

String theory and the uncertainty principle

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String theory has been pursued as the prime possibility towards the fundamental theory of matter, space-time and all natural forces, including gravity. It suggests a way by which the traditional notion of continuum space-time is modified in the short distance regime such that relativity and quantum theory might finally be unified. We discuss the possible significance of this aspect of string theory from a historical perspective, focusing on the uncertainty principle.

Quantum theory and ultraviolet difficulty

IN the history of modern physics, theoretical physicists have often been struggling with ‘infinities’ or ‘divergences’. The problem goes back to the time when the so-called black-body radiation was a topic of great interest, more than hundred years ago. A black body is an ideal body which absorbs radiation, electromagnetic waves, of all possible wavelengths. A hollow cavity behaves as a black body, since it can absorb all possible ranges of radiation falling on it through a small opening in its wall. By the end of the 19th century, it had become known that the properties of black-body radiation in thermal equilibrium are universal in the sense that they depend only on temperature. The questions were, for example, what is the nature of the radiation from a black body when heated, what is the specific heat, and so on. To answer these questions amounts to evaluating the energy density of the radiation in a cavity at a given temperature. According to classical statistical mechanics, the average energy distribution for an arbitrary body in thermal equilibrium is determined essentially by the number of degrees of freedom contributing to thermal energy. The average energy associated with each degree of freedom is equal to $kT/2$, where T is the absolute temperature of the system under consideration and k is a universal constant known as the Boltzmann constant, which depends only on the unit of temperature. This is known as the equipartition law. According to Maxwell’s theory of electromagnetism, electromagnetic radiation can be represented in general as a superposition of harmonic (plane) waves of all possible wavelengths, and the energy is the sum of contributions

from these harmonic waves of definite wavelengths. Now since the wavelengths of electromagnetic waves can become arbitrarily short, it is obvious that the degrees of freedom of the radiation are simply continuously infinite in number. Therefore, on the basis of the equipartition law, the classical theory of radiation predicts that the energy density and hence the specific heat of a black body would necessarily be infinite. If this were really true, a black body would never reach thermal equilibrium, in obvious contradiction with experiments.

As is well known, this difficulty, called the ‘ultraviolet catastrophe’, is one of the most important motivations which led to the development of quantum theory in the early 20th century by many great physicists like Max Planck, Albert Einstein, Niels Bohr and others. The resolution was that electromagnetic waves are actually made of light-quanta, called ‘photons’, particles whose energy, $h\nu$, is proportional to the frequency ν and which propagate with the velocity of light. The proportionality constant h , called the Planck’s constant, plays a fundamental role in modern quantum physics. The formula $h\nu$ shows that, as the wavelength becomes shorter, the energy of the corresponding light-quanta increases since the frequency is inversely proportional to the wavelength. As the energy of the photons exceeds the definite energy of order kT per degree of freedom set by the classical equipartition theorem, they cease to contribute to the thermal equilibrium because the corresponding degrees of freedom cannot be ‘excited’. Thus the components of the radiation with too short wavelengths can be ignored, and hence the energy density is now finite.

Uncertainty principle and quantum infinity

Physicists’ struggle with infinities did not end by the establishment of quantum theory. As soon as quantum mechanics was applied to the field theories of interacting electromagnetic waves and matter particles, physicists encountered the problem of divergences whose origin is purely quantum mechanical. This time again the basic reason for the divergences was the existence of arbitrarily short wavelengths. To understand this, let us recall the nature of quantum theory.

One of the distinguishing features of quantum theory compared with classical physics is the inherent exist-

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tence of *quantum fluctuations*. In classical physics, a physical state can in principle be determined exactly such that, with sufficient knowledge of the state at a given time, one can exactly predict the precise values of various physical quantities at any other time by solving the equations of motion. In contrast to this, in quantum theory, we can predict only the probabilities of possible values of physical quantities, even when we know the state at a given time as precisely as possible in the theory. This situation is characterized by the famous uncertainty principle of Heisenberg. For example, suppose we wish to measure the value of the position of a particle with an error of order Δx along some direction. Also, let the error of the value of momentum of the same particle along the same direction in the same state be Δp . Quantum mechanics tells us that there is a strict restriction for the smallness of these errors: The restriction can be expressed as $\Delta x \Delta p \geq h$; namely if we wish to determine the position of a particle with the uncertainty of order Δx , the uncertainty Δp with respect to the momentum can never become smaller than $h/\Delta x$. That is to say, we can never make both uncertainties small *simultaneously*, beyond this restriction.

A similar restriction holds also for the measurements of energy and time:

$$\Delta T \Delta E \geq h.$$

For example, suppose we wish to measure the time of arrival of a particle with an error ΔT at some particular position. According to this uncertainty principle, the uncertainty ΔE of the measured energy of the arriving particle cannot be smaller than $h/\Delta T$. Actually, the interpretation of the time–energy uncertainty relation has been somewhat controversial compared to the case for position and momentum, and we must be careful in applying it and in each application we must properly define the uncertainties in energy and time. Nevertheless, this uncertainty principle is more important in characterizing the dynamical development of arbitrary quantum mechanical systems. In contrast, the position–momentum uncertainty principle characterizes physical states only at a given time.

These uncertainty relations are a direct consequence of the most essential feature of quantum theory. Both of these relations have their origin in the fact that the two entirely different classical concepts, particle and wave, are united as mutually complementary aspects of the physical degrees of freedom in the microscopic domain of nature. The resolution of the apparent conflict in this unification of particle and wave was precisely due to these complementary properties of quantum uncertainties.

The uncertainty principle signifies the existence of quantum fluctuations in any physical state and in arbitrary physical processes, which always exist in such a

way that we can never have complete control on them by any means. This is true even in the absolute vacuum at absolute zero temperature, $T=0$. In particular, if we consider any physical system at very small scales of space–time interval, the uncertainty principle tells us that the fluctuations of energy and momentum can become arbitrarily large as we go to shorter and shorter space–time intervals. Of course, our experimental apparatus always have limitations in their precision of measurements and hence cannot probe space–time structure at arbitrarily short intervals. Does this mean that the quantum fluctuations associated with arbitrarily short intervals which exceed the order of precision of our measuring apparatus can be ignored? According to the presently established standard gauge field theories of matter, the answer to this question is partially yes and partially no. Yes, if we are interested only in the relationship among the results of various measurements, *provided* we neglect the existence of gravity. But no, if we take gravity properly into account.

After the advent of quantum mechanics, physicists tried to construct a fully consistent theory of interacting matter and electromagnetic radiation. This is called quantum electrodynamics (QED). If we apply a standard computational method, perturbation theory, to QED, beyond its classical approximation, we encounter various divergences. This problem of divergences was the main trouble of QED until the end of the 1940s. The quantum fluctuations of radiation associated with arbitrarily short wavelengths indeed caused various infinities in computations using perturbation theory. Theorists like Tomonaga, Schwinger, Feynman and others, however, realized that these infinities could be eliminated in the final results if all the formulae are expressed as relations among physically observable quantities *and* if we supply a few undetermined constants phenomenologically, by fitting predictions to experimental data. Thus, our ignorance of the quantum fluctuations in the very short distance scales is completely absorbed in these constants. It turns out that, reinterpreted in this way, QED can explain experimental results to an amazing degree of accuracy. This method of dealing with the infinities is called ‘renormalization’. The method of renormalization has been extended to other fundamental interactions as well, and we now have a rather firm grasp of physics up to the distance scales of order $10^{-16} \sim 10^{-17}$ cm. The theory established in this way until the mid 1980s, both experimentally and theoretically, is now known as the Standard Model.

This happy status, however, is ruined once we take into account gravity quantum-mechanically. The reason is that gravity directly couples to energy and momentum. This universal nature of gravitation was the great discovery of Newton, forming the basis for classical physics, and leading Einstein to the formulation of his general theory of relativity. All other interactions,

which can be treated successfully using the method of renormalization, do *not* couple to energy and momentum directly. For example, the electromagnetic force directly couples only to particles with nonzero electric charges, and the nuclear force only to particles with particular charges, called ‘colours’ which are analogous to electric charges. The main reason why the method of renormalization works for a force which does not directly couple to energy–momentum is that the strength of such a force is affected only mildly as we go into the very short distance regime. This is because the large energy–momentum associated with the short-distance quantum fluctuations does not directly influence the strength of such a force.

However, the renormalization method loses its power for gravity since the quantum fluctuations of energy and momentum, which increase without limit at arbitrarily short-distance scales, directly affect the strength of gravitational interaction. For this reason, it turns out that when we apply the renormalization method to Einstein’s theory of general relativity, we are forced to introduce an infinite number of undetermined constants, which must be fitted to experimental data. Any theory with an infinite number of undetermined constants would be a nonsensical theory. But, general relativity as classical theory has been firmly confirmed by many nontrivial experiments and various astrophysical observations. Furthermore, for theoretical physicists, the general relativity theory is so beautifully and tightly constructed that it seems no exaggeration to say that general relativity is one of a few most important intellectual triumphs of humankind. Classically, only two unknown constants need to be supplied from experimental data for making predictions from the general relativity theory. These are Newton’s constant of gravitation and the so-called cosmological constant. Apart from this, one must of course also supply the many parameters which characterize the matter existing in the universe, since these are necessary to apply general relativity concretely to the actual phenomena.

If general relativity is quantized, however, renormalization requires an infinite number of undetermined constants in order to make predictions from quantum gravity itself. From the mid seventies to the mid eighties, various attempts towards the extension of general relativity by enlarging its symmetry using the so-called ‘super’ symmetry had been made. But it turned out that such extensions, known as supergravity, are not sufficient to resolve the divergence difficulty.

Brief history towards string theory

Now what does this failure of general relativity in quantum domain mean? Basically, there have been two contrasting standpoints. One is to suppose that it is only the

failure of the usual method of renormalization based on perturbation theory and that it does not mean the failure of the general relativity theory itself. Another is to suppose that general relativity should be modified in the short-distance regime, irrespective of the validity of perturbation theory, where the quantum fluctuations of energy and momentum become large. After various attempts towards the possible resolution of the problem, most of the theorists in this field now believe that the second standpoint should be the right direction, although the first point is also recognized to be important anyway in deepening our understanding of quantum theory in general beyond perturbation theory. After all, the perturbation theory is just a systematic way of solving any theory approximately using expansion with respect to a small parameter. So logically, both attitudes are possible. Unfortunately, we cannot conceive of experiments which might directly decide which standpoint is correct. This is because the energy scale of the order of 10^{28} eV called Planck energy, at which the quantum gravitational effects can be manifested, is vastly greater than what is achievable by the present techniques (at best, of the order 10^{12} or 10^{13} eV using the largest accelerator at present). Moreover, we do not actually have any reliably established predictions from either of the two approaches. However, the significance of the conflict between general relativity and quantum theory is so profound that these difficulties cannot prevent us from concentrating our endeavour towards its resolution.

String theory, as understood now, can be regarded as a sort of the final outcome of many essential ideas springing from various attempts towards the fundamental theory of interactions, although the way string theory was first discovered in the end of the sixties was something peculiar, when viewed from the present standpoint. Though string theory has not yet given the final answer to the above fundamental question, it is at least true that, through exploring string theory, we are uncovering a multitude of facets of the theory which are useful for deepening our understanding of gauge theory and general relativity in quite an unexpected way, putting aside the progress of our understanding of strings themselves. The basic reason why it has been made possible is that both of gauge field theory and general relativity are united inextricably in one and the same framework of string theory. Before string theory, attempts towards unifying the framework of general relativity and gauge theory have never been really successful. Before achieving the ultimate unified theory finally, string theory has already provided a remarkable arena where various physical ideas and mathematical structures that have been regarded as being entirely unrelated, are unified.

At this juncture, it seems appropriate to outline the history of string theory, since actually we do not yet have the fundamental definition of string theory, and

therefore it is useful to recall how we arrived at the theory for evaluating its significance appropriately at the present stage of development. Briefly, the origin of string theory was the discovery by Veneziano¹ and others of simple formulas as a model for describing the scattering of hadrons. Hadrons are particles interacting strongly among them via the ‘strong force’ which is responsible for nuclear binding. At that time, the issue was the possibility of certain dual descriptions of hadron-scattering. One is the ‘exchange’ picture (often called the Regge picture): Particles can interact with others via exchange of particles. The other is the ‘resonance’ picture: Particles can be united during short time interval as a single particle (called a ‘resonance’) and then are scattered again into a system of several particles. Phenomenologically, both descriptions appeared equivalent. This was known as the s -channel– t -channel duality. So the construction of theory satisfying this duality property was a challenge for theorists. The discovered formula realized the s – t duality in a very strict and idealized form. Also, the formula revealed a rather novel mathematical structure which was soon interpreted by the physical picture based on the relativistic dynamics of strings by Nambu, Nielsen, Susskind² and others.

In parallel to this initial development of string theory, the gauge field theory of strong interactions had been vigorously studied by the mid seventies, and it soon turned out that the basic theory for hadrons should be the gauge field theory of colour charges, called quantum ‘chromo’dynamics (QCD), which could be treated appropriately by the method of renormalization based on perturbation theory. Because of the success of QCD of strong interaction, string theory as a model for hadrons had been abandoned by most particle physicists, before entering the eighties.

Actually, there were other reasons why string theory had been discarded during this period. The most important among them is that the theory is too stringently self-contained. For example, for string theory to be unitary and relativistically invariant, the dimensionality of space–time must be restricted to particular ‘critical dimensions’, 26 or 10, depending on the choice of particle spectra. The ones with the critical dimension 10 are more preferable than those with 26, because they contain fermions and exhibit supersymmetry, which is the origin of the popular word ‘superstring’. Supersymmetry ensures that the vacua of the theories are stable. The vacuum of the theory with critical dimension 26, known as the bosonic string theory, is actually unstable, at least in perturbation theory. In addition to this, it was established by the present author and by Scherk and Schwarz³ that string theories automatically contain gravity. These properties are clearly unwanted as in the theory of strongly interacting hadrons for which gravity can be ignored and the space–time dimensions must be four.

However, if string theory is reconsidered from the viewpoint of unified quantum theory of all interactions, it turns out that the theory is astonishingly rich and has many features which are desirable in an ultimate unified theory. In particular, the fact that gravity is an essential part of the theory is most remarkable from this point of view. In fact, after the establishment of the existence of gravity in string theory as mentioned above, Scherk and Schwarz boldly expressed a suggestion that string theory should be regarded as a fundamental theory rather than as a model for strong interaction. In the mid seventies, such a proposal was received as a quite premature suggestion. But as the failure of various attempts towards building a consistent quantum theory of gravity within the framework of the ordinary local field theories had gradually become a common understanding, the idea that string theory might be the fundamental theory began to be taken more seriously. In this short exposition, I cannot explain all the promising features of string theory as the fundamental unified theory. Some of them must be discussed in other articles. I only mention that the extreme self-containedness of the theory should not be regarded as a defect. On the contrary, it is now reinterpreted as the most important signature for ultimate unification.

For example, string theory includes not only gravity but also gauge-like forces, which, in the low-energy limit where the length of the string can be ignored, are approximately described by appropriate gauge-field theories of ordinary type, like Maxwell’s electromagnetism. The gravitational interaction contained in string theory is also described in the low-energy limit by the supergravity field theories which, as mentioned earlier, were actually constructed in the attempts towards a generalization of general relativity by extending the symmetry of the latter. The mathematical structure of the theory shows that all the parameters of the theory, apart from the fundamental unit of length, even including the space–time geometry itself, can in principle be determined by the dynamics of the theory itself. The appearance of the critical space–time dimensions can be regarded as a special case of this general feature of the theory. Unfortunately, however, we do not actually think that the meaning and the content of string theory are fully grasped at the present stage of development. Although string theory indeed resolves the problem of the divergence associated with the earlier attempts at quantizing gravity, it is achieved only within the limitation of perturbation theory. The reader might be somewhat surprised to hear such a statement in view of the 30-year history of string theory⁴. But, in my opinion, this itself shows how deep string theory could be, and how difficult it is to find the really appropriate mathematical language to formulate the principles behind String theory. It seems quite probable that we need a new mathematical framework in order to satisfactorily

express the whole content of string theory and the principles behind it, without using perturbation theory.

Space–time uncertainty relation

Now that we have briefly explained the history, let us go back to our main subject, the relation between the quantum uncertainties and divergences.

In order to see how the fundamental difficulty of the divergence associated with quantum gravity is resolved in string theory, we first have to understand the basic nature of the string dynamics. A string is simply a one-dimensional extended object. However, it is not like just an ordinary string, say a violin string. The energy density along a string in the fundamental string theory is assumed to be a universal *constant* which is usually denoted as $1/2\pi\alpha'c$, where c is the velocity of light and α' is the new fundamental constant characterizing string theory. Thus, even when the string stretches or shrinks, the energy per unit length does not change. In other words, the total mass of a string is essentially determined by its total length. This means that the length of the string in the states with lowest energy is zero, at least classically, and so the masses of these states vanish.

If we treat the string quantum-mechanically, we have to take into account quantum fluctuations and therefore we cannot say that the length of the lowest energy states of the string is strictly zero in the classical sense. However, the fact that their masses are vanishing is still valid. It also turns out that the states have in general nonzero ‘spin’, namely they are rotating. The massless states of closed strings, strings which close upon themselves, necessarily contain a spin-2 state, using some appropriate unit for measuring the strength of rotation (called angular momentum). The massless states of open strings, strings with open ends, similarly contain a spin-1 state. The spin-2 massless closed string state behaves as graviton, which is responsible for the universal gravitational force. In the low-energy limit, it exactly coincides with the graviton one expects from the quantization of general relativity. The spin-1 open string state coincides with the gauge particles, like photon corresponding to the electromagnetic interaction. Basically, this is why string theory in general contains gravity and/or gauge forces. In particular, even if we start only from open strings, the consistency of the theory requires that closed strings must always coexist with open strings and interact with open and closed strings in a manner which is dictated, at least in the low-energy limit, by general relativity or by its extension, supergravity.

The secret of how string theory manages to avoid divergences lies in the very peculiar way in which strings interact. If we imagine the ordinary violin

strings, they can collide at an arbitrary point and recoil from each other. Then it would be meaningful to talk about the distances between two violin strings measured along directions which are transverse to them, since the distances and momenta measured along the directions perpendicular to the strings are important to characterize the collision. But this is not the case in string theory. Scattering of strings in string theory is quite different. Roughly speaking, there are only three types of interactions. Splitting or joining at the end points of open strings and the rejoining of two crossed strings at arbitrary points along both open and closed strings. Using a somewhat more abstract language, this can be rephrased in the following way: Imagine the trajectories of these strings by tracing them in space *and* time. The trajectory of a one-dimensional line is a two-dimensional surface, which is called a ‘world sheet’. The above property of the string interactions amounts to the statement that sufficiently small portion of each world sheet at an arbitrary point on it is always equivalent dynamically to the segment of a one-sheeted plane.

The uniformity of the world sheet in this sense, which is mathematically formulated by a characteristic symmetry known as conformal invariance, is intimately connected to the universal nature of the energy density of the string, as we emphasized in the beginning of this section. Because of this crucial property, we need not take into account the possible intersections and collisions of the world sheets. Then it is not important to talk about the distances along the transversal directions among strings. Instead, the meaningful way of measuring distances is now along the strings themselves. This is not saying that the transverse directions can totally be neglected. The momentum along the transversal directions can of course affect the behaviour of string scattering. But the most decisive directions of distances and hence the velocity or momentum in the string dynamics, from the viewpoint of probing the short distance space–time structure, are those along the strings themselves. We may call the spatial directions along the strings and hence space–time distances along the world sheet as ‘longitudinal’ directions, as distinct from the transverse directions. If the longitudinal distance in a particular direction in space–time becomes large, the string cannot probe short distances along such a direction. Although it is not easy to explain this without using appropriate mathematical language, this property is actually at the heart of the s – t duality which was the basic motivation for the original discovery of strings, as explained earlier. Both physical pictures mentioned there are united in these properties of the string world sheet. Roughly speaking, the exchange picture corresponds to the situation when the world sheet stretches very far in spatial directions, while the resonance picture corresponds to the situation when the world sheet stretches far in the temporal directions.

Here we may ask whether there exists any simple and universal characterization of the above ‘stringy’ property with respect to the space–time distance scales. Since the dynamics of the strings obey the usual rules of quantum mechanics, it is natural to try to seek such possibilities using the uncertainty relations by taking into account the special characteristics of string theory. Since the uncertainty relation for the momentum and coordinate is expressed in the form which is appropriate for point particles, it does not seem convenient for our purpose. It is of course true that the center-of-mass coordinate and the corresponding momentum of the string satisfy the usual coordinate–momentum uncertainty relation, but that does not tell us about the crucial role of the extendedness of the string. The one for the energy and time, on the other hand, is valid for arbitrary dynamical processes if the uncertainty ΔT with respect to time is appropriately interpreted. Indeed, since the energy of strings is basically given by just the length of a string measured along the string because of the universal nature of the constant energy density, it is more than natural to identify the precision scale ΔE of energy to the extension ΔX of the string along its longitudinal direction as $\Delta E \sim \Delta X / 2\pi\alpha' c$. Therefore the time–energy uncertainty relation can be reinterpreted as the uncertainty relation of the space–time in the form

$$\Delta T \Delta X \geq \ell_s^2 / c,$$

where we have defined a constant ℓ_s called the string-length parameter by $\ell_s^2 = 2\pi\alpha' \hbar c^2$. It clearly indicates that the strings cannot probe short-distance scales to arbitrary precision with respect to both time-like and space-like distances, simultaneously. It is appropriate to call the relation as the space–time uncertainty relation.

To appreciate the significance of this simple-looking relation, we should remember that, due to the self-containedness of string theory, the space–time structure itself should be determined self-consistently by using the dynamics of strings themselves. The above relation must then be interpreted as the qualitative characterization of space–time itself, if one believes string theory as the fundamental unified theory. We may express this situation by claiming that space–time is ‘quantized’. In addition to this, the proportionality between energy and the longitudinal length indicates that the large quantum fluctuations of energies associated with short time measurement are actually reinterpreted as the fluctuations of long strings and hence turn into a long-distance phenomenon. Thus the structure of quantum fluctuations of string theory is drastically different from that in the ordinary field theory of point particles. This is precisely the physical mechanism which is hidden behind the mathematical proof that the string perturbation the-

ory has no divergences associated to the short distance space–time structure.

We have explained the space–time uncertainty relation as a reinterpretation of the ordinary time–energy relation in terms of the space–time lengths. That was in fact the way how the above relation was originally proposed for the first time by the present author⁵ in 1987. (Related considerations were done independently by other workers⁶ about the same time, on the basis of the high-energy behaviour of string scattering.) Actually it is also possible to directly derive the relation by using the conformal symmetry of the world sheet, without relying upon the time–energy uncertainty relation, and to check its validity for string scattering. Furthermore, by combining with the existence of gravity, the space–time uncertainty relation enables us to derive the most interesting (spatial) length scale of string theory, which is expected to play a key role in the formulation beyond perturbation theory, as first emphasized by Hull, Townsend, Witten⁷ and others. That is known as the ‘*M*-theory’ scale and is given by $\ell_M \sim g_s^{1/3} \ell_s$, where the constant g_s , called the string coupling, is related to the gravitational constant G_{10} in 10 space–time dimensions by $G_{10} \sim g_s^2 \ell_s^8$. The *M*-theory scale is usually derived by postulating the existence of a hidden 11-dimensional theory, the *M*-theory. The space–time uncertainty relation, however, reveals that it is actually intrinsic to superstring theory in 10 dimensions.

The same *M*-theory scale appears also as the characteristic scale in the dynamics of the important point-like excitation called the ‘*D*-particle’. The *D*-particle is a special case of ‘Dirichlet branes’, a new class of physical excitations in string theory. In general, the *D*-branes are objects to which the end points of open strings are attached. The modifier ‘Dirichlet’ comes from the boundary condition in treating such open strings. They can have a variety of different dimensionalities, and, as the name ‘*D*-particle’ suggests, even the point-like objects are possible. In general, the mass of *D*-branes is inversely proportional to the string coupling g_s . Given the fact that point-like physical objects can exist in string theory, one may wonder whether our considerations which led to the space–time uncertainty relation are valid for them. Actually, since the dynamics of *D*-branes is completely governed by the open strings attached to them, the properties as signified by the space–time uncertainty relation are equally valid for *D*-branes⁸. The only difference is that the spatial scale ΔX must now be interpreted as the one measured along directions transverse to the *D*-brane volume, since it is these directions that are the longitudinal directions for the open strings attached to them. It is generally expected that *D*-branes would play a fundamental role in future nonperturbative reformulation of string theory or *M*-theory. The reason is that the whole structure of string theory is expected to be invariant under the inter-

change $g_s \leftrightarrow 1/g_s$ of the string coupling by its inverse. The possibility of this ‘S-duality’ symmetry has been pioneered by Sen⁹ and others in the mid nineties. The validity of the perturbative string theory, which is the only formulation known to us, is restricted to the limit of small string coupling $g_s \rightarrow 0$, where the D -branes are very massive and hence can usually be ignored in studying string scattering. If the S-duality symmetry is true, however, we should be able to express the whole content of the theory using the D -branes, since by the interchange $g_s \rightarrow 1/g_s$, the masses of D -branes become vanishingly small in the limit of strong string coupling, and hence at strong coupling these objects must play a dominant role in arbitrary physical processes. We can show that the space–time uncertainty relation is compatible with the S-duality symmetry. Therefore it is not unreasonable to hope that the space–time uncertainty relation could be one of the key features which the future nonperturbative formulations should exhibit.

Conclusion

To summarize, the quantum mechanical uncertainty relations are useful for understanding some crucial qualitative properties of short-distance divergences in quantum field theory, especially the one associated with quantum gravity. String theory automatically unifies gravity with other gauge forces and resolves the divergence difficulty by exhibiting a promising new structure which could not be envisaged if one remains within the framework of the ordinary local field theories. The basic mechanism why the divergences are resolved can be formulated by reinterpreting the time–energy uncertainty relation in terms of the space–time distance scales. This leads to the proposal of a new uncertainty relation in space–time.

Finally, let me mention some more recent developments related to the issue discussed in this article. The space–time uncertainty relation suggests that the classical space–time itself may be something analogous to the phase space of classical mechanics. A physical state in classical mechanics is fixed by determining the (generalized) coordinates and momenta of particles at a given time. Thus the space of all possible states is the continuum $3 \times 2N$ -dimensional Euclidean space for an N -particle system in the ordinary 4-dimensional space–time. This $6N$ -dimensional space is called phase space. After quantization, the coordinate and the corresponding momentum cannot be specified simultaneously because of the uncertainty relation and therefore the notion of phase space as the total space of physical states must be modified for nonzero Planck constant. In quantum mechanics, the space of states is reformulated

as the Hilbert space of wave functions, which is characterized by the principle of superposition of quantum-mechanical states. We should perhaps expect similar modification of the notion of the space–time, in view of the validity of our space–time uncertainty relation. Presumably, string theory may be understood as some sort of ‘quantum geometry’ along this line of thought. At present, it is very difficult to formulate this kind of ideas concretely. In fact, there have been several attempts in the past towards generalization of local field theories based on similar ideas, before string theory. For example, we may assume that the space–time coordinates are operators instead of ordinary numbers, mimicking quantum mechanics where the coordinates and momenta can be treated as operators acting in the Hilbert space. Recently, this idea is revived in the context of string theory. String theory exhibits similar behaviour as a ‘noncommutative’ field theory in a certain limit, with some assumptions on certain background fields allowed in string theory. Unfortunately, these recent discussions have not yet provided notable new insight on the space–time uncertainties which are characterized by the string-length parameter ℓ_s . This is because such limits or assumptions usually adopted in recent investigations tend to neglect the crucial extendness of the strings along the longitudinal directions. Hopefully, however, investigations of toy models of this sort might be useful for seeking the right direction towards our final goal, the development of appropriate mathematical framework and the construction of the really ultimate theory of everything on its basis.

1. Veneziano, G., *Nuovo Cimento*, 1968, **A57**, 190–197.
2. Nambu, Y., Proc. Int. Conf. on Symmetries and Quark Models, Wayne State Univ., 1969; Nielsen, H. B., Fifteenth Conf. on High Energy Physics, Kiev, 1970; Susskind, L. *Nuovo Cimento*, 1970, **69**, 457–496.
3. Yoneya, T., *Lett. Nuovo. Cimento*, 1973, **8**, 951–955; *Prog. Theor. Phys.*, 1974, **51**, 1907–1920; Scherk, J. and Schwarz, J., *Nucl. Phys.*, 1974, **B81**, 118–144; *Phys. Lett.*, 1975, **B57**, 463–466.
4. For a detailed account of string theory including an extensive list of references till 1987, see Green, M., Schwarz, J. and Witten, E., *Superstring Theory*, Cambridge Univ. Press, 1987, vols 1 and 2; For a more recent account, see Polchinski, J., *String Theory*, Cambridge Univ. Press, 1998, vols 1 and 2.
5. Yoneya, T., *Wandering in the Fields* (eds Kawarabayashi, K. and Ukawa, A.), World Scientific, Singapore, 1987, pp. 419–428; Yoneya, T., *Mod. Phys. Lett.*, 1989, **A4**, 1587–1595.
6. Gross, D. and Mende, P., *Nucl. Phys.*, 1988, **B303**, 407–454; Amati, D., Ciafaloni, M. and Veneziano, G., *Phys. Lett.*, 1987, **B197**, 81–88.
7. Hull, C. and Townsend, P., *Nucl. Phys.*, 1955, **B438**, 109–137; Witten, E., *Nucl. Phys.*, 1995, **B443**, 85–126.
8. Li, M. and Yoneya, T., *Phys. Rev. Lett.*, 1997, **78**, 1219–1222; For a recent comprehensive review, see Yoneya, T., *Prog. Theor. Phys.*, 2000, **103**, 1081–1125.
9. Sen, A., *Int. J. Mod. Phys.*, 1994, **A9**, 3707–3750.