Fun and Fundamentals of Mathematics. Jayant V. Narlikar and Mangala Narlikar. Universities Press (India) Ltd, 3-5-819 Hyderguda, Hyderabad 500 029. 2001. 194 pp. Price: Rs 200.

There is an urgent need today for books in mathematics that are not of the textbook variety – books that are both fun and serious at the same time. This is particularly so in a country like ours, where the examination-cum-marks culture has taken root at such depths that school teachers are actually nervous of doing anything that is 'out of the syllabus', or anything that smacks of 'fun'. The book under review, written by authors who are gifted teacher-expositors (the senior one in addition being a renowned astrophysicist) is therefore to be welcomed enthusiastically. It is described on the back cover as designed to arouse interest in mathematics for readers in the 12–18 age group; it should certainly succeed in this aim. (We have to hope however that the book does actually get into children's hands! Given the current extremely strong inhibition towards reading non-mainstream material, this cannot be taken for granted.) Many themes of mathematics are presented, in an engaging, informal and reader-friendly style; the authors write with a certain élan, and are quite happy to toss in a cartoon or two where appropriate! Here is a list of some of the topics covered.

Network theory – an old favourite for recreational mathematics buffs – is introduced via the Königsberg bridges problem (requiring travellers to traverse a system of bridges and roads so that every portion is crossed just once) and the four-colour problem (dealing with map-colouring). Brief analyses are given (for the former problem, Euler's original analysis), and some comments on the computer-assisted approach used by Appel and Haken to prove their famous theorem.

Non-Euclidean geometry is discussed ('Can the sum of the angles of a triangle exceed 180°?'), and its significance for cosmology given a brief mention. The mysteries and apparent paradoxes of the one-sided Möbius strip are discussed, and also the possibility of similar paradoxes in higher-dimensional space. (For instance, could a left shoe taken on a closed path around the universe somehow turn into a right shoe? (Read the book to find out!) The discussion here is presented through matrix algebra and is on the harder side, but written with clarity.

The impossibility of certain kinds of geometric constructions is considered, e.g. trisection of an arbitrary angle, using only compasses, straight-edge and pencil. What laymen generally find astonishing is that the impossibility can actually be proved; the logic of the proof is described, in brief. The discussion here is densely packed, with ideas from field theory; but there is no harm in young readers being exposed to such logic.

The counter-intuitive nature of probability theory is presented via the story of a mother and two daughters who live in separate houses on one street, with the mother in between. She visits one daughter each Sunday, travelling by a randomly chosen bus, yet she finds that she visits one daughter more often than the other one; how can this be so? A solution is presented, along with that of the famous Buffon needle problem: here one computes the probability that a matchstick thrown at random on ruled paper crosses one of the lines on the paper.

Cantor's theory of equivalence in sets is discussed, and the question (the 'continuum hypothesis') of whether there exists a set with cardinal number between that of the natural numbers and the real numbers is posed. Some paradoxes of modern set theory and logic are then taken up, e.g. Russell's paradox, followed by speculations on whether there can be 'limits' to mathematics. *En route* the authors discuss Gödel's theorem, and wonder whether some currently unsolved problems are of the unprovable type. (They could have mentioned that the continuum hypothesis is now known to be of this type.) The topic is introduced through the game of cricket in a nice way. In the 'Fallacies' chapter, some evergreen fallacies of elementary geometry and algebra are presented; for example, the 'theorems' that $1 = 0$, $90° = 100°$, and that all triangles are equilateral. As everywhere else, the approach is very engaging.

An extremely unusual chapter is the one on the mathematics of warfare. It is shown, using differential equations, how it can happen in a battle that the numerically smaller army loses much more of its forces than the numerically larger army, even if all combatants involved happen to possess equal skill. This fact invites a 'divide and rule' strategy for dealing with an enemy whose forces outnumber yours.

Many of the topics are introduced through problems – a good pedagogic device. In 'Noughts and crosses' the claim that the number of winning lines (i.e. of length 3) in the $3 \times 3 \times 3$ version of the game is $\frac{1}{2}(5^3 - 3^3)$ is proved most elegantly. The book closes with a useful 'Hints and solutions' chapter.

All in all, the book is well written. However there are a few areas where a tidier treatment was possible. The recent proof of Fermat's Last Theorem is by Andrew Wiles and not 'Andrew Weil' (clearly, an unfortunate mix-up between Andrew Wiles and André Weil, who coincidentally works in the same field). In the 'Fallacies' chapter, the resolution of the geometrical fallacies is incomplete. For instance, in proving that 'every triangle is equilateral' one considers the point D where an angle bisector meets the perpendicular bisector of the opposite side; if D is taken to lie inside the triangle, then a paradox results – one finds that all angles of the triangle have become equal. The resolution now lies in seeing that D must lie outside the triangle. But can one show directly that D *must* lie outside the triangle? Indeed one can, and this convincingly resolves the paradox once for all: but without doing this, the resolution is incomplete.

The only item that this reviewer found quite misplaced is the inclusion of Euclid's classic proof for the infinitude of primes in the chapter on paradoxes. As noted in that chapter, a paradox arises when a statement as well as its opposite both lead to absurdity. In this sense, there is nothing 'paradoxical' at all about Euclid's proof, which remains to this day one of the finest examples of proof by contradiction.

These few flaws apart (and some stray typographical errors, fortunately of no consequence), the book is a fine present for a child who has potentially a deeper interest in mathematics. May there be more such books, and may they triumph over the syllabus!

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