Structure of the lowest octupole states in $N = 82$ magic nuclei

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Properties of the lowest octupole states with spin parity $I^{\pi} = 3^-$ in $N = 82$ magic nuclei ranging from the neutron-rich, doubly-magic nucleus ^{132}Sn ($Z = 50$), through another doubly-magic nucleus ^{146}Gd ($Z = 64$), up to the proton drip-line nucleus ^{154}Hf ($Z = 72$) are examined. A comparative systematics of the excitation energies for the lowest quadrupole, octupole and hexadecapole states is presented. Variation of the $B(E3)$ values with proton number is examined. Results from the microscopic structure analysis for the octupole states in these nuclei are discussed. A simple formula is proposed to describe the excitation energies of the lowest 3^- level in these nuclei in the Sn–Gd region. The unidentified 3_1^- state in ^{134}Te is predicted to lie around 3950 keV.

IN the 1991 White Paper[1], aimed at 'making out the scientific case for a dedicated North American facility for intense high-quality radioactive nuclear beams (RNB)', the Iso-Spin Laboratory (ISL) Steering Committee pointed to 'the fortunate aspect of nuclear physics that often most readily observables, namely the excitation energies of the lowest states and a few key reduced transition probabilities $B(E\lambda)$, are excellent signposts of inherent structure'. For even–even nuclei, the relevant states are the lowest ones with spin-parity 2^+, 4^+ and 3^-. The transitions connecting these states to the ground state reveal respectively the quadrupole, the hexadecapole and the octupole features of shape oscillations. For magic or near-magic nuclei, the higher multipole ($\lambda = 3$ and $\lambda = 4$) transitions correspond to the evolution of the corresponding exotic shapes from the spherical ground state. Global surveys[2–4] of the $E3$ and $E4$ transition rates provide valuable databases for such investigations. Recently we have reported[5] the distinctive features of $E4$ transitions in closed shell (magic) nuclei. Presently we look at the properties of the lowest 3^- (octupole) state in the 12 even–even $N = 82$ nuclides. This sequence ranges from the exotic neutron-rich, doubly-magic nucleus ^{132}Sn ($Z = 50$), through another doubly-magic nucleus ^{146}Gd ($Z = 64$), up to the proton-rich nucleus ^{154}Hf ($Z = 72$) which touches the proton-drip line. A simple formula for describing the excitation energy of the 3_1^- state in nuclei bounded by the $Z = 50$ and $Z = 64$ doubly-magic structures is presented. The expected location of the unidentified 3_1^- state in the two-proton nucleus ^{134}Te is also predicted based on estimates from four different approaches.

In an earlier study of the octupole features of nuclei in the $_{56}$Ba–$_{62}$Sm region, Sheline and Sood[6] had observed that in the region beyond the double closed shell at ^{132}Sn, the $d_{5/2}$ and $h_{11/2}$ proton orbitals are quite close together and lie near the Fermi surface; the combination of these orbitals is expected to give rise to low-lying 3^- states in these nuclei. However that study[6] addressed the question of onset of 'stable' octupole deformation and hence focused on the $N = 88$–90 nuclei. We presently focus on the long sequence of $N = 82$ isotones having spherical equilibrium shape; this sequence includes on either end nuclei off the stability line with unusual $N:Z$ ratios and is expected to present a rich variety of shape evolution processes, as briefly mentioned in the preceding paragraph. In this communication, we discuss the characteristics of the lowest 3^- states in these nuclei.

Over the past decade, considerable effort[7–21] has been made to experimentally explore and theoretically understand the structure of these low-lying states in $N = 82$ nuclei with Z values ranging from 50 to 72. The presently available experimental information[7,8] on the excitation energies of the lowest 2^+, 4^+ and 3^- states in these nuclei is shown in Figure 1. It is seen that the 3_1^- remains unidentified only in one case, namely ^{134}Te (wherein we predict its location later in this report), and the 4_1^+ is not seen in the two heaviest isotones, namely ^{152}Yb and ^{154}Hf. The doubly-closed shell nucleus ^{132}Sn shows 'exceptional stiffness' with respect to excitations, indicating 'the strongest shell closure of any nucleus – about 35% more than ^{16}O or ^{208}Pb'[13]. Investigation of the octupole collectivity of its 4352 keV 3_1^- state in terms of neutron particle-hole configurations was found inadequate, leaving the question of its complex structure still open[13]. For nuclei with $Z > 50$, the excitation energies of the 2_1^+ and 4_1^+ states show a gradual smooth rise, with a perceptible kink at ^{140}Ce corresponding to the $g_{7/2}$ filled sub-shell, up to ^{146}Gd wherein the 'stiffness', related to the doubly-closed shell structure, yields an abrupt 'peak' for both these states; their energies fall-off beyond $Z = 64$ with $E(2_1^+)$ levelling-off around 1.5 MeV for $Z \geq 68$. In contrast, the $E(3_1^-)$ exhibits sharp, almost linear drop between ^{132}Sn and ^{146}Gd, and a slower linear increase for $Z > 64$. In the Sn–Gd region, this behaviour reflects the similar sharp drop of the $h_{11/2}$ single-proton state in the odd–Z, $N = 82$ odd-mass nuclei[22].

The 3^- state in these spherical (magic) nuclei arises from surface octupole vibrations which are normally of collective origin. The degree of collectivity is generally determined by the observed enhancement of the reduced $E3$ transition probability $B(E3)$ over the single-particle estimate; this is achieved by quoting $B(E3)$ values in

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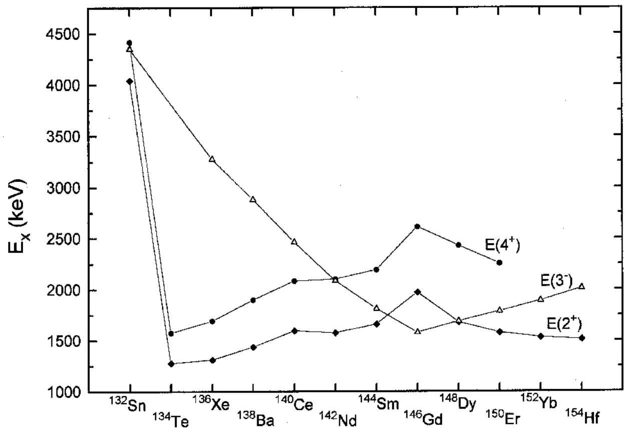

Figure 1. Experimental excitation energies E_x (in keV) for the lowest 2^+, 4^+ and 3^- levels in the $N = 82$ nuclei ranging from ^{132}Sn to ^{154}Hf.

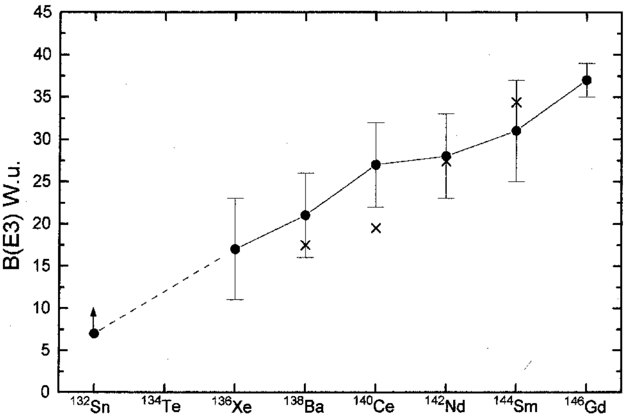

Figure 2. Experimentally measured reduced transition probabilities $B(E3;\ 3^-_1 \rightarrow 0^+$ ground state) in Weisskopf units (W.u.) for presently known cases in $N = 82$ isotones shown by filled circles with indicated uncertainties; crosses denote the calculated values using quasiparticle phonon model[18].

Weisskopf (single-particle) units (W.u.). The experimentally deduced $B(E3)$ values (in W.u.) for nuclei under discussion are shown in Figure 2. The $E3$ transition rate is presently unknown for any $Z > 64$ nucleus. The $B(E3)$ values are experimentally determined from the inelastic scattering of electrons or protons, Coulomb excitation, or combination of level lifetime with decay branching ratios; these processes yield values with large uncertainties (e.g. 31 ± 6 W.u. for ^{144}Sm by Coulomb excitation) or widely varying values from different processes; for example, for ^{133}Ba, (e, e′) yields (24.6 ± 1.5) W.u., while Coulomb excitation yields (16.8 ± 1.7) W.u. The average values drop from 37 W.u. to 17 W.u. from ^{146}Gd to ^{136}Xe, with only a lower limit of 7 W.u. indicated[13] for ^{132}Sn.

Detailed theoretical calculations of these transition rates using quasiparticle-phonon model (QPM)[10–12,18] yield satisfactory agreement with the experiment for the stable nuclei in the Ba–Sm region, as seen in Figure 2. These model calculations also specify the microscopic composition of individual states. For the lowest experimental 3^- state, they conclude around 90% one-phonon contribution for each case with about 60–70% contribution to the one-phonon RPA state arising from the $\pi(2d_{5/2} \otimes 1h_{11/2})$ two-quasiparticle configuration.

Shell model calculations with realistic interactions have been reported[19–21] for the two-proton nucleus ^{134}Te and the doubly-magic nucleus ^{146}Gd. It was concluded[21] that the $(d_{5/2} \otimes h_{11/2})$ configuration exhausts about 67% of the norm in the wavefunction of the 3^-_1 state in ^{146}Gd. For the $N = 82$ nuclei beyond $Z = 64$, the valence protons start occupying the non-normal parity deformation-driving high–spin proton orbital $h_{11/2}$. It has been concluded[9] that $h_{11/2}$ subshell is close to being half-filled in the six valence proton nucleus ^{152}Yb and it is more than half–filled in ^{154}Hf. Thus the simple prescription of describing 3^-_1 as a coupled state of $h_{11/2}$ with the core orbitals $d_{5/2}$ and/or $g_{7/2}$ is no longer valid for $Z > 64$ nuclei.

Based on the above observations, we propose the following simple formula for describing the excitation energy of 3^-_1 state in the nuclei beyond ^{132}Sn, wherein the valence protons occupy $g_{7/2}$ or $d_{5/2}$ orbitals, i.e. from ^{134}Te up to ^{146}Gd:

$$E(3^-; Z, A) = \alpha[E_x(h_{11/2}) + E_x(d_{5/2}) + E_{pair}] + (1 - \alpha)[E_x(h_{11/2}) + E_x(g_{7/2}) + E_{pair}], \quad (1)$$

where E_x on the right side denotes the experimentally observed excitation energy of the corresponding orbital state in the $(Z - 1,\ A - 1)$ odd-mass neighbour, α denotes the fractional contribution from the $(h_{11/2} \otimes d_{5/2})$ configuration and the pairing term E_{pair} is half the value of the pairing energy gap evaluated using the atomic mass data[23]. The calculated values using this formula with $\alpha = 0.65$, as estimated in microscopic calculations[10–12,18,21] discussed above, are shown in Figure 3 in comparison with the experimental results. Remembering that all microscopic calculations indicate about 10% contribution to 3^-_1 energy from other configurations and two-phonon structures, the agreement between the calculated and the experimental values is found to be quite satisfactory.

Out of the 12 isotones in this sequence, 3^-_1 has been experimentally identified in all but one nucleus; the single missing value is for the two-proton nucleus ^{134}Te. We use[24] four different approaches to estimate the missing datum. First is the empirical approach which uses the known experimental data shown in Figure 1. Assuming a pure linear variation, an interpolation between the nearest neighbours ^{132}Sn and ^{136}Xe suggests $E_x(3^-,\ ^{134}\text{Te}) = 3.81$ MeV. However, it is well known that $E_x(3^-)$ drops down significantly for doubly-magic

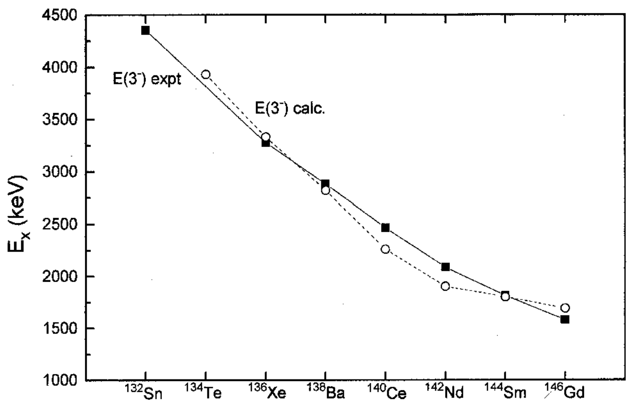

Figure 3. Comparison of the experimental (filled squares joined by full line) and calculated (open circles joined by dashed line) excitation energies for the lowest 3^- levels in $A = 132–146$, $N = 82$ isotones. Calculated values are from eq. (1) given by us in the text with $\alpha = 0.65$.

nuclei; in ^{132}Sn, 3^- has dropped even below the first 4^+ level. Accordingly, the empirical approach should employ backward extrapolation of the $Z \geq 54$ data curve; this procedure yields $E_x(3^-, {}^{134}\text{Te}) \approx 3.95$ MeV. The second approach uses our simple formula, eq. (1) in this report; with a value of $\alpha = (0.70 \pm 0.05)$ we predict $E_x(3^-, {}^{134}\text{Te}) = (3.98 \pm 0.05)$ MeV. The third approach is based on the well-established[14] analogous nature of few-particle nuclei beyond the doubly-closed shell structures in ^{132}Sn and ^{208}Pb; it is seen that each (n, l, j) single particle state in the post – ^{132}Sn region has an analogous $(n, l+1, j+1)$ state in the post – ^{208}Pb region in the same order and with very similar spacings. The recent experimental study[14] of ^{134}Te level structures has specifically brought out in detail the observed one-to-one correspondence of the various two-proton configuration states in ^{134}Te and its analogous two-proton nucleus ^{210}Pb. The 3^- level in ^{210}Pb is placed 3.2 keV below the $I_{max} = 11^-$ level of the $(h_{9/2}\ i_{13/2})$ configuration. Taking into account the $A^{1/3}$ dependence of level spacings, analogy between ^{134}Te and ^{210}Pb spectra predicts the 3^- in ^{134}Te to be about 10 keV below the analogous $I_{max} = 9^-$ level of the $(g_{7/2}\ h_{11/2})$ configuration which is observed[14] at 4013 keV. Finally, the position of the missing 3^- in ^{134}Te is estimated using the microscopic shell model calculations with realistic interaction. Sarkar and Sarkar[20] conclude that their CW5082 interaction 'is undoubtedly a better choice' for predicting spectra of the $N = 82$ isotones. Using this interaction, they place the missing 3^- level in ^{134}Te 20 keV lower in energy than the 9^- level; with the observed $E_x(9^-) = 4.013$ MeV, the predicted relative location of the 3^- is just below 4.00 MeV. Combining the results from these four approaches discussed above, we place the missing 3_1^- state at (3.97 ± 0.03) MeV excitation energy in ^{134}Te. Experimental confirmation of this result may be sought by looking for an $E2$ γ feeding of this level from the 4323 keV 5^- level and its expected decays to the ground state through the 2462 keV 2_2^+ or 1279 keV 2_1^+ levels.

In summary, we have examined the systematics of the lowest octupole 3_1^- state in the $N = 82$ isotones in the $A = 132–154$ region through its observed excitation energies and $B(E3)$ transition rates in comparison with theoretical calculations. A simple formula for $E_x(3_1^-)$ is proposed and the expected location of the only missing 3_1^- in this long isotonic sequence has been predicted.

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