Joint modelling of gravity and magnetic fields – A new computational approach

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It is shown that the gravity and magnetic fields resulting from an arbitrary three-dimensional object bounded by plane polygonal facets having uniform volume density and magnetization, can be expressed as the field due to an equivalent distribution of surface mass density and surface pole density over the bounding surface of the source body. This provides a new relation between the components of the corresponding gravity and magnetic fields and facilitates construction of a unified algorithm for the computation of gravity and magnetic anomalies separately or simultaneously. This algorithm appears to be computationally more efficient than the well-known Poisson's relation, commonly used for the transformation of one field to another.

THEORETICAL computation of gravity and magnetic (GM) anomalies due to arbitrary-shaped discrete sources, forms an integral part of any interpretation scheme used in the modelling of potential field data. The most convenient and useful geometry adopted for representing the discrete sources is a polyhedron bounded by plane polygonal facets. Over the years, a number of algorithms have been developed based on analytical solutions provided by various workers[1–11]. These formulations are based on the assumptions that the source region has uniform volume density and uniform magnetization distribution, which seems to prevail in a great majority of practical cases. It is observed that there is a general improvement in the analytical approach of evaluating the required volume integral over the polyhedral source body. The step involves transformation of the required volume integral into an equivalent surface integral by the use of Gauss's divergence theorem and subsequently the surface integral is either evaluated analytically over the entire boundary surface or transformed into line integral along the edge of the polygon using Stoke's theorem. Most of these workers have addressed the computation of the gravity and magnetic fields independently or employed the Poisson's relation to transform the gravity field into components of magnetic fields.

In this paper, we present a new relationship between the components of gravity and magnetic fields using the concept of surface mass-density and surface pole-density distribution over the surface of the polyhedral source having uniform density and magnetization. This has resulted in the construction of a unified algorithm for computing the gravity or the magnetic field or both simultaneously.

Let the arbitrary object be represented by a polyhedron bounded by plane polygonal facets having uniform volume magnetization **m** and uniform volume density ρ. Transforming these volume distributions into an equivalent surface distribution, the components of the anomalous magnetic field vector **H** and gravity field vector **F** in the direction of the surface element ds having position vector **r** (Figure 1) may be written in the form of a surface integral[10,12] as,

$$H_r \cong -C_{\mathrm{m}} \sum_{i=1}^{N} \sigma_i \iint_s \frac{1}{r^2} \mathrm{d}s, \tag{1}$$

$$F_r \cong G \sum_{i=1}^{N} \sigma_i' \iint_s \frac{1}{r^2} \mathrm{d}s, \tag{2}$$

where r is the distance between the point of observation at P and the source at Q, C_{m} is a constant used to balance units and has a value that depends on the system in use. In the EMU system $C_{\mathrm{m}} = 1$ and is dimensionless, whereas in SI units $C_{\mathrm{m}} = (\mu_0/4\pi)$ Henry/meter, where μ_0 is the permeability of the free space. G is the universal gravitational constant and N is the number of polygon facets bounding the object. To simplify calculations, the origin is placed at the observation point P.

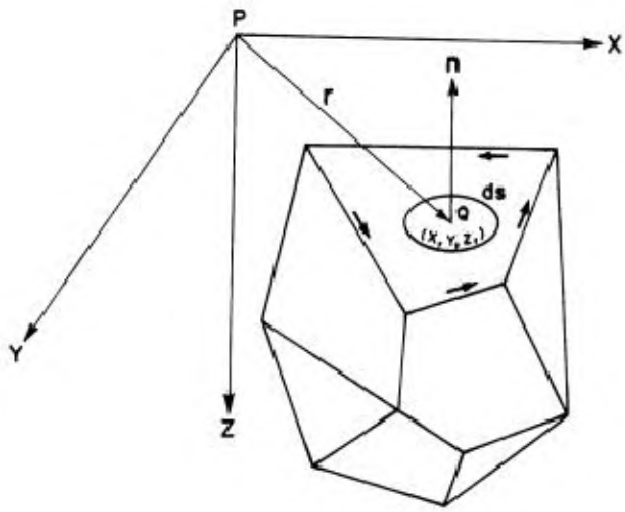

Figure 1. Three-dimensional body approximated by a surface composed of flat polygonal facets. Small solid arrows show the direction of line integration (counter-clockwise) around the edge. The scalar product of the unit outward normal vector **n** with the radius vector **r** of any point (x, y, z) on the polygon surface is negative if the surface is seen from outside, but is positive otherwise.

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σ in eq. (1) is the surface pole density and is numerically equal to $\mathbf{m}\cdot\mathbf{n}_i$. It is a constant on each facet and has the unit of pole strength per unit area. The symbol (.) represents the scalar product of vectors and $\mathbf{n}$ is the outward-directed unit vector normal to the surface element ds. Thus, the problem reduces to the calculation of magnetic fields caused by distribution of magneto static charges (magnetic poles) on the surface of the object. Therefore, a three-dimensional body having uniform volume magnetization can be completely represented by a distribution of magnetic charges on the surface of the object. Although visualization of magnetic charge is fictive, the conceptual model is simple and very useful in computing the magnetic fields. Similarly, σ_i' in eq. (2) is the surface mass-density and is equal to $\rho(\mathbf{r}\cdot\mathbf{n}_i)$. It is also a constant on each facet and has the dimension of mass per unit area. It may be noted that the value of σ_i' can be positive or negative depending on $(\mathbf{r}\cdot\mathbf{n}_i)$. It is merely an artifice introduced to simplify the calculations.

In the above formulation, the magnetic field due to a positive surface pole density is directed away from the surface element ds while the gravity field due to positive surface mass density is directed towards the surface element ds. Then, the x-, y- and z-components of the gravity and magnetic fields may be obtained by multiplying the integrand in eq. (1) and eq. (2) by the ratio (x/r), (y/r) and (z/r) respectively.

The surface integrals in eqs (1) and (2) can be evaluated by converting them into line integrals around the boundary of each polygonal facet using Stoke's theorem. The results thus obtained are combined with the outward normal component of the field for that facet to get the x, y, and z components of the anomalous magnetic and gravity fields[11] as,

$$H_x \cong -C_{\mathrm{m}} \sum_i \sigma_i S_{xi},$$

$$H_y \cong -C_{\mathrm{m}} \sum_i \sigma_i S_{yi}, \qquad (3)$$

$$H_z \cong -C_{\mathrm{m}} \sum_i \sigma_i S_{zi},$$

and

$$F_x \cong \mathrm{G} \sum_i \sigma_i' S_{xi},$$

$$F_y \cong \mathrm{G} \sum_i \sigma_i' S_{yi}, \qquad (4)$$

$$F_z \cong \mathrm{G} \sum_i \sigma_i' S_{zi},$$

where S_{xi}, S_{yi} and S_{zi} are geometrical factors, which depend on the attitude of the polygon surface with respect the observation point and are given as,

$$S_{xi} = l_i \ddot{Y}_i - n_i Q_i + m_i R_i,$$

$$S_{yi} = m_i \ddot{Y}_i - l_i R_i + n_i P_i, \qquad (5)$$

$$S_{zi} = n_i \ddot{Y}_i + l_i Q_i - m_i P_i,$$

where l_i, m_i and n_i are direction cosines of the unit outward normal vector for the ith facet, Ω_i is the solid angle subtended by the ith facet as viewed from the observation point and is numerically equal to the normal field component for that facet. The value of Ω is calculated from any standard procedure, such as the one given by Guptasarma and Singh[10]. P_i, Q_i and R_i are simply the line integrals of the vectors $\mathbf{i}/r$, $\mathbf{j}/r$ and $\mathbf{k}/r$ taken around the edge of the ith facet in the counter-clockwise direction and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the x, y and z axes. The contributions, P_{ij}, Q_{ij} and R_{ij} from the jth edge of the ith facet may be obtained from refs 9–10.

The above procedure has been implemented in a computer code and tested for different polyhedral models, including those given by Coggon[3], Plouff[12] and Xiong and Chouteau[13]. The gravity field can be computed everywhere, including points on the edge or at a corner of the body where more than two surfaces meet, because in such a case $\sigma' = 0$ and all the facets meeting there can be omitted from the summation process indicated in eq. (4). However, the magnetic field cannot be computed when the observation point happens to be at a corner or on the edge of the body.

By comparing eqs (1) and (2), we find that the components of the gravity and magnetic fields have the same form and therefore a direct relation can be established between the components of the corresponding gravity and magnetic fields. Combining eqs (4) and (5) we get,

$$H_x = -\left(\frac{C_{\mathrm{m}}}{G}\right) \sum_i \left(\frac{\sigma_i}{\sigma_i'}\right) F_{xi}, \qquad (6)$$

where $H_x = \sum_i H_{xi}$.

Equation (6) provides a new relation between the x component of the anomalous gravity and magnetic fields. A similar relation can also be obtained between the other field components.

The above equation allows the construction of a unified algorithm for the simultaneous computation of gravity and magnetic fields of an arbitrary three-dimensional object approximated by a polyhedron.

From eq. (6) it is clear that the magnetic field may be obtained by carrying out the same numerical calculations as those needed for the gravity field, except that $(-G\sigma)$ needs to be replaced by $(C_m\sigma)$. Therefore, the same computer code can be used for computing the gravity and magnetic fields separately or simultaneously.

Keeping the geometry and physical properties same as above, the Poisson's relation[14] which is commonly used for the transformation of gravity to magnetic anomalies may be given as,

$$H_x = \frac{M}{G\rho}\sum_i \frac{\partial F_{xi}}{\partial I}, \tag{7}$$

where M is the intensity of magnetization and I is the direction of magnetization.

It is interesting to note that the new relation derived between gravity and magnetic field components (eq. (6)) differs from the well-known Poisson's relation (eq. (7)). The new approach provides a direct relation between the corresponding field components, whereas Poisson's approach gives a relation between the components of magnetic field to the gradient of the gravitational attraction in the direction of magnetization. It is apparent therefore that the new algorithm is computationally more efficient in the transformation of gravity field components to the magnetic fields and vice versa.

A new relation is derived between the components of the gravity and magnetic fields resulting from an arbitrary three-dimensional object bounded by plane polygonal facets having uniform volume density and magnetization. Gravity and magnetic field components have been expressed as the field due to an equivalent distribution of surface mass-density and surface pole-density over the bounding surface of the source body. This facilitates construction of a unified algorithm for the computation of gravity or magnetic anomalies separately or simultaneously. This algorithm will substantially simplify and speed-up the numerical modelling of gravity and magnetic anomalies and may find wide applications in the joint inversion of potential field data. The gravity field can be computed at all points, including points on the surface of the body or at any of its corners. However, the magnetic field has a singularity at the corner. Our formulation does not require coordinate transformation, as with all previously published schemes. It appears to be computationally efficient compared to the well-known Poisson's relation normally used for the transformation of one field to another.

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Differential influence of pollen and stylar genotypes on lifespan of pistillate flowers in a monoecious herb, *Momordica tuberosa* (Cogn.) Roxb. (Cucurbitaceae)

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The floral lifespan of pistillate flowers of *Momordica tuberosa* is terminated by abscission of petals, intentionally, when they are still fully turgid within sixty-five minutes following their pollination; the staminate flowers do not abscise their petals, but progressively wilt. Petal abscission was found to depend on the type of pollination: hand pollination using outcross pollen induced an early petal abscission than the self-pollen (geitanogamy). At least six pollen grains were critical to initiate successful petal abscission in freshly-bloomed flowers, below which the petals did not abscise even at the end of the day. There exists a quantitative relationship between the number of pollen grains deposited onto the stigma and the time taken for petal abscission; greater the pollen grain deposition, quicker the rate of abscission. The time required to induce petal abscission seems to be controlled by pollen genotype, genotype of pollen recipient and the interaction between the two. By invoking the theory of sexual selection it is explained that strategies of pollen genotype and those of stylar genotype may not always coincide,

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