A soft mathematical model for brain drain

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It is a widely held belief, even among senior people in the government, that India is a country with vast human resources and that even if about 10% goes abroad after higher qualifications, it would not make a dent in the country's total productive potential. Implied in this argument is the assumption that if 10% of the human resources goes abroad, it would take away only 10% of the intellectual energy in the population. Is there any scientific basis for this? If a scientific, or a mathematical model were to be sought for this, how should this be done? In this article, based on some well-known power-law models used in complex systems like ecology, economics, scientometrics and seismology, one can argue through a soft mathematical model that a small per cent of the cream at the top can take away a disproportionately large amount of intellectual resources.

Some preliminaries

THE origins of this essay go back nearly thirty years to September 1974. The occasion was the convocation function where I was to receive my first degree in engineering. The Chief Guest was the Prime Minister of India, Indira Gandhi. The Chairman of the Board of Governors of IIT Madras, in his speech, assured her that only 25% of the graduates of that institute left for greener pastures abroad; what has come to be known as the brain drain. Again, it was implied that 75% of the output in numbers was available to address and solve the problems of our country.

This seemingly simple formula that if 25% of the numbers leaves, it takes away only 25% of the intellectual potential (energy?) in the age cohort, is a deceptive one. There was no scientific basis for it, which could be expressed as a simple mathematical model so that one could assert quantitative relationships describing the distribution of intellectual potential in a population. Personally, as a student of the graduating class of 1974, it was clear to me that this was obviously wrong. The 25% of my batch that migrated in 1974 were arguably the best of that batch, the crème de la crème, as it were, of the crème de la crème that was the pool from which they were selected through a punishing process. It was also obvious to me that they took away with them more than 25% of the talent in the batch, but how much more than that was not something that could be easily factored out of a simple formula.

Before a formula could be found, it was necessary to assemble the basic underlying principles, so that a phenomenological model can be proposed. I shall elucidate on this search for first principles below, as these principles were not gathered in one place, nor did they appeal to me at one point of realization in time. What seems apparent is that there are patterns in the ecology of intellectual energy. It has been well known for quite some time now that power-law distributions are useful descriptors for a variety of complex natural, social and economic systems[1]. It is tempting to apply this to the problem of brain drain. There are models from economics (the Pareto law), biology (the Eltonian pyramid), scientometrics (the Zipf and Lotka laws) and seismology (the power laws that describe the distribution of seismic events and the energies released) which can be combined to give a soft mathematical model for the ecology of intellectual activity. (A hard mathematical model is typically like one using the laws of Newton to predict the trajectory of a rocket launch vehicle, or the use of Navier–Stokes equations to predict the lift on an aircraft wing).

It has taken nearly thirty years for this search to come to a point where a written account of it could be satisfyingly made. Yet, many doubts remain and much more refinement of the arguments is called for.

The rank-ordering statistics

Almost immediately after the 1974 convocation, I came across something the historian Will Durant had written. He said that one of the greatest lies ever invented was that all men are born equal. Equal before the law, perhaps. Equally deserving of opportunity for personal growth and realization, perhaps. But definitely not born equal from the point of view of pelf or privilege; let alone from the point of view of ability. Thus, one could find an Einstein or an Edison, who is the equal of a hundred thousand or a million men and women, as far as his potential to transform the world through his ideas was concerned. How does one draw up this distribution of ability in a cohort?

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Or with much more difficulty, the distribution in a large community, like that of a whole subcontinent?

The question is, how frequently is intellectual ability or energy of extremely high order distributed among the general population? The factor that determines this is a complex and dynamic interaction of biology, sociology, economics and psychology, in a large distributed system, an ecology of intellectual energies. Thus, the systemic behaviour and its patterns must have evolved from billions of individual interactions over millions of years. We are hopeful that patterns do exist and can be sensed. What we are looking for is definitely not a Gaussian or normal distribution. Rank-order statistics[2] based on power-law distributions are obviously required, as we are concerned with establishing how frequently large events (an Einstein or an Edison) which are assumed to be rare, appear among the small events (the man in the street) which are common. While the statistics of cumulative distributions is well suited to describe the small events which are frequent, rank-ordering statistics seems to be the only way to describe the infrequent large events.

One area of sociology which has been systematically studied from this point of view is that called scientometrics, where an attempt is made to measure scientific activity. But, before that, there are some interesting lessons to be learnt from economics.

Lessons from econometrics

The Italian engineer-turned-economist and political sociologist, Vilfredo Pareto, realized that wealth is not evenly distributed[3]. Some of the people have most of the money. In fact, a fairly consistent minority, about 20% of people, controlled the large majority, about 80%, of a society's wealth. If we examine this distribution with an even finer microscope, we would find that of the top 20% which owns 80% of the wealth, the 80–20 formula still applies reasonably consistently, so that the following pyramid can be set up as shown in Table 1. Thus, less than 1% or so of the population may account for 50% or so of the wealth.

Table 1. The Pareto principle and the pyramid of wealth distribution

80–20 rule	:	20% has 80%	0.800 has 0.200
80–20 rule on this 20%	:	0.2 × 0.2 = 0.04 has 0.8 × 0.8 = 0.64	0.160 has 0.160
80–20 rule on this 4%	:	0.2 × 0.04 = 0.008 has 0.8 × 0.64 = 0.512	0.008 has 0.512

so that

Pyramid of numbers	Pyramid of wealth
0.008	0.512
0.480	0.480
0.512	0.008

That the same distribution is true for many other areas has been frequently noticed and is now termed the Pareto principle. In fact, Juran has extended this idea to an argument that 80% of all effects is produced by only 20% of the possible causes. One could project from this that 80% of all major intellectual and social revolutions originates from about 20% of the protagonists, and that even in this, maybe a small fraction (less than 1%) accounts for most of the major developments.

Lessons from scientometrics

An area of intellectual activity that is most easily amenable to quantification is the production of research output as measured by publications in the open literature. The ecology of this enterprise, where a large number of scientists work, about half of them publish, but only a few account for the highly cited work, is complex. It is by no means clear that iron-clad laws have emerged, but the semblance of power-law distributions is easily noticed. Norbert Wiener (I am a mathematician, *Science*, 1964) is said to have argued that 95% of the original work is made by less than 5% of all scientists. Here, we see something like Pareto's law at work. Two laws that are well known in this field go by the names of Zipf[4] and Lotka[5].

Zipf's is the law of rank frequency, which postulates that rank r occurs with a frequency which is inversely related to r. Note that a large number of variables are hidden in the system, but the rank-to-frequency relationship is captured in a simple way. Thus, if an author of the first rank has a 100 papers, an author of the second rank may have 50 (= 100/2) or 25 (= $100/2^2$) papers, depending on the power of the inverse relationship. In this simple relationship that Zipf postulated, some kind of 'principle of least effort' was operating.

More useful in our context is Lotka's law of scientific productivity, whereby the number of authors making n contributions is about $1/n^2$ of those making 1. In grossly simplified terms, this means that if we find a 1000 authors with 1 paper each, about 10 authors may have 10 papers each, and several thousands may participate in the intellectual process associated with scientific discovery but never get to publish. However, the law is not accurate at extreme tails, and it is perfectly possible that we may find an author with a 1000 papers and another with a 100 or more papers. This is not unlike the Pareto law expressed for economics, where 1 person may have 10 billion dollars, another 10 may have a billion dollars each, and billions may live below the poverty line. Recently, press reports indicated that the three richest families in the world have as much wealth as that of the total population of the poorest 46 countries of the world! Later, I will argue that in seismology, earthquakes are distributed in a similar pyramidal fashion, and that they release energy in a similar inverted pyramidal way.

What if human ability and genius are not distributed in a Gaussian fashion, but in the highly skewed manner that we see in other complex systems, like those examined above? This leads to the frightening conclusion that a small fraction of the population can take with it a disproportionate amount of the singular genius in a population. If this group is creamed-off in the emigration process, the loss to the donor nation is huge, even if the numbers concerned are small.

Elton's pyramid for ecological systems

In complex ecosystems, e.g. as found in most food chains, the number of individuals decreases at each stage, with huge numbers of tiny individuals at the base and a few large individuals at the top[6]. A simple example often given is that displayed by millions of plankton, a moderate number of large fish, and a few eagles. Animals high on the food chain are both larger and rarer than animals lower down, as in the predator–prey relationship, where the predator must be larger than the prey. This is referred to as the 'pyramid of numbers' or Eltonian pyramid. This model has an order of magnitude of 10. The Eltonian pyramid thus portrays the relationships among the trophic levels of such ecosystems and can be based on numerical abundance, biomass or energy. A complex interplay between energetic principles, parameters and processes gives rise to the Eltonian pyramid. We shall see below that a soft model will capture this pyramidal structure.

The seismic analogy

Another excellent example of the presence of power-law distributions in the natural sciences is the frequency–magnitude distribution of earthquakes. On the Richter scale, magnitude is a quantitative measure of the size of an earthquake. Earthquakes of higher magnitudes are rarer than those of lower magnitudes. Table 2 shows the average annual number of the global occurrence of earthquakes. Note a pyramidal sequence in these numbers. However, the energy released scales up according to a $10^{3/2}$ law; i.e. an increase in magnitude by 1 indicates about 31.6 times higher energy released. The pyramid of energies is now inverted. We see that only 0.1% of the major earthquakes (here chosen as $M > 4$), accounts for more than half of the energy released and that about 2% of these accounts for more than 90% of all energy released.

The pyramid for intellectual energy

We start by recognizing that the intellectual energy in a cultural system is built up in a complex way from nature (biology of the genotype and phenotype) and nurture (the sociology and psychology of the information process and the meme) and the dynamics of the socio-economic system and its interactions. There are obviously too many variables for any meaningful hard mathematical model or formula to be obtained. However, we can take comfort from the foregoing examples that the ecology of the intellectual process will throw up outstanding scientists and inventors in the same pyramidal and power-law fashions shown above. If we assume, in one simple model that the numbers scale by factors of ten (say, the Einsteins and the Edisons are ten times rarer than a Nobel Prize winner, etc.) and that intellectual energies scale the way energy is released in an earthquake, we get Table 3.

One per cent of the population (1 + 10 + 100 + 1000) takes away almost 99% of the supremely original intellectual energy of the population. One can play around

Table 2. The pyramid of numbers and energies of earthquakes

Magnitude	Annual average number	Energy per event	Total energy
> 8	1	3.16×10^{10}	3.16×10^{10}
7–8	18	1×10^{9}	1.80×10^{10}
6–7	120	3.16×10^{7}	0.38×10^{10}
5–6	800	1×10^{6}	0.08×10^{10}
4–5	6200	3.16×10^{4}	0.02×10^{10}
3–4	49000	1×10^{3}	
2–3	365000	3.16×10^{1}	
1–2	2920000	1	
			5.44×10^{10}

Pyramid of numbers	Cumulative numbers	Cumulative numbers as percentage	Pyramid of energy as percentage	Cumulative energy as percentage
1	1	0.1	58	58
18	19	2	33	91
120	139	15	7	98
800	939	100	1.5	99.5

Table 3. A pyramid of intellectual energies

Number	Energy	Total energy	Percentage
1	32000000	32000000	68.64
10	1000000	10000000	21.45
100	32000	3200000	6.864
1000	1000	1000000	2.145
10000	32	320000	0.686
100000	1	100000	0.215
111111		46620000	100

Table 4. A pyramid of intellectual energies

Number	Energy	Total energy	Percentage
1	100000	100000	16.67
10	10000	100000	16.67
100	1000	100000	16.67
1000	100	100000	16.67
10000	10	100000	16.67
100000	1	100000	16.67
111111		600000	100

with these terms. For example, if the energies also scale as factors of ten, then 1% of the population takes away two-thirds of the energy in that population, as shown in Table 4. In fact, we find that Pareto's distribution is more charitable, where the top 1% accounts for roughly 50% of the intellectual wealth.

The economics of brain drain

The Indian diaspora has dispersed over two centuries. Since the mid-19th century, the migrations were mainly of semi- or unskilled labour to South Africa, Mauritius, the West Indies, Fiji, Sri Lanka, Malaysia and Singapore, Myanmar, East Africa and more recently, to the Gulf countries. However, since Independence, several million people have been economic emigrants to North America and other Western countries like Australia, Canada, etc. These are highly trained people. More recently, since the IIT system was put in place, a significant number has left for greener pastures abroad. Even more recently, with India being recognized as a cheap source of English-speaking IT talent, an equally significant number of highly trained youngsters (high-technology specialists, as they are sometimes called) goes abroad. Even the recent trend where multinationals set-up Centres of Excellence in India, employing our most highly qualified youngsters, at salaries which are several times the prevailing compensations in the government or public sector, is not without worry. A *Nature* news feature (19 October 2000) had indicated that our 'most valuable scientific assets [are] being used as cheap labour to address the problems of multinational companies, rather than the issues facing India's developing economy'. These may be only a minuscule fraction (less than 1%, i.e. about 250,000 each year of an age cohort of about 25 million). Yet, I have argued that these are invariably the brightest and the best of those trained in through the system, and would carry away with them not 1% of the intrinsic genius in the population, but maybe 50% or thereabouts of the singular ability in that population.

We recall once again that it is known through empirical evidence collected over several centuries that both natural phenomena and complex social and economic systems are governed by power-law size distributions. The largest events dominate these processes. Thus, we have seen from seismology that a few large earthquakes account for most of the energy released throughout the year by millions of seismic events, and therefore for most of the changes that take place at plate boundaries.

In a similar fashion, one can argue that human progress is dominated by the appearance of a few individuals of rare, exceptional abilities. If the brain drain is predicated on a filtration process that ensures that the highest trained individuals find it profitable to leave to donee nations (in recent years, almost invariably the United States), the loss to the productivity potential of donor nations would be huge. The gain to the donee nations is also incalculable. The brain drain is something that cannot be ignored or wished away.

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