratio of the Drude and Hall resistivities. As resistance is the basic manifestation of dissipation, it is evident that large values of \( X \) will result in stronger decoherence. On the other hand, the persistence of coherence is more pronounced for larger values of the magnetic field \( \vec{B} \). Figure 7 illustrates how the system makes the transition from the coherent \textit{Landau regime} to the (apparently) classical Bohr–van Leeuwen regime, as dissipation increases.

We conclude by stating that a quantum system is characterized by unitary time evolution and therefore, displays phase coherence. Dissipation, which occurs due to interaction with a (quantum) environment, leads to decohe-
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Figure 7. Magnitude of the dimensionless magnetization vs scaled resistance $r$ for different values of the dimensionless cyclotron frequency $v_c = 1.0, 10.0$. The case $X = 0$ corresponds to the Landau limit, and $X = X_{BvL}$ corresponds to the Boltz-van Leuwen limit.

This interplay of coherence and decoherence is an important issue in the present context of wave-packet dynamics in atomic physics, electron-transfer reactions in the chemistry of biological systems, tunnelling centres in condensed matter physics and above all, in quantum computing. Indeed, it is a matter of active research on how to control the ubiquitous presence of decoherence in order to make quantum computing effective. The other issue is in relation to the question: can there be decoherence without dissipation? This question can be best addressed by considering a subsystem in contact with a (quantum) environment at zero temperature, because at $T = 0$, there cannot be any energy transfer and hence any dissipation. However, ‘zero-point’ fluctuations cannot be switched off and as it turns out, they lead to decoherence due to what may be termed as ‘dephasing’.


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