String theory and tachyons

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In the first part of this article, I give a brief review of string theory and its goal. In the second part I outline the study of tachyons in string theory and an attempt at a non-perturbative formulation of string theory based on these studies.

Although most physicists might have by now heard both words – string theory and tachyons – neither of these topics may be very familiar to a majority of the physicists. Keeping this mind, I shall spend a major part of the article explaining the role of string theory in the formulation of a unified theory of all matter and their interactions, and also briefly explain the role of tachyons in ordinary quantum field theories. Then I shall turn to the role of tachyons in string theory.

String theory and gravity

RELATIVISTIC quantum field theory – the theory that unifies special theory of relativity and quantum mechanics – has been extremely successful in providing a description of elementary particles and their strong, weak and electromagnetic interactions. The currently accepted theory of elementary particles, known as the 'standard model', works extremely well in explaining all the observed experimental phenomena involving elementary particles. The notable exception is the discovery of neutrino mass; according to the standard model the neutrinos are massless, but recent experiments suggest that neutrinos have mass. This, however, can be easily accommodated using minor modification of the standard model within the framework of quantum field theory.

However, the techniques of quantum field theory, at least its perturbation expansion that has been so successful in the context of particle physics, do not seem to work for quantizing gravity. If we quantize gravity using the standard techniques of perturbative quantum field theory, we encounter a new kind of particle, known as the graviton, which mediates gravitational interaction just as photons mediate electromagnetic interaction in quantum electrodynamics. However, while calculating probability amplitudes for processes involving exchange of the graviton, we run into infinite answers. These infinities come from a region of loop momentum integration, where the momenta carried by the intermediate virtual particles is large (or equivalently, the distance between the interaction points is small) and are known as ultraviolet divergences. Figure 1 is a particular example of such a divergent Feynman diagram in quantum gravity. Such ultraviolet divergences also occur in more conventional quantum field theories, including the standard model, but there all these infinities can be absorbed into a redefinition of the various coupling constants, and once the amplitudes are expressed in terms of physically observed coupling constants rather than the original parameters used in writing down the theory, all expressions become finite. This procedure breaks down in the case of quantum gravity since the strength of the gravitational interaction grows with energy/momentum, and as a result the ultraviolet divergences in quantum gravity are much more severe.

Since ultraviolet divergences are controlled by the behaviour of the theory at short distances, one might hope that one could cure the ultraviolet divergences of quantum gravity by modifying the theory at short distances, while keeping the behaviour of the theory at large distances, the same as that of general relativity. String theory is one of the attempts to achieve this goal. The basic idea in string theory is quite simple. One simply postulates that different elementary particles are different vibrational states of a string, rather than point-like particles. As we shall see, starting from this simple hypothesis, and requiring that string theory is consistent with the rules of quantum mechanics and special theory of relativity, we get a remarkable set of results. However before we describe them, we should first address the basic question: if all elementary particles are really states of a string, then why do we not see this stringy structure inside various elementary particles like electrons, quarks, etc.? The answer to this is that the

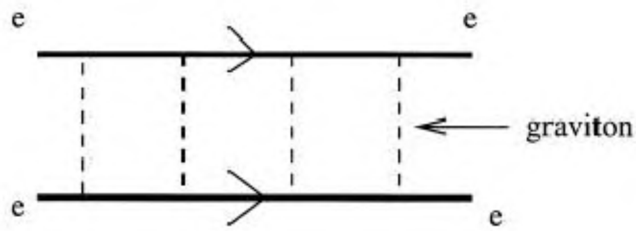

Figure 1. Graviton exchange contribution to the scattering of two electrons.

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typical size of a string is much smaller than the resolution of the most powerful microscopes available today. The strongest such microscope – the tevatron accelerator at Fermilab – can probe distances of the order of 10^{-17} cm, whereas the most standard scenarios give a string size of the order of 10^{-33} cm. Due to this reason, even if the elementary particles are states of a string, for all practical purposes, they appear to be point-like. (More recently, modifications of the standard scenario have been suggested that give a much larger string size, but it is still smaller than 10^{-17} cm.)

To begin with, one could think of formulating various kinds of string theories depending on what kind of vibrational degrees of freedom the string possesses. However, once we require that the quantum string theory is consistent with the principles of special theory of relativity, we have severe constraints on the kind of string theories that we can have. One finds that

- String theory is consistent only in (9 + 1)-dimensional space–time.
- In (9 + 1) dimensions, there are precisely five consistent string theories which differ from each other in the kind of internal degrees of freedom that the string has. These five theories are called type IIA, type IIB, $SO(32)$ heterotic, $E_8 \times E_8$ heterotic and type I string theories, respectively. Of these, the first four string theories have only closed strings – those without any boundaries – whereas the type I string theory has both closed and open strings. Some vibrational modes of closed and open strings have been shown in Figure 2.

The fact that consistent string theories can only be formulated in (9 + 1)-dimensional space–time rather than the (3 + 1)-dimensional space–time in which we live, seems to be a bad news. Also the existence of five different consistent string theories rather than a unique consistent string theory should be a cause of concern. After all, nature should be described by one single theory. We shall address both these questions later. But let us now proceed ignoring these problems and see what string theory has to offer us.

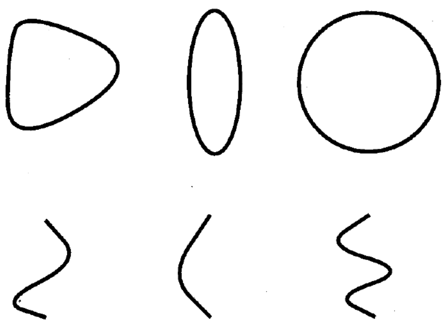

Figure 2. Vibrational modes of closed and open strings.

It turns out that each of these five consistent string theories has two remarkable properties:

- First of all, one finds that quantum string theories do not suffer from any ultraviolet divergence. Thus it is on an even better footing than renormalizable quantum field theories which do suffer from ultraviolet divergences (which can eventually be absorbed into a redefinition of the coupling constants).
- Spectrum of a string theory contains a particle which has all the properties of a graviton. In particular, interaction mediated by this particle gives rise to gravitational interactions identical to that predicted by general theory of relativity at long distances.

Thus string theory gives a finite quantum theory of gravity. Besides gravity, string theory also has other particles/interactions. Indeed a string, being a collection of infinite number of harmonic oscillators, has infinite number of vibrational states, and each of these vibrational states corresponds to an elementary particle. Fortunately, most of these particles are extremely heavy and beyond the range of observation of present-day experiments. Otherwise string theory would be in immediate contradiction with experiments which have only seen a finite number of elementary particles so far.

We now turn to the two problems of string theory listed earlier. It turns out that the first problem, that of having too many dimensions, is resolved via a mechanism called compactification. The second problem, that of having more than one consistent string theory, is partially resolved via a mechanism known as duality.

Compactification

The basic idea of compactification is very old, and was proposed by Kaluza and Klein in the 1920s. The idea is best illustrated by taking a simpler system like a (2 + 1)-dimensional world, rather than the (9 + 1)-dimensional world which is the case of interest. We consider a (2 + 1)-dimensional world in which one of the two special dimensions is curled up into a circle of radius R. Thus the space is the surface of an infinite cylinder, as shown in Figure 3. Now if R is very large, say much larger that the range of the most powerful

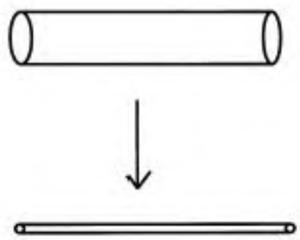

Figure 3. Compactification of a 2-dimensional world.

telescope available to the two-dimensional experimentalists living in this world, then to them the world will appear to be an ordinary two-dimensional Euclidean space with infinite extent in all directions. For smaller values of R (in particular, if R is within the visual range), the world will look very different. But if we now consider the other limit, namely where R is very small, then the world will appear to be one-dimensional. In particular, if R is smaller than the resolution of the most powerful microscopes available to the experimentalists in this world, then these experimentalists with their sophisticated tools will be led to believe that their world is one-dimensional.

This mechanism, that makes an intrinsically two-dimensional world appear one-dimensional, can be easily generalized to make a 9-dimensional world look like a 3-dimensional world. All we need to do is to take 6 of the 9 spatial directions to be small and compact. As long as the sizes of the compact directions are smaller than the reach of the present-day accelerators, the world will appear to be $(3+1)$-dimensional. But unlike in the case of one compact dimension, where the only choice of the compact space is a circle, there are many choices for the compact six-dimensional space. Thus each string theory in $(9+1)$ dimensions gives rise to many different string theories in $(3+1)$ dimensions, after compactification. Some of these theories, where the six-dimensional compact space is of a special kind known as Calabi–Yau manifolds and the string theory that we compactify is the $E_8 \times E_8$ heterotic string theory, come tantalizingly close to the observed universe. In particular, at distance scale large compared to the string scale, and sizes of the compact directions, these theories behave as $(3+1)$-dimensional theories with:

- Gauge interactions (the kind of interactions which are responsible for strong, weak and electromagnetic interactions);
- Gravitational interactions;
- Fermionic particles which are not left–right symmetric (a property of electrons, quarks, neutrinos and other elementary particles).

However we should note that at present:

- We do not know of any compact six-dimensional space such that a string theory on such a space exactly reproduces all the observed properties of elementary particles, including their masses.
- We do not have a theoretical principle which tells us precisely which compact space we should use.

More recently a variation of this compactification scheme has been proposed in which all the observed elementary particles, including the mediators of strong, weak and electromagnetic interactions, live on a subspace of the full 9-dimensional space, whereas the graviton lives in the full space. Such subspaces are known as branes, since they are analogous to membranes in ordinary three-dimensional space. However, unlike membranes which have only two spatial directions, the brane on which the observed particles live must have three large spatial directions and in addition can have other dimensions which are curled up into small manifolds. Such branes appear naturally in string theory. This scenario allows us to have the compact directions transverse to the brane to have relatively large size, since these directions are only seen by gravitational interactions and the level of our current knowledge of how gravity behaves at short distance is much lower than that for the other interactions. For example, we know that the inverse square law for Coulomb interactions holds at least to the atomic scale, since we can explain the atomic spectrum so well using this law. In fact the success of quantum electrodynamics, and more generally the 'standard model', down to a distance scale of 10^{-16}–10^{-17} cm, shows that the laws of strong, weak and electromagnetic interactions given by this model hold at least to this scale. On the other hand, for gravity the inverse square law has been tested only down to a distance of about a millimetre.

Duality in string theory

Usually all present-day attempts to describe the properties of matter and their interactions, including the description of string theory that we have given so far, follow the so-called reductionist hypothesis. According to this philosophy we try to describe the properties of matter in terms of the properties of its 'elementary' constituents, with the idea that the basic interaction between these constituents is described by a simple set of rules, and the more complex behaviour of the composite objects (like the atom or the molecule) can be explained in terms of the simpler interactions between their constituents. This gives a special role to the elementary particles in a given theory. While this philosophy seems to be valid at the level of atoms and molecules, and even the nucleus, proton and neutron, the discovery of duality symmetries has shown us that this may not be the right approach to studying the theory at a much smaller scale, where stringy nature of matter begins to play a role.

In simple terms, a duality conjecture is a statement of equivalence between two or more apparently different string theories. Under a duality map, often an elementary particle in one string theory gets mapped to a composite particle in a dual string theory and vice versa. Thus classification of particles as elementary and composite loses significance, as it depends on which particular theory we use to describe the system. Also, duality often relates a weakly-coupled theory to a strongly-coupled theory. In particular in many examples the coupling constants g and $\tilde{g}$ in a dual pair of theories are related by the relation $g = (\tilde{g})^{-1}$. Thus duality provides a powerful tool for analysing a theory at strong coupling – by mapping the problem to a weakly-coupled dual theory where perturbation theory is applicable.

Some examples of duality are as follows:

- $SO(32)$ heterotic string theory is dual to type I string theory in ten dimensions.
- $SO(32)$ heterotic string theory compactified on a four-dimensional torus T^4 (which is a simple four-dimensional compact manifold in which each of the four directions are periodically identified with some fixed periods) is dual to type IIA string theory compactified on a highly non-trivial four-dimensional compact manifold called $K3$.
- Type IIB string theory is self-dual in the sense that the theory at coupling constant g is dual to the same theory at coupling constant g^{-1}.

The discovery of duality symmetry in string theory gives a unified picture of all string theories. According to this picture, different weakly-coupled (compactified) string theories are all different limits of a single theory, labelled by a large number of parameters. A sketch of the parameter space of this theory is shown in Figure 4. Inside any of the shaded regions, where the theory can be represented by one of the string theories in the weak coupling domain, the parameters correspond to the value of the coupling constant, the parameters labelling the geometry of the compact manifold, the vacuum expectation values of various other background fields, etc. The white region is the non-perturbative domain which cannot be explored by any of the weakly-coupled string theories. Note also that besides the five shaded regions containing the weakly-coupled string theories, there is a sixth corner of the diagram labelled 11-dimensional supergravity. This is a special limit of the parameters of M-theory. One can argue that in this limit the theory has eleven-dimensional Lorentz invariance. Furthermore in this limit the low energy dynamics of massless fields is described by a well-known field theory in eleven dimensions, the $N = 1$ supergravity theory.

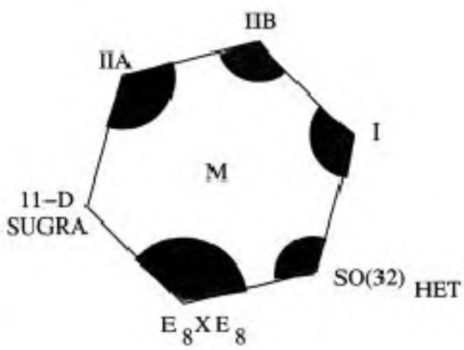

Figure 4. Parameter space of 'M-theory'. Different weakly-coupled string theories occupy the shaded regions of this parameter space.

From this picture it should be clear that we could try to understand M-theory starting from any of the corners, *provided we could understand the theory associated with that corner at finite/large value of coupling constant.* This, of course, is easier said than done. Even in a quantum field theory it is difficult to study the behaviour of the theory at finite/strong coupling. Typically, in order to calculate some physical quantity 'A', one uses perturbation theory to calculate the coefficients of the Taylor series expansion

$$A(g) = A_0 + A_1 g + A_2 g^2 + \dots .$$

The coefficients A_0, A_1, A_2,… are reasonably straightforward to calculate using Feynman diagrams. But this does not give us much information about $A(g)$ for large or finite g. In string theory, the problem is more severe. Unlike in quantum field theory, where $A(g)$ can be defined even for finite g (e.g. using path integral formulation), in string theory we only know how to define and calculate the coefficients A_n. In other words, we only know the rules for calculating an amplitude to any order in the perturbation theory. Thus even in principle we do not know how to define string theories beyond the shaded regions of the M-diagram. Various attempts have been made to define string/M-theory beyond perturbation theory. Among these are string field theory – an analogue of the second quantized version of string theory – and Matrix theory.

Tachyons in string theory

I shall now turn to the study of tachyons in string theory. The theories under study will be weakly-coupled type IIA/IIB string theories. Elementary excitations in these string theories are closed strings. However, these theories also contain extended objects known as Dirichlet p-branes (D-p-branes). A p-brane is a p-dimensional extended object. Thus a 0-brane is a particle, a 1-brane is a string, a 2-brane is a membrane and so on. Such extended objects do often arise in quantum field theories as classical solutions of the field equations. Notable examples in $(3 + 1)$-dimensional field theories are 't Hooft–Polyakov monopoles describing 0-branes, Nielsen–Olesen vortices describing 1-branes, and domain wall solutions describing 2-branes. The D-branes are however described somewhat differently. Instead of saying what a D-brane is, we describe what happens in the presence of a D-brane. This is an indirect way of describing an object. But it turns out that starting with this description we can find out all the properties of these objects.

So what happens in the presence of a D-brane? Let us take a D-p-brane lying along a p-dimensional subspace of the full 9-dimensional space. In the presence of such a p-brane, the theory contains open strings with its two ends forced to move on the brane, besides containing the usual closed-string states. This has been shown in Figure 5. These open strings describe the dynamics of the D-brane. In particular, certain massless open-string states represent the degrees of freedom associated with transverse motion of the D-brane; other massive modes describe internal vibrations of the D-brane.

Some properties of D-brane which we shall need to know for our analysis are described below:

- Type IIA theory contains stable (BPS) D-p-branes for even p. On the other hand, type IIB theory contains stable (BPS) D-p-branes for odd p.
- These D-branes are oriented. A D-brane with opposite orientation will be called an anti-D-brane and will be denoted by a $\overline{D}$ -brane.
- These D-branes have certain amount of energy per unit p-volume, known as the brane tension $\mathcal{T}_p$.$\mathcal{T}_p$ is known for every p.

Our object of study will be a system of parallel and *coincident* D-p – $\overline{D}$ -p system. There are four different kinds of open strings living on this system, as shown in Figure 6. Each of the open string types A, B, C or E can exist in different states of vibration. It turns out that the states of vibration of A or B type strings give rise to states with mass$^2 \geq 0$. On the other hand, states of vibration of C or E type string each contain a state of negative mass^2. Such states are known as tachyons!

Normally we think of a tachyon as a particle travelling faster than light, but in quantum field theory we do have a different interpretation. Here, to every particle we associate a *field* ϕ The mass^2 of the particle is the second derivative of the potential $V(\phi)$ at the origin $\phi = 0$. Thus negative mass^2 simply implies that the potential $V(\phi)$ has a maximum at the origin. This has been shown in Figure 7. In other words, the system is unstable.

Since on a D–$\overline{D}$ system there are two tachyonic states, we can associate with each of them a real scalar field. It is however often more convenient to combine the two real scalars into a complex scalar field T. One can show that in this description the tachyon potential $V(T)$ has a phase symmetry, so that $V(T)$ is really a function of $|T|$. The question that we shall be interested

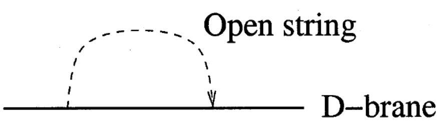

Figure 5. Open strings in the presence of a D-brane.

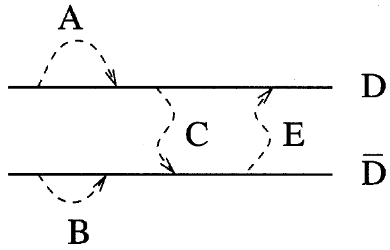

Figure 6. Open strings living on a system of parallel and coincident D-branes. Although for clarity we have shown the D-brane and $\overline{D}$ -brane as separated in space, we shall study the case when they are on top of each other.

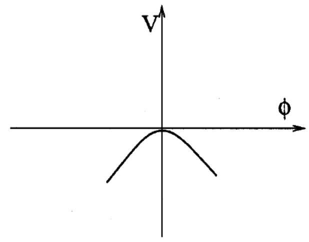

Figure 7. Behaviour of the potential of a tachyon field near the origin.

in is: Does the tachyon potential $V(T)$ have a minimum? The answer to this question is summarized in the following set of conjectures:

1. The tachyon potential does have a family of minima at $|T| = T_0$, and at the minima

   $$V(T_0) + \mathcal{T}_p + \overline{\mathcal{T}_p} = 0.$$

   Thus the total energy of the system at the bottom of the potential is identically zero.
2. The bottom of the potential ($|T| = T_0$) corresponds to the vacuum without any D-brane. This is consistent with the first conjecture that the total energy density vanishes at the bottom of the potential. Since open strings can only end on a D-brane, this conjecture implies that around this vacuum there are no physical *open-string* excitations in this theory.
3. One can also consider other *classical solutions* involving this tachyon field which describe lower dimensional D-branes. Thus, for example, a vortex-like configuration where the tachyon depends on two spatial coordinates, and has the following behaviour in the polar coordinate system (ρ, Θ):

   $$T = f(\rho)e^{i\theta}, \quad f(\rho) \to T_0 \text{ as } \rho \to \infty, \quad f(0) = 0,$$

   represents a D-brane of two lower dimensions. This gives a conventional description of D-branes, as classical solutions, something that we are familiar with in a (quantum) field theory.

All of these conjectures have been tested using a variety of techniques in string theory. They include indirect approaches like the study of the behaviour of an open string in the presence of a tachyon background and seeing how the behaviour changes as we change the background, and more direct approaches of string field theory, where we describe the theory of open strings on the D-brane as a field theory with infinite number of fields, and numerically evaluate the value of the tachyon potential at the minimum of the potential.

Can these conjectures help in a non-perturbative formulation of type IIA/IIB string theory? For this, let us first consider type IIB string theory. This theory has stable D-p-branes for all odd p. In particular, it has space-filling D-9-branes, branes which fill the whole of space–time. Let us start with a certain number (N) of $D9-\overline{D}9$ pair in this theory. The dynamics of this system is described by that of open strings living on the system. Unlike in the case of closed strings where we know how to define the theory only by giving the rules for perturbation expansion, for open strings there is an action principle where we start from an action and derive the Feynman rules using path integral quantization of this action. This description is known as open-string field theory, and can, in principle, be used to study non-perturbative aspects of open-string theory.

Now, according to the conjectures described earlier, this open-string field theory has a classical solution describing the conventional type IIB string theory without any background $D9$-brane, and this describes the standard type IIB string theory that we want to study. Around this vacuum the theory has no open string states. In the language of standard quantum field theory, we would say that there are no perturbative elementary particles in this theory. Yet this theory has soliton solutions which describe the known D-branes in the theory. In particular, one can show that if we start with a sufficiently large number of brane–antibrane pairs, then all the lower dimensional D-brane configurations can be regarded as classical solutions in the corresponding open-string field theory. Thus it is natural to propose that this open-string field theory be taken as the definition of full type IIB string theory. The advantage of doing this, as we have already mentioned, is that for open-string theories, there is an action principle that gives a non-perturbative formulation of string theory.

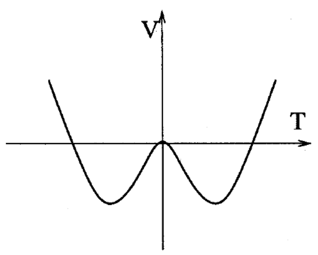

Figure 8. Conjectured form of the tachyon potential on a $D-\overline{D}$ system.

In the language of conventional quantum field theory, this is a field theory with infinite number of fields. The additional advantage is that the D-branes are regular soliton solutions in this field theory and can be treated by the standard techniques which one uses in dealing with regular solitons in a quantum field theory.

Of course, for this proposal to work we must make sure that the theory contains not only D-branes, but other known objects in type IIB string theory, e.g. closed strings. There are indirect hints that this is indeed the case. In particular, while studying the interaction between a pair of D-branes in this theory, one automatically sees the effect of closed-string exchange interaction between the D-branes without having to introduce the closed strings explicitly. This is a strong indication that closed-strings are already included in this theory. However, a more direct verification of this is still awaited.

A similar formulation can be given for type IIA string theory. This theory does not have a stable space-filling $D9$-brane, but turns out to have unstable $D9$-branes. One can repeat constructions similar to the type IIB theory by starting with the background of multiple space-filling (unstable) $D9$-branes. The vacuum without D-branes, as well as all lower dimensional D-branes, then arise as classical solutions in this string field theory.

To summarize, we have a peculiar quantum field theory that has no conventional elementary particles. Yet, we are using this to formulate the full type IIB (or type IIA) string theory which has many physical states. In this description all physical states are composite objects of some kind. If this procedure does give a full non-perturbative definition of type IIA or IIB string theories, then, according to Figure 4, this will also provide a description of M-theory in (at least part of) the white region. This will be a really novel way of describing our universe – building the whole theory out of ‘nothing’.

Even if the grand proposal of obtaining a non-perturbative formulation of string theory using open-string field theory does not fully succeed, the study of open-string tachyons on D-branes has already given us valuable insight into some aspects of string theory. Tachyons have been present in string theory since its early days. The original version of string theory – the bosonic string theory – has both open and closed string tachyons. It has long been suspected that the presence of these tachyonic modes does not represent an inherent sickness of the theory, but is simply a reflection of the fact that we are working in the wrong vacuum, i.e. near the maximum of the potential, and once we find the correct vacuum and quantize the theory around this new vacuum, the problem will disappear. From the study of tachyon dynamics on the D-brane, we now know that at least for open string tachyons, this is indeed the case; with the correct vacuum representing the disappearance of the *D*-brane system on which the original open strings lived. Many people have suggested that a similar mechanism may work also for the closed string tachyon, although what exactly would represent the tachyon vacuum is not entirely clear. (There has been some suggestion that it might involve two-dimensional non-critical string theory, as this theory is free from closed-string tachyons.) If we succeed in understanding the fate of the tachyon vacuum in closed bosonic string theory, then eventually it may also be possible to realize this theory as another corner of the *M*-theory parameter space. This will be a major step towards the grand unification of all string theories.

Summary

To summarize, string theory is a promising candidate theory that has the potential to explain the origin of all matter and their interactions by starting from a simple set of postulates. Although there has been spectacular progress in this subject since its birth more than thirty years ago, we are still quite far from achieving the final goal of using string theory to describe nature in a completely quantitative manner. One of the stumbling blocks to our progress is the lack of a complete formulation of the theory. What we have at present is a set of rules for doing various computations, without a complete understanding of where these rules come from.

There have been several attempts to overcome this difficulty. In this article I described briefly one such attempt. In the past the subject has developed rapidly, and in unexpected directions. Only time can tell if the approach described here will eventually allow us to learn aspects of string theory which are not accessible using the current techniques.

1. For textbooks on string theory, see, Green, M. B., Schwarz, J. H. and Witten, E., *Superstring Theory. Vol. 1: Introduction*, *Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies and Phenomenology*, Cambridge, UK, Univ. Press, 1987, Cambridge Monographs on Mathematical Physics; Polchinski, J., *String Theory. Vol. 1: An Introduction to the Bosonic String*, *String Theory. Vol. 2: Superstring Theory and Beyond*, Cambridge, Univ. Press, 1998.
2. For a review of tachyon condensation in string theory, see http://online.itp.ucsb.edu/online/mp01/sen1/(also/sen2/,/sen3/and/sen4.)
3. For elementary reviews of different aspects of string theory, see the special section on string theory (*Curr. Sci.*, 1999, **77**, no. 12).