String theory

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This article presents the basic concepts of string theory followed by an overview of the peculiar history of how it arose.

MANY of the major developments in fundamental physics of the past century arose from identifying and overcoming contradictions between existing ideas. For example, the incompatibility of Maxwell's equations and Galilean invariance led Einstein to propose the special theory of relativity. Similarly, the inconsistency of special relativity with Newtonian gravity led him to develop the general theory of relativity. More recently, the reconciliation of special relativity with quantum mechanics led to the development of quantum field theory. We are now facing another crisis of the same character, namely general relativity appears to be incompatible with quantum field theory. Any straightforward attempt to 'quantize' general relativity leads to a nonrenormalizable theory. In my opinion, this means that the theory is inconsistent and needs to be modified at short distances or high energies. The way that string theory does this is to give up one of the basic assumptions of quantum field theory, the assumption that elementary particles are mathematical points, and instead to develop a quantum field theory of one-dimensional extended objects, called strings. There are very few consistent theories of this type, but superstring theory shows great promise as a unified quantum theory of all fundamental forces, including gravity. There is no realistic string theory of elementary particles that could serve as a new standard model, since there is much that is not yet understood. But that, together with a deeper understanding of cosmology, is the goal. This is still a work in progress.

Even though string theory[1,2] is not yet fully formulated, and we cannot yet give a detailed description of how the standard model of elementary particles should emerge at low energies, there are some general features of the theory that can be identified. These are features that seem to be quite generic irrespective of how various details are resolved. The first, and perhaps most important, is that general relativity is necessarily incorporated in the theory. It gets modified at very short distances/high energies but at ordinary distances and energies, it is present in exactly the form proposed by Einstein. This is significant, because it is arising within the framework of a consistent quantum theory. Ordinary quantum field theory does not allow gravity to exist; string theory requires it! The second general fact is that Yang–Mills gauge theories of the sort that comprise the standard model, naturally arise in string theory. We do not understand why the specific $SU(3) \times SU(2) \times U(1)$ gauge theory of the standard model should be preferred, but (anomaly-free) theories of this general type do arise naturally at ordinary energies. The third general feature of string theory solutions is supersymmetry. The mathematical consistency of string theory depends crucially on supersymmetry, and it is very hard to find consistent solutions (quantum vacua) that do not preserve at least a portion of this supersymmetry. This prediction of string theory differs from the other two (general relativity and gauge theories) in that it really is a prediction. It is a generic feature of string theory that has not yet been discovered experimentally.

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Supersymmetry

As we have just said, supersymmetry is the major prediction of string theory that could appear at accessible energies, that has not yet been discovered. A variety of arguments, not specific to string theory, suggest that the characteristic energy scale associated with supersymmetry breaking should be related to the electro-weak scale, in other words in the range 100 GeV–1 TeV. The symmetry implies that all known elementary particles should have partner particles, whose masses are in this general range. This means that some of these superpartners should be observable at the CERN Large Hadron Collider (LHC), which will begin operating in the middle part of this decade. There is even a chance that Fermilab Tevatron experiments could find superparticles earlier than that.

In most versions of phenomenological supersymmetry, there is a multiplicatively conserved quantum number called *R*-parity. All known particles have even *R*-parity, whereas their superpartners have odd *R*-parity. This implies that the superparticles must be pair-produced in particle collisions. It also implies that the lightest supersymmetry particle (or LSP) should be absolutely stable. It is not known with certainty which particle is the LSP, but one popular guess is that it is a 'neutralino'. This is an electrically neutral fermion that is a quantum-mechanical mixture of the partners of the photon, Z^0, and neutral Higgs particles. Such an LSP would interact very weakly, more or less like a neutrino. It is of considerable interest, since it is an excellent dark-matter candidate. Searches for dark-matter particles called WIMPS (weakly interacting massive particles) could discover the LSP some day, though current experiments might not have sufficient detector volume to compensate for the exceedingly small cross-sections.

There are three unrelated arguments that point to the same mass range for superparticles. The one we have just been discussing, a neutralino LSP as an important component of dark matter, requires a mass of order 100 GeV. The precise number depends on the mixture that comprises the LSP, what its density is, and a number of other details. A second argument is based on the famous hierarchy problem. This is the fact that standard model radiative corrections tend to renormalize the Higgs mass to a very high scale. The way to prevent this is to extend the standard model to a supersymmetric standard model and to have the supersymmetry be broken at a scale comparable to the Higgs mass, and hence to the electro-weak scale. The third argument that gives an estimate of the susy-breaking scale is grand unification. If one accepts the notion that the standard model gauge group is embedded in a larger gauge group such as $SU(5)$ or $SO(10)$, which is broken at a high mass scale, then the three standard model coupling constants should unify at that mass scale. Given the spectrum of particles, one can compute the evolution of the couplings as a function of energy using renormalization group equations. One finds that if one only includes the standard model particles, this unification fails quite badly. However, if one also includes all the supersymmetry particles required by the minimal supersymmetric extension of the standard model, then the couplings do unify at an energy of about 2×10^{16} GeV. For this agreement to take place, it is necessary that the masses of the superparticles are less than a few TeV.

There is further support for this picture, such as the ease with which supersymmetric grand unification explains the masses of the top and bottom quarks and electro-weak symmetry breaking. Despite all these indications, we cannot be certain that this picture is correct until it is demonstrated experimentally. One could suppose that all this is a giant coincidence, and the correct description of TeV-scale physics is based on something entirely different. The only way we can decide for sure is by doing the experiments. As I once told a newspaper reporter, in order to be sure to be quoted: discovery of supersymmetry would be more profound than life on Mars.

Basic concepts of string theory

In conventional quantum field theory the elementary particles are mathematical points, whereas in perturbative string theory the fundamental objects are one-dimensional loops (of zero thickness). Strings have a characteristic length scale, which can be estimated by dimensional analysis. Since string theory is a relativistic quantum theory that includes gravity, it must involve the fundamental constants c (the speed of light), $\hbar$ (Planck's constant divided by 2π), and G (Newton's gravitational constant). From these one can form a length, known as the Planck length

$$\ell_p = \left(\frac{\hbar G}{c^3}\right)^{3/2} = 1.6 \times 10^{-33} \text{ cm}. \tag{1}$$

Similarly, the Planck mass is

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} = 1.2 \times 10^{19} \text{ GeV} / c^2. \tag{2}$$

Experiments at energies far below the Planck energy cannot resolve distances as short as the Planck length. Thus, at such energies, strings can be accurately approximated by point particles. From the viewpoint of string theory, this explains why quantum field theory has been so successful.

As a string evolves in time it sweeps out a two-dimensional surface in space–time, which is called the world sheet of the string. This is the string counterpart of the world line for a point particle. In quantum field theory, analysed in perturbation theory, contributions to amplitudes are associated to Feynman diagrams, which depict possible configurations of world lines. In particular, interactions correspond to junctions of world lines. Similarly, pertubative string theory involves string world sheets of various topologies. A particularly significant fact is that these world sheets are generically smooth. The existence of interaction is a consequence of world-sheet topology rather than a local singularity on the world sheet. This difference from point-particle theories has two important implications. First, in string theory the structure of interactions is uniquely determined by the free theory. There are no arbitrary interactions to be chosen. Second, the ultraviolet divergences of point-particle theories can be traced to the fact that interactions are associated to world-line junctions at specific space–time points. Because the string world sheet is smooth, string theory amplitudes have no ultraviolet divergences.

Perturbation theory is useful in a quantum theory that has a small dimensionless coupling constant, such as quantum electrodynamics, since it allows one to compute physical quantities as power series expansions in the small parameter. In QED, the small parameter is the fine-structure constant $\alpha \sim 1/137$. Since this is quite small, perturbation theory works very well for QED. For a physical quantity $T(\alpha)$, one computes (using Feynman diagrams)

$$T(\alpha) = T_0 + \alpha T_1 + \alpha^2 T_2 + \ldots. \tag{3}$$

It is the case generically in quantum field theory that expansions of this type are divergent. More specifically, they are asymptotic expansions with zero radius convergence. Nonetheless, they can be numerically useful if the expansion parameter is small. The problem is that there are various nonperturbative contributions (such as instantons) that have the structure

$$T_{NP} \sim \mathrm{e}^{-(\mathrm{const.}/\alpha)}. \tag{4}$$

In a theory such as QCD, there are regimes where perturbation theory is useful (due to asymptotic freedom) and other regimes where it is not. For problems of the latter type, such as computing the hadron spectrum, nonperturbative methods of computation, such as lattice gauge theory, are required.

In the case of string theory the dimensionless string coupling constant, denoted g_s, is determined dynamically by the expectation value of a scalar field called the dilaton. There is no particular reason that this number should be small. So it is unlikely that a realistic vacuum could be analysed accurately using perturbation theory. More importantly, these theories have many qualitative properties that are inherently nonperturbative. So one needs nonperturbative methods to understand them. Until 1995, it was only understood how to formulate string theories in terms of perturbation expansions.

A brief history of string theory

The dual resonance model

String theory grew out of the *S*-matrix approach to hadronic physics, which was a very hot subject in the 1960s. Some of the relevant concepts were Regge Poles, the bootstrap conjecture, and 'duality' between direct channel and crossed-channel resonances. The bootstrap/duality programme got a real shot in the arm in 1968, when Veneziano found a specific mathematical function that explicitly exhibits the features that people had been discussing in the abstract[3]. Within a matter of months Virasoro found an alternative formula with many of the same duality and Regge properties[4]. Later it would be understood that whereas Veneziano's formula describes scattering of open-string ground states, Virasoro's describes scattering of closed-string ground states.

In 1969, several groups independently discovered *N*-particle generalizations of the Veneziano four-particle amplitude[5]. The *N*-point generalization of Virasoro's four-point amplitude was constructed by Shapiro[6]. In short order, it was shown that the Veneziano *N*-particle amplitudes could be consistently factorized in terms of a spectrum of single-particle states described by an infinite collection of harmonic oscillators[7]. This was a striking development, because it suggested that these formulas could be viewed as more than just an approximate phenomenological description of hadronic scattering. Rather, they could be regarded as the true approximation to a full-fledged quantum theory. I do not think that anyone had anticipated such a possibility one year earlier.

Once it was clear that we were dealing with a system with a rich spectrum of internal excitations, and not just a bunch of phenomenological formulas, it was natural to ask for a physical interpretation. The history of who did what and when is a little tricky to sort out. As best I can tell, the right answer – a one-dimensional extended object (or 'string') – was discovered independently by three people: Nambu, Susskind and Nielsen[8]. The string interpretation of the dual resonance model was not very influential in the development of the subject until the appearance of the 1973 paper by Goddard *et al.*[9]. It explained in detail how the string action could be quantized in light-cone gauge.

The RNS model and world-sheet supersymmetry

The original dual resonance model (bosonic string theory), developed in the period 1968–70, suffered from several unphysical features: the absence of fermions, the presence of a tachyon, and the need for 26-dimensional space–time. These facts motivated the search for a more realistic string theory. The first important success was achieved in January 1971 by Pierre Ramond, who had the inspiration of constructing a string analogue of the Dirac equation[10]. A bosonic string $X^\mu(\sigma,\tau)$ with $0 \le \sigma \le 2\pi$ has a momentum density $P^\mu(\sigma,\tau) = \frac{\partial}{\partial\tau} X^\mu(\sigma,\tau)$, whose zero mode

$$p^\mu = \frac{1}{2\pi}\int_0^{2\pi} P^\mu(\sigma,\tau)\mathrm{d}\sigma$$

is the total momentum of the string. Ramond suggested introducing an analogous density $\Gamma^\mu(\sigma,\tau)$, whose zero mode

$$\gamma^\mu = \frac{1}{2\pi}\int_0^{2\pi} \Gamma^\mu(\sigma,\tau)\mathrm{d}\sigma$$

is the usual Dirac matrix. He then defined Fourier modes of the product $\Gamma{\cdot}P$:

$$F_n = \frac{1}{2\pi}\int_0^{2\pi} e^{-in\sigma}\Gamma\cdot P\,\mathrm{d}\sigma,\ n\in\mathbf{Z}.$$

The zero mode,

$$F_0 = \gamma p + \text{oscillator terms}$$

is an obvious generalization of the Dirac operator, suggesting a wave equation of the form

$$F_0|\psi\rangle = 0$$

for a free fermionic string. By postulating the usual commutation relations for X^μ and P^μ, as well as

$$\{\Gamma^\mu(\sigma,\tau),\Gamma^\nu(\sigma',\tau)\} = 4\pi\eta^{\mu\nu}\delta(\sigma-\sigma'),$$

he discovered the super-Virasoro (or $N = 1$ superconformal) algebra

$$\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}\left(m^2 - \frac{1}{4}\right)\delta_{m+n,0},$$

$$[L_m, F_n] = \left(\frac{m}{2} - n\right)F_{m+n},$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0},$$

extending the well-known Virasoro algebra (given by the L_n's alone).

Neveu and I developed a new bosonic string theory containing a field $H^\mu(\sigma, \tau)$ satisfying the same anticommutation relations as $\Gamma^\mu(\sigma, \tau)$, but with boundary conditions that give rise to half-integral modes. A very similar super-Virasoro algebra arises, but with half-integrally moded operators

$$G_r = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{-ir\sigma} H\cdot P\,\mathrm{d}\sigma,\ r\in\mathbf{Z}+1/2$$

replacing the F_n's (ref. 11). This model contains a tachyon that we identified as a slightly misplaced 'pion'. We thought that our theory came quite close to giving a realistic description of nonstrange mesons, so we called it the 'dual pion model'. This identification arose because only amplitudes with an even number of pions were nonzero. Thus we could identify a G-parity quantum number for which the 'pions' were odd. It was obvious that one could truncate the theory to the even G-parity sector, and then it would be tachyon-free. However, we did not emphasize this fact, because we wanted to keep the pions. Our hope at the time was that a mechanism could be found that would shift the tachyonic pion and the massless rho to their desired masses.

In August 1971, Gervais and Sakita presented a paper proposing an interpretation of the various operators in terms of a two-dimensional world-sheet action principle[12]. Specifically, they took the $X^\mu(\sigma, \tau)$, which transform as scalars in the world-sheet theory, together with free Majorana (2-component) fermions $\psi^\mu(\sigma, \tau)$. The action is

$$S = \frac{1}{2\pi}\int \mathrm{d}\sigma\,\mathrm{d}\tau\{\partial_\alpha X^\mu\partial^\alpha X_\mu - i\bar{\psi}^\mu\rho^\alpha\partial_\alpha\psi_\mu\},$$

where ∂_α are world-sheet derivatives $\left(\frac{\partial}{\partial\tau},\frac{\partial}{\partial\sigma}\right)$ and ρ^α are two-dimensional Dirac matrices. They noted that this has a global fermionic symmetry: The action S is invariant under the supersymmetry transformation

$$\delta X^\mu = \bar{\varepsilon}\psi^\mu$$

$$\delta\psi^\mu = -i\rho^\alpha\varepsilon\delta_\alpha X^\mu,$$

where ε is a constant infinitesimal Majorana spinor. So this demonstrated that the theory has global world-sheet supersymmetry. I think that this was the first consistent supersymmetric action to be identified. However, it did not occur to us at that time to explore whether the corresponding string theory could also have space–time supersymmetry. Perhaps the presence of the tachyonic 'pion' in the spectrum prevented us from considering the possibility. A few years later, this theory was also explored by Zumino[13], a fact which I think was historically important in setting the stage for his subsequent work with Wess[14] on supersymmetric field theory in four dimensions.

Gravity and unification

Among the massless string states, there is one that has spin two. In 1974, it was shown by Scherk and me[15], and independently by Yoneya[16], that this particle interacts like a graviton, so the theory actually includes general relativity. This led us to propose that string theory should be used for unification rather than for hadrons. This implied, in particular, that the string length scale should be comparable to the Planck length, rather than the size of hadrons (10^{-13} cm) as we had previously assumed.

In the context of the original goal of string theory – to explain hadron physics – extra dimensions are unacceptable. However, in a theory that incorporates general relativity, the geometry of space–time is determined dynamically. Thus one could imagine that the theory admits consistent quantum solutions in which the six extra spatial dimensions form a compact space, too small to have been observed. The natural first guess is that the size of this space should be comparable to the string scale and the Planck length.

Space–time supersymmetry

In 1976 Gliozzi *et al.*[17] noted that the RNS spectrum admits a consistent truncation (called the GSO projection), which is necessary for the consistency of the interacting theory. In the NS sector, the GSO projection keeps states with an odd number of b-oscillator excitations, and removes states with an even number of b-oscillator excitations. (This corresponds to projecting onto the even G-parity sector of the dual pion model.) Once this rule is implemented, the spectrum of allowed masses is integral

$$M^2 = 0, 1, 2, \ldots .$$

In particular, the bosonic ground state is now massless, so the spectrum no longer contains a tachyon. The GSO projection also acts on the R sector, where there is an analogous restriction that amounts to imposing a chirality projection on the spinors. The claim is that the complete theory now has space–time supersymmetry.

If there is space–time supersymmetry, then there should be an equal number of bosons and fermions at every mass level. Let us denote the number of bosonic states with $M^2 = n$ by $d_{NS}(n)$ and the number of fermionic states with $M^2 = n$ by $d_R(n)$. Then we can encode these numbers in generating functions

$$f_{NS}(w) = \sum_{n=0}^{\infty} d_{NS}(n) w^n = \frac{1}{2\sqrt{w}} \left(\prod_{m=1}^{\infty} \left(\frac{1+w^{m-1/2}}{1-w^m} \right)^8 - \prod_{m=1}^{\infty} \left(\frac{1-w^{m-1/2}}{1-w^m} \right)^8 \right)$$

and

$$f_R(w) = \sum_{n=0}^{\infty} d_R(n) w^n = 8 \prod_{m=1}^{\infty} \left(\frac{1+w^m}{1-w^m} \right)^8 .$$

The 8's in the exponents refer to the number of transverse directions in ten dimensions. The effect of the GSO projection is the subtraction of the second term in f_{NS} and reduction of coefficient in f_R from 16 to 8. In 1829, Jacobi discovered the formula (he used a different notation, of course)

$$f_R(w) = f_{NS}(w).$$

For him this relation was an obscure curiosity, but we now see that it provides strong evidence for supersymmetry of the GSO-projected string theory in ten dimensions.

A complete proof of supersymmetry for the interacting theory was constructed by Green and me five years after the GSO paper[18]. We developed an alternative world-sheet theory to describe the GSO-projected theory[19]. This formulation has as the basic world-sheet fields X^μ and Θ^a, representing ten-dimensional superspace. Thus the formulas can be interpreted as describing the embedding of the world-sheet in superspace.

The first superstring revolution

In the 'first superstring revolution' which took place in 1984–85, there were a number of important developments[20–22] that convinced a large segment of the theoretical physics community that this is a worthy area of research. By the time the dust settled in 1985 we had learned that there are five distinct consistent string theories, and that each of them requires space–time supersymmetry in the ten dimensions (nine spatial dimensions plus time). The theories are called type I, type IIA, type IIB, $SO(32)$ heterotic, and $E_8 \times E_8$ heterotic. Calabi–Yau compactification, in the context of the $E_8 \times E_8$ heterotic string theory, can give a low-energy effective theory that closely resembles a supersymmetric extension of the standard model. There is actually a lot of freedom, because there are very many different Calabi–Yau spaces, and there are other arbitrary choices that can be made. Still, it is interesting that one can come quite close to realistic physics. It is also interesting that the number of quark and lepton families that one obtains is determined by the topology of the Calabi–Yau space. Thus, for suitable choices, one can arrange to end up with exactly three families. People were very excited by the picture in 1985. Nowadays, we tend to make a more sober appraisal that emphasizes all the arbitrariness that is involved, and the things that do not work exactly right. Still, it would not be surprising if some aspects of this picture survive as part of the story when we understand the right way to describe the real world.

The second superstring revolution

Around 1995 some amazing discoveries provided the first glimpses into nonperturbative features of string theory[23–26]. These included 'dualities' that were quickly recognized to have several major implications. First, they implied that all five of the superstring theories are related to one another. This meant that, in a fundamental sense, they are all equivalent. Another way of saying this is that there is a unique underlying theory, and what we had been calling five theories are better viewed as perturbation expansions of this underlying theory about five different points (in the space of consistent quantum vacua). This was a profoundly satisfying realization, since we really did not want five theories of nature. That there is a completely unique theory, without any dimensionless parameters, is the best outcome one could have hoped for. However, it should be emphasized that even though the theory is unique, it is entirely possible that there are many consistent quantum vacua. Classically, the corresponding statement is that a unique equation can admit many solutions. It is a particular solution (or quantum vacuum) that ultimately must describe nature.

A second crucial discovery was that the theory admits a variety of nonperturbative excitations, called *p*-branes, in addition to the fundamental strings. The letter *p* labels the number of spatial dimensions of the excitation. Thus, in this language, a point particle is a 0-brane, a string is a 1-brane, and so forth. The reason that *p*-branes were not discovered in perturbation theory is that they have tension (or energy density) that diverges as $g_s \to 0$. Thus they are absent from the perturbative theory. A special class of *p*-branes, called *D*-branes, are especially tractable, because they are described by the theory of open strings. The third major discovery was that the underlying theory also has an eleven-dimensional solution, which is called *M*-theory. Later, we will explain how the eleventh dimension arises.

One type of duality is called *S*-duality. (The choice of the letter *S* is a historical accident of no great significance.) Two string theories (let us call them *A* and *B*) are related by *S*-duality, if one of them evaluated at strong coupling is equivalent to the other one evaluated at weak coupling. Specifically, for any physical quantity *f*, one has

$$f_A\,(g_s) = f_B\,(1/g_s). \tag{5}$$

Two of the superstring theories – type I and $SO(32)$ heterotic – are related by *S*-duality in this way. The type IIB theory is self-dual. Thus *S*-duality is a symmetry of the type IIB theory, and this symmetry is unbroken if $g_s = 1$. Thanks to *S*-duality, the strong-coupling behaviour of each of these three theories is determined by a weak-coupling analysis. The remaining two theories, type IIA and $E_8 \times E_8$ heterotic, behave very differently at strong coupling. They grow an eleventh dimension!

Another astonishing duality, which goes by the name of *T*-duality, was discovered several years earlier. It can be understood in perturbation theory, which is why it was found first. But, fortunately, it often continues to be valid even at strong coupling. *T*-duality can relate different compactifications of different theories. For example, suppose theory *A* has a compact dimension that is a circle of radius R_A and theory *B* has a compact dimension that is a circle of radius R_B. If these two theories are related by *T*-duality, this means that they are equivalent provided that

$$R_A R_B = (\ell_s)^2, \tag{6}$$

where ℓ_s is the fundamental string length scale. This has the amazing implication that when one of the circles becomes small, the other one becomes large. Later, we will explain how this is possible. *T*-duality relates the two type II theories and the two heterotic theories. There are more complicated examples of the same phenomenon involving compact spaces that are more complicated than a circle, such as tori, K3, Calabi–Yau spaces, etc.

Concluding remarks

This article has sketched some of the remarkable successes that string theory has achieved over the past 30 years. There are many others that did not fit in this brief survey. Despite all this progress, there are some very important and fundamental questions whose answers are unknown. It seems that whenever a breakthrough occurs, a host of new questions arise, and the ultimate goal still seems a long way off. To convince you that there is a long way to go, let us list some of the most important questions:

- What is the theory? Even though a great deal is known about string theory and *M*-theory, it seems that the optimal formulation of the underlying theory has not yet been found. It might be based on principles that have not yet been formulated.
- We are convinced that supersymmetry is present at high energies and probably at the electro-weak scale, too. But we do not know how or why it is broken.
- A very crucial problem concerns the energy density of the vacuum, which is a physical quantity in a gravitational theory. This is characterized by the cosmological constant, which observationally appears to have a small positive value – so that the vacuum energy of the universe is comparable to the energy in matter. In Planck units this is a tiny number ($\Lambda \sim 10^{-120}$). If supersymmetry were unbroken, we could argue that $\Lambda = 0$, but if it is broken at the 1 TeV scale, that would seem to suggest $\Lambda \sim 10^{-60}$, which is very far from the truth. Despite an enormous amount of effort and ingenuity, it is not yet clear how superstring theory will conspire to break supersymmetry at

the TeV scale and still give a value for Λ that is much smaller than 10^{-60}. The fact that the desired result is about the square of this, might be a useful hint.

- Even though the underlying theory is unique, there seem to be many consistent quantum vacua. We would very much like to formulate a theoretical principle (not based on observation) for choosing among these vacua. It is not known whether the right approach to the answer is cosmological, probabilistic, anthropic or something else.

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