Physics with large extra dimensions: String theory under experimental test

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A theory with such a mathematical beauty cannot be wrong: this was one of the main arguments in favor of string theory, which unifies all known physical theories of fundamental interactions in a single coherent description of the universe. But no one has ever observed strings, not even indirectly, neither the space of extra dimensions where they live. However, there is a hope that the 'hidden' dimensions of string theory are much larger than what we thought in the past and they become within experimental reach in the near future, together with the strings themselves.

LOOK in front of you. Now on your side. Next on the top. These are the known spatial dimensions of the Universe: there are just three. Have you ever wondered about the origin of this number? Have you ever thought if there are new dimensions that can escape our observation?

In all physical theories, the number of dimensions is a free parameter fixed to three by observation, with one exception: String theory, which predicts the existence of six new spatial dimensions. This is the only known theory today that unifies the two great discoveries of 20th century: quantum mechanics, describing the behaviour of elementary particles, and Einstein's General Relativity, describing gravitational phenomena in our Universe.

String theory replaces all elementary point-particles that form matter and its interactions with a single extended object of vanishing width: a tiny string. Thus, every known elementary particle, such as the electron, quark, photon or neutrino, corresponds to a particular vibration mode of the string (see Figure 1). The diversity of these particles is due to the different properties of the corresponding string vibrations.

Until now, there is no experimental confirmation of string theory. String theorists feel though that the situation may be similar to the one of Einstein's General Relativity before 1919, when its first experimental test has arrived in the occasion of a total eclipse of sun. String physicists believe today in string theory, mainly for theoretical reasons, because it provides a framework for unification of all interactions including gravity. However, some precise experimental tests are necessary to decide whether this theory describes the physical reality.

Small dimensions

How can it be tested? If our universe has really six additional dimensions, we should observe new phenomena related to the existence of these dimensions. Why has nobody detected them until now? String theorists had an answer for a long time: because the size of the new dimensions is very small, in contrast to the size of the other three that we know, which is infinitely large.

An infinite and narrow cylinder for example is a two-dimensional space, with one dimension forming a very small cycle: one can move infinitely far away along the axis, while one returns back at the same point when moving along the orthogonal direction (see Figure 2). If one of the three known dimensions of space was small,

Figure 1. In string theory, the elementary constituent of matter is a miniscule string, having vanishing width but finite size. It can be open with free ends (upper part), or closed (lower part). Its vibration modes, like the ones shown above in two dimensions, correspond to various elementary particles. *Cordes ouvertes* means open strings in French and cordes fermées means closed strings.

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Figure 2. A small dimension of space may have two different forms. It may close back to itself: in the surface of a cylinder, the dimension transverse to its axis forms a closed cycle. But it may also form an interval, like the thickness of a box: it is impossible to go out of the box, since there is nothing there, not even vacuum. Obviously, it is impossible to draw really extra dimensions within our space.

say of millimeter size, we would be flat and, while we could move freely towards left or right, forward or backward, it would be impossible to do more than a few millimeters up or down where space ends.

For a long time, string physicists thought that the six extra dimensions were extremely small, having the smallest possible size of physics, associated to the Planck length $\sim 10^{-35}$ m. In fact, strings were introduced to describe gravitation whose strength becomes important and comparable to the strength of the other three fundamental interactions (electromagnetic, nuclear strong and weak) at very short distances, of the order of the Planck length. It was then natural to assume that the size of the extra dimensions should be of the same order. In this case, the manifestation of new phenomena associated to the extra dimensions are by far out of experimental reach, at least in particle accelerators. Indeed, the Large Hadron Collider (LHC) which is the biggest accelerator under construction at CERN (European Center of Nuclear Research) in Geneva (Switzerland) will explore short distances, only up to 10^{-19} m.

The situation changed drastically recently. During the last three years, more and more theorists examine the possibility that the new dimensions of string theory may be much larger than we thought in the past. The increasing number of scientific articles studying this possibility and the high positions they occupy in the list of most cited papers testify this fact[1]. These ideas lead in particular to experimental tests of string theory that can be performed at TEVATRON (current collider of protons and anti-protons at Fermilab in Chicago, USA) and LHC.

Supersymmetry breaking and TeV dimensions

The first indication of large extra dimensions in string theory came in 1988 from studies of the problem of supersymmetry breaking[2]. Supersymmetry is a new fundamental symmetry of matter which renders the masses of elementary particles compatible with gravitation. In fact, in a quantum theory without supersymmetry, the presence of gravity, which is much weaker than the other interactions, introduces a new high energy scale, the Planck mass $\sim 10^{19}$ GeV (gigaelectronvolts), which attracts all masses of elementary particles to become 10^{16} times heavier than their observed values: this is the so-called mass hierarchy problem.

One of the main predictions of supersymmetry is that every known elementary particle has a partner, called superparticle. However, since none of these superparticles have ever been produced in accelerators, they should be heavier than the observed particles. Supersymmetry should therefore be broken. On the other hand, protection of the mass hierarchy requires that its breaking scale, i.e. the mass splitting between the masses of ordinary particles and their partners, cannot be larger than a few TeV (teraelectronvolts). They can therefore be produced at LHC, which will test the idea of supersymmetry.

Assuming that supersymmetry breaking in string theory arises by the process of compactification of the extra dimensions (i.e. from their intrinsic geometry and topology), one can show that its energy breaking scale is tied to the size of these dimensions. Thus, a breaking scale in the TeV region would imply the existence of an extra dimension of size $\sim 10^{-18}$ m (ref. 2). This was the only general prediction of string theory which had the chance to be testable in the next generation of particle experiments in the near future.

At that time however, this result was not taken seriously. The reason was rather due to a theoretical prejudice to evade invalidating the only computations we could do. Even now, string theory is so complicated that it can be studied in most cases only approximately, namely in perturbation theory. More precisely, computations can be performed if strings interact weakly. The strength of string interactions is controlled by a parameter, called string coupling, which increases when extra dimensions become large. Thus, when their size is larger than 10^{-35} m the approximations of perturbative computations do not hold. In practice, nobody took the risk and the above result has been interpreted negatively. Namely, that supersymmetry breaking could not arise by compactification and remains as an open question.

Two years later, in 1990, I proposed specific models where perturbative computations were possible in part of the theory, even in the presence of large extra dimensions of size of 10^{-18} m (ref. 3). In this class of models, the new dimensions form small intervals with our world localized at their ends. The mediators of interactions on the other hand can propagate in the bulk (along the intervals). The study of the physical consequences of these models was performed subsequently in ref. 4. Their main characteristics is the production of the so-called Kaluza–Klein states in particle accelerators (see Figure 3). If for instance the photon propagates along an extra dimension, one should observe a tower of massive particles with the same properties as the photon but with a mass that becomes higher as the size of the extra dimension is getting smaller. It follows, that for a size of order 10^{-18} m, an energy of order of a few TeV would be sufficient to produce them. Thus, the existence of such dimensions is testable in LHC. Moreover, these models contain a very light particle that mediates new attractive forces at short distances of the order of a fraction of millimeter, which can be tested in table-top experiments that measure the Newton's law[5] (see Figure 5). Despite this progress, there was little interest in models with large dimensions because of theoretical reasons related to the large string coupling problem.

String dualities

We thus arrived in 1996, when Edward Witten proposed that the string size is a free parameter of the theory, with *a priori* no relation to the Planck length[6]. In particular, it could be as large as 10^{-18} m which is just below the limiting distance that can be probed by present experiments[7]. In order to understand the change of situation, let us return a couple of years earlier. All the works discussed until now were in the context of the so-called Heterotic string theory. On the other hand, there were in total five consistent string theories! Four of them contain only closed strings that form closed loops; the other contains also open strings with ends that move freely with the speed of light; besides, all these theories do not have the same amount of supersymmetry.

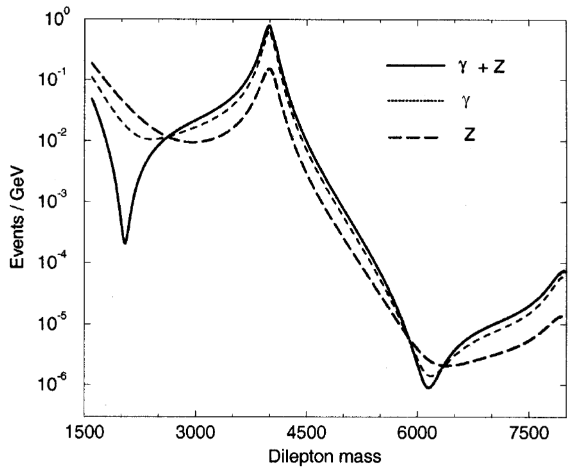

Figure 3. If there is an extra dimension of size 10^{-18} m, felt by the electroweak interactions, LHC should produce the first Kaluza–Klein states of the photon and of the *Z* boson (one of the mediators of weak interactions). We can then detect the electron–positron pairs produced by the desintegration of these states. The number of the expected events is computed as a function of the energy of the pair in GeV (gigaelectronvolts). From highest to lowest: excitation of photon + *Z*, photon and *Z* boson.

Figure 4. How can we draw an elephant if we have never seen it but only certain different parts? In the same way, the five string theories describe some particular corners of *M* theory, which no one knows yet.

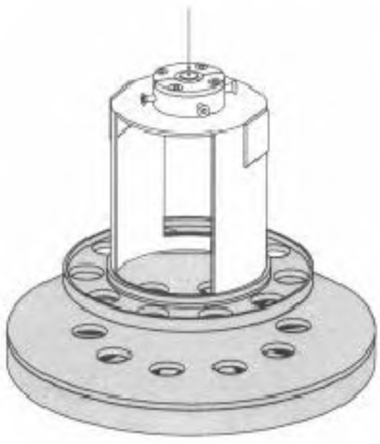

Figure 5. In *Principia* of 1687, Isaac Newton explained that the gravitational attraction between two bodies is proportional to the product of their masses and the inverse square of their distance. The success of this law was spectacular, in particular for the description of the planets motion. At short distances, however, the validity of this law is not tested experimentally. In February 2001, C. D. Hoyle and his colleagues from Washington University published the strongest bounds at present: using the torsion pendulum shown in the figure, they have tested the validity of Newton's law down to 0.2 millimeters (ref. 11). Thus, the size of extra dimensions where gravity propagates is constrained by this value, which allows them to exist at shorter distances, e.g. at 0.01 millimeter. By improving the sensitivity of measure, one should then see violations of Newtonian gravity at these distances. The weakness of gravity complicates considerably the experiments: there are several sources of background noise due to other forces that should be eliminated using appropriate devices. At very short distances, one should consider even the Casimir attraction due to fluctuations of the vacuum.

This multiplicity of theories was creating a problem, since string theory was supposed to provide a unified framework of all physical theories. Thus, one of the five string theories had to be the right one to describe nature but which one? The answer has been provided in 1994: all of them simultaneously. Following the works of several groups and in particular of Witten, it was realized that every known string theory describes a particular limit of an underlying more general fundamental theory that can be defined in eleven dimensions of space, called *M*-theory (Figure 4).

This discovery made an important progress but did not solve all problems. The main achievement was the connection of the five string theories due to the existence of duality symmetries. One type of these symmetries relates two theories with mutually inverse string couplings. Thus, to solve a problem in the context of some theory with large coupling, it is sufficient to perform an appropriate duality transformation. One obtains then a new problem in the context of a dual theory which has a small coupling, the inverse of the former. The new problem can be solved in perturbation theory of the small coupling. Finally, the resulting solution can be transformed back using the inverse duality transformation that takes us in the first theory. Since computations with large coupling became effectively possible, the road was open to study models with extra dimensions much larger than the Planck length.

Figure 6. In the type I string framework proposed in 1998, our Universe contains, besides the three known spatial dimensions (reduced here to a single orange line), some extra dimensions longitudinal to our world brane (here only one is shown on the grey plane) along which the light described by open strings propagates, as well as some transverse dimensions (here only one again is shown) where only gravity described by closed strings can propagate. Moreover, matter is localized everywhere and propagates only in our three dimensions. The longitudinal extra dimensions have the string size of the order of 10^{-18} m, while the size of the transverse dimensions varies in the range of 10^{-12} m and a fraction of a millimeter. Lumière means light in French.

On the other hand, in February 1998, a new idea was proposed for explaining the mass hierarchy problem: why the gravitational force remains much weaker than the other fundamental forces up to very high energies, of the order of the Planck scale ~ 10^{16} TeV? As we discussed above, a possible solution is provided by the introduction of supersymmetry at electroweak energies. N. Arkani-Hamed, S. Dimopoulos and G. Dvali have observed that if at the TeV scale gravity was 10^{32} times stronger than what we believed in the past, then all interactions would have comparable strengths[8]. Thus, in this case, there is no hierarchy problem to explain and there is no need to introduce supersymmetry. This idea can be realized without experimental conflict in models possessing large extra dimensions along which only gravity propagates: gravity appears to us very weak at macroscopic scales because its intensity is spread in the 'hidden' extra dimensions. In order to increase the gravitational force without contradicting present observations, one has to introduce at least two such extra dimensions of size that can be as large as a fraction of a millimeter. At these distances, gravity should start deviate from Newton's law, which may be possible to explore in laboratory experiments (Figure 5).

Large dimensions and TeV strings

These developments led us a few weeks later to propose a new string theory framework for particle physics[9]. The hierarchy problem can be explained if the fundamental string size is fixed to 10^{-18}–10^{-19} m. A convenient perturbative framework realizing this idea is one of the five string theories, called type I, that contains simultaneously closed and open strings. Our universe should be localized on a hypersurface, i.e. a membrane extended in p spatial dimensions with $p \leqslant 7$, called p-brane (see Figure 6). Closed strings describe gravity and propagate in all nine dimensions of space: in those extended along the p-brane, as well as in the transverse ones. On the contrary, the endpoints of open strings describing the other interactions are confined on the p-brane.

Obviously, our p-brane world must have at least the three known dimensions of space. But it may contain more: as opposed to the transverse dimensions that interact with us only gravitationally, the 'longitudinal' to the brane extra dimensions can be 'seen' by the light at sufficiently high energies, giving rise to the production of massive Kaluza–Klein particles in accelerators (see Figure 3).

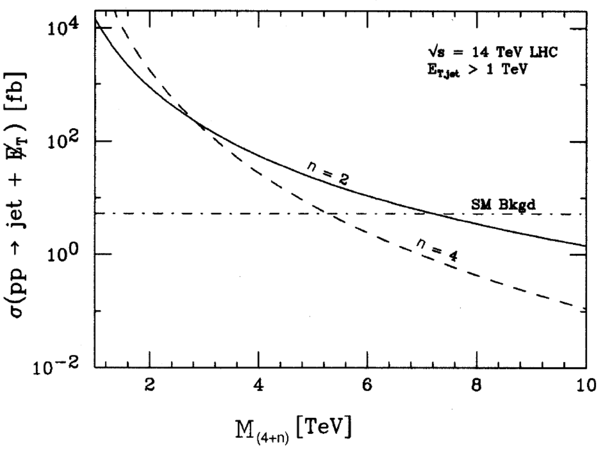

Figure 7. Missing energy due to graviton emission in the LHC experiment, as a function of the fundamental scale $M_{(4+n)}$ of quantum gravity that propagates in n large transverse dimensions. It is produced together with a hadronic jet that one detects in the collision of the two proton beams. The figure shows the expected cross-section for $n = 2$ and $n = 4$ extra dimensions, together with the background (horizontal dotted-dashed line) coming from other known sources[12].

On the other hand, the existence of the extra large (sub)millimeter dimensions, transverse to our p-brane universe, guarantee that gravitational interactions appear to us very weak at macroscopic distances, larger that a millimeter. The size of these transverse dimensions varies from a fraction of millimeter (in the case of two) to a fermi (10^{-14} m in the case of six). Their characteristic signal in particle colliders is graviton emission into the bulk, leading to missing energy that escapes detection (see Figure 7).

A year later, in 1999, we reconsidered the old models proposed in 1990, in the context of the heterotic string. As mentioned before, at that time, the size of the extra dimensions was much larger than the heterotic string length, making the string coupling strong and invalidating most of the perturbative computations. Now these models could be studied upon an appropriate duality transformation in perturbation theory. Surprisingly, we found out that most of the old models are in fact equivalent to the recent ones with open strings and transverse dimensions[10]. In addition, by varying the number of large dimensions in the old models, we discovered, due to duality, an alternative way to lower the string scale at the TeV, and thus, accounting for the hierarchy without supersymmetry. In these models, there are no extra large transverse dimensions but still the gravitational force is very weak: it is fixed by the value of the string coupling, which in this case is an independent free parameter of the theory.

Clearly, today, these theories exist only in our imagination. However, we look forward at the next generation of high energy experiments and in particular at the most powerful machine, the LHC, which will start working around 2006 at CERN. It consists of two beams of protons that will collide at a centre of mass energy of 14 TeV, almost a hundred times bigger than the energy of LEP, the previous accelerator of electrons and positrons at CERN, and about ten times higher than the energy of TEVATRON. I am convinced, as the majority of my colleagues, that LHC will play a very important role for the future of high-energy physics of fundamental interactions. In fact, it is designed since last decade to explore the origin of mass of elementary particles and to test, in particular, the idea of supersymmetry, looking for the production of superparticles. We now hope that this accelerator may discover more spectacular and 'exotic' phenomena, such as the existence of large extra dimensions of space and of fundamental strings.

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