Musical notes and their perception

In an earlier correspondence, we have examined the musical notes produced in a veena and any other such instrument[1,2]. Eq. (1) there related to the lengths l_n, l_{n+1}, etc. of the wires from the bridge to the frets on the veena for production of the consecutive notes n, $n + 1$, etc. with the number of notes per octave N as reproduced below:

$$l_{n+1}/l_n = (1/2)^{(1/N)}. \qquad (1)$$

In a conventional veena, $N = 12$ and n are the serial orders '1 2 3 4 . . . 12 13 14' etc. of the notes "sa ri ga ma . . . sa' ri' ga' . . ." etc. respectively. A closer analysis of the above equation has shown that only for a few specific values for N of 12, 14, 17, 22, 24, etc. over 26 ignored, numerous higher harmonics of each note n happen to be fundamentals or harmonics of the notes higher up the scale to a 'fairly good accuracy'. Thereby no harmonic of any note fringes out of the scale. Thus the notes together with their harmonics form a family of concordant or harmonious tunes and a 'musical scale'.

The human ear[3] is composed of a stapes at the tip of a three-bone lever mechanism, ending at an oval window of a cochlea, a fluid-filled, 35 mm long, folded tube with over 30,000 basilar reeds wedged into a tissue at the joint of the folds. The response of the cochlea for incident musical signal is such that the high frequency (~ 8000 Hz) signals are attenuated near its base and the low frequency (~ 200 Hz) signals at the far end near its tip. They are transduced as electrical pulse trains into the nerve fibres to be subsequently conducted to the brain mind system for analysis and perception as perhaps in a holograph. The response of human ear improves by three to six decades when signal frequency increases from 30 to 8000 Hz.

Neural activity in brain, as any textbook on physiology[3] shows, is invariably frequency-based. Hence an analysis of the above notes is simpler with a frequency-based approach, via an inverse transform of eq. (1), obtained by the well-known laws of physics of the stretched wires

$$\{f_n/f_{n+1}\} = (1/2)^{(1/N)}. \qquad (2)$$

This equation relates to the notes within any octave and the associated frequencies f_n over the entire audio range where the veena can be operated, i.e. about 20 to 15,000 Hz.

Table 1 lists a few frequencies of interest for our discussions here highlighting the significance of the integer ratios between them and their harmonics or overtones[4]. Row 1 lists a few sequential integer ratios attributed to Pythagoras to be representing the integeral relationship between the notes, thereby contributing to the aesthetics and appreciation of any musical presentation. The row also presents a few idealized frequencies based on the standard frequency of 240 Hz, corresponding to sa above and satisfying the Pythagorean integeral requirement for any value of N. For example, at row 1 and column 6 with the integer ratio 4/3, the fourth harmonic of the standard frequency 4 × 240 Hz is identical with the third harmonic of the frequency 3 × 320 indicated here, or in general for any value of N, $4 \times f_{io}^N = 3x f_{in}^N$, and so on.

On the other hand an analysis of the eqs (1) and (2) as pursued below, shows that a total adherence of the notes to the above requirements is impossible under practical conditions. Nevertheless computational and experimental pursuances with the veena for various values of N and n have shown that one can approach the ideal solution as follows:

For example, the rows 2 to 6 in the table, list the frequency values f_n^N for various values of N notes per octave and n the order of the note within each octave as computed by eq. (2). It is seen that the values of f_n^N approach the ideal value f_{io}^N along row 1 only for specific values of N of 10, 14, 12, 17 and 22, in that order.

It is obvious that the difference between the ideal and practical values of the frequencies expressed as a percentage of the former is a measure of the extent of closeness of the two values and are termed 'concordance d' between the responses of an ideal and practical veena and it is given by the ratio

$$d = [(f_{io}^N - f_n^N)/(f_{io}^N)].$$

These values have also been listed in Table 1. The f_n^N here represents the notes obtainable in a veena under the combined, linear or spatial constraints eq. (1) or frequency or temporal constraint eq. (2). The 'concordance d' is now a quantized parameter, which is given by the modular difference between the 'free' and 'constrained' frequencies of the notes as discussed above and expressed as a percentage of the former.

The significance of the 'concordance d' as described above is that it represents closeness-to-perfection of the veena as one can obtain for the specific combinations of N and n under consideration. We see that the lowest value for this

Table 1. Concordance between various notes in the musical instrument veena

Pythagorean ratios std. frequency (Hz)	Integer ratios 240	7/6 280	6/5 288	5/4 300	4/3 320	3/2 360	2/1 480
$N = 10$	n			2	3	5	10
	f_n^N			295.5	316.7	363.8	480
	d%			1.54	1.04	1.04	0
$N = 12$	n	3	4		5	7	12
	f_n^N	285.4	302.4		320.4	359.6	480
	d%	1.93	1.52		0.112	0.114	0
$N = 14$	n		4		6	8	14
	f_n^N		292.6		323.0	356.6	480
	d%		1.6		0.94	0.93	0
$N = 17$	n	4	5	6	7	10	17
	f_n^N	282.5	294.3	294.3	319.8	360.8	480
	d%	1	2.18	1.9	0.23	0.27	0
$N = 22$	n	5	6	7	9	13	22
	f_n^N	280.95	289.9	299.2	318.7	361.5	480
	d%	0.34	0.6	0.26	0.41	0.41	0

parameter is as low as 0.113 in practical veenas and it indicates possible 'closeness to perfection' of the conventional veena that can be constructed. However, in a veena with $N = 22$ the concordance d remains reasonably low over a large number of overtones as partially shown in the table. This indicates the nearness to perfection that Bharatha[2,4] has achieved in the design of a veena over a millennium ago. That he could invent this value for N of 22 over its nearest contenders, viz. 24 and 17 using perhaps only his trained ears is, we consider, really very commendable.

The evolution of the cochlear reeds is such as to separate out, i.e. Fourier transform, the frequency components in the incident sonar signal harmonics[3]. This involves propagation of the pressure waves in the cochlear fluid and the vibrations of the resonating reeds whose persistence duration could be 40 ms or more which is of the same order as photochemical transitions in our eyes. Besides, it is known that the low frequency, ~ 200 Hz or less, harmonics penetrate deep into the cochlear fluid while the high frequency ones penetrate only near its base. Besides, at less than about 200 Hz, the depth of penetration of the sonar signals gets saturated at the far end of the cochlea and it appears that the sonar signals are communicated via the cerebral cortex to the perception centers by a different mechanism; details of which are not very clear as yet.

Table 1 shows that the harmonics generated by the fundamentals at each note are either fundamentals or harmonics and hence are in good harmony with other notes within the scale. In other words, when any of the 72 Janaka Ragas or their numerous derivatives forming an extended musical tune and spanning over multitudes of octaves is played, only a specific fraction of the responding resonating reeds are called upon to participate in the associated transduction; the remaining reeds remain practically stationary contributing to the overall harmony and perception of the musical performance, i.e. each Raga above is associated with a specific narrow band of reeds for its perception, while the rest of the reeds act as silent 'spectators'.

To conclude, in view of the fact that the perception in the human brain is frequency based, here we considered a similar approach for the analysis of the notes. It was found that only when the octaves are divided into a specific number of notes, viz. 12, 17, 22, 24 etc., the veena produced concordant, melodious and harmonious tunes. The reasons for this specificity are ascribed to the requirements to be fulfilled by the active lengths of the wires that (i) these lengths from the bridge to the corresponding frets shall follow a geometric relation given by eq. (1) above and that (ii) the various tunes generated by these wires shall follow faithfully the laws of vibrating strings which state that for a uniform, stretched string, the frequency of the sound generated when it is plucked is inversely related to its length.

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Rediscovery of *Podocarpus wallichianus*: A rare gymnosperm from tropical rain forests of Great Nicobar Island

During our fieldwork in the tropical rain forests of Great Nicobar Island (Figure 1) we came across a plant in a seedling stage (about 40 cm in length), on the East-West Road, 13 km from the Campbell Bay. It was identified as *Podocarpus wallichianus* (*P. latifolia*).

In 1888, J. D. Hooker while treating the gymnosperms has mentioned three species under the tribe Podocarpae, viz. *P. cupressina* R. Br., *P. neriifolia* D. Don. and *P. latifolia* Wall. Phytogeographically *P. latifolia* is known to occur in Khasia mountains, South Deccan, Myanmar and Malay peninsula and Java. *P. neriifolia* is distributed in the tracts of tropical Himalayas and Malaya peninsula. *P. cupressina* is found to occur in the upper Myanmar[1]. So there was no record of the distribution of this plant from the Andaman and Nicobar Island until 1953 when K. C. Sahni collected the plant during a joint expedition to the island headed by B. S. Chengappa. The plant specimen was collected in the interior of the Great Nicobar Island in vegetative stage from a small tree on the hillside above Alexandra River near the Shompen Hut[2]. Since then, several expeditions were undertaken in Great Nicobar Island but this species could not be located. Hence the present report is of special significance.

The collection of the plant *P. wallichianus* (Figure 2) (family Podocarpaceae) is interesting due to its unique distribution in the Nilgiri Hills in Tamil Nadu, Assam, Martaban, Tanasserim usually at the altitude of 900–1500 m and complete absence from the rest of the Andaman and Nicobar islands. Podocarpaceae experienced drastic reduction in number and geographical extent sometime during the later part of the Cretaceous period, but continued to survive in