Coefficient of variation in field experiments and yardstick thereof – An empirical study

The experimental data provide information on controlled (treatment effect) and uncontrolled variation. The uncontrolled variation is expressed as experimental error, which could be quantified as an estimate called 'coefficient of variation (CV)'. Besides, fertility variation among experimental units (plots), the factors contributing toward uncontrolled variation are climatic and experimental. Therefore CV of field experiments varies with the situation. Lower magnitude of CV is the reflection of reliability (precision) of the experimental results. The acceptable range of CV advocated by various workers is based on the experience with very limited number of experiments. There is a need to develop a yardstick (critical value) for CV based on theory and also on a large number of experiments conducted under different situations. The present paper deals with this aspect.

Johnson and Welch[1] reported that for a normal distribution, the ratio of mean to standard deviation should be of the order of 3 or more. Further, they mentioned that 33% has often been stated as the permissible upper fiducial limit of CV. Tyagi *et al.*[2] and Patel *et al.*[3] pointed out that CV obtained for the crops under study was found to be considerably higher than those reported from the uniformity trials. They stated that the yardstick for accepting experimental results should be worked out using CV observed in the experiments rather than in the uniformity trials. Bajpai and Nigam[4] suggested a working rule for deciding the value of $W2$ (weight corresponds to precision of the experiment) and developed an index to evaluate agricultural field experiments statistically. Gomez and Gomez[5] reported that CV varies greatly with the type of experiment, the crop grown and the character measured. They opined that the acceptable range of CV is 6 to 8% for varietal trials, 10 to 12% for fertilizer trials and 13 to 15% for insecticidal and herbicidal trials on rice. No other information (except reviewed and reported here), is available in the literature on the yardstick of CV for field experiments on crops.

Plot-wise yield data were collected for 906 experiments laid out in RCBD on six important pulse crops, viz. arhar (*Cajanus cajan* L.), cowpea (*Vigna sinesis* L.) urid (*Vigna mungo* L. Wilczek), mung (*Vigna radiata* L. Wilczek), guar (*Cymopsis tetragonaloba*) and gram (*Cicer erietinum* L.) at 25 research stations of the Gujarat Agricultural University during the years 1988–1989 to 1992–1993. Since more than 90% of the total experiments on pulse crops was conducted in RCBD, the data related to this design were collected. Other supplementary information on each experiment, i.e. season, year, discipline, crop, number of treatments and replications, plot size, etc. was also collected.

The plot-wise yield data were subjected to statistical analysis and CV was estimated for individual experiments. The same was assumed as random variable in further analysis. CV is a function of square root of mean square (S) and mean ($\bar{X}$).

$$\mathrm{CV} = S/\bar{X}. \tag{1}$$

The distributions of $\bar{X}$ and S have simple forms and Student's t distribution provides complete solution for testing the hypothesis or estimating fiducial limits relating to either μ or σ, singly. But t-distribution cannot be used for CV ($S/\bar{X}$). Mckay[6] used non-central t-distribution for providing fiducial limits of CV.

Let z be a quantity distributed normally about zero mean with unit standard deviation and let w be a quantity distributed independently as χ^2, with degrees of freedom of χ^2. Then, if t is defined by the equation:

$$t = \frac{z+\delta}{\sqrt{w}}, \tag{2}$$

where δ is some constant, then t is distributed in a manner depending only on δ and f. This distribution is a non-central t-distribution. When δ equals zero, the distribution is the familiar Student's t.

Let an estimate of V be $v = S/\bar{X}$, the sample coefficient of variation.

Now, one may write

$$\frac{\sqrt{n}}{V} = \frac{\sqrt{n}\bar{X}}{S} = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} + \frac{\sqrt{n}\mu}{\sigma} + \frac{S}{\sigma}\cdots \tag{3}$$

It appears from comparison with eq. (2) that $\sqrt{n}/V$ is distributed as non-central t with $f = (n-1)$ and $\sigma\sqrt{n}/V$ (ref. 6). This distribution can be used for test of significance and for providing fiducial limits of V (i.e. CV), as is done for μ. Since the objective was to work out the yardstick based on CV, the upper fiducial limit of CV using non-central t-distribution was estimated following the procedure given by Johnson and Welch[1]. The procedure is briefly explained below.

Let,

(i) $\mathrm{CV} = \dfrac{\sqrt{\text{Error mean square}}}{\text{General mean}}$,

n = number of treatments × number of replications in a given experiment,

$f = n - 1$ degrees of freedom.

Now, the upper fiducial limit of CV is,

$$\mathrm{CV_{UL}} = \sqrt{n}/\delta(f, t_o, \varepsilon),$$

where, $t_o = \sqrt{n}/\mathrm{CV}$.

(ii) Find

$$y = \left(1+\frac{t_o^2}{2f}\right)^{-1/2}$$

or

$$y' = \frac{t_o}{\sqrt{2f}}\left(1+\frac{t_o^2}{2f}\right)^{-1/2}.$$

according to whether $|t_o/\sqrt{2f}|$ is greater than or less than 0.75. Consider Y', if $t_o/\sqrt{2f}$ lies between -0.75 and 0.75, otherwise consider Y.

(iii) If $f > 9$, calculate $12/\sqrt{f}$.

(iv) Select desired probability level of confidence, i.e. ε and obtain $\lambda(f, t_o, \varepsilon)$ from the table in ref. 1 interpolating with respect to the quantities obtained in (ii) and (iii).

(v) Calculate $\delta(f, t_o, \varepsilon)$ =

$$t_o - \lambda\left(1+\frac{t_o^2}{2f}\right)^{-1/2}.$$

In the present investigation, the yardstick of CV for field experiments was worked out using two concepts: (i) average upper fiducial limit of CV for each of the 906 experiments was worked out separately and then average of these upper fiducial limits was computed, and (ii) upper fiducial limit of CV based on average

Table 1. Upper fiducial limits (yardstick) for CV%

	Mean CV (%)	Upper fiducial limit 0.05	0.10
Average of 906 experiments	17.97	22.54	21.41
Based on average degree of freedom (48)	17.97	21.78	20.86

Table 2. Power of F-test as influenced by CV

Class (CV in %)	Number of experiments	F-test Significant	Non-significant	Ratio
<3	23	23	0	0.00
3–6	77	76	1	0.01
6–9	111	105	6	0.06
9–12	102	92	10	0.11
12–15	118	106	12	0.11
15–18	91	70	21	0.30
18–21	117	86	31	0.36
21–24	56	36	20	0.56
24–27	53	34	19	0.56
27–30	47	29	18	0.62
30–33	24	9	15	1.67
33–36	21	7	14	2.00
36–39	19	7	12	1.71
39–42	9	2	7	3.50
42–45	13	6	7	1.17
45–48	5	2	3	1.50
48–51	3	2	1	0.50
51–54	1	0	1	*
54–57	4	1	3	3.00
57–60	2	1	1	1.00
60–63	1	0	1	*
63–66	2	0	2	*
66–69	1	0	1	*
69–72	2	0	2	*
72–75	2	0	2	*
75–95	1	0	1	*
95–115	1	0	1	*
Total	906	694	212	0.31

*Infinite.

size of experiments, i.e. average degree of freedom.

The upper fiducial limits of CV at 95% and 90% confidence were worked out using the theory of truncated t-distribution as described by Johnson and Welch[1] and are presented in Table 1.

The average fiducial limits of CV at 95% and 90% confidence were worked out to be 22.54% and 21.41%, respectively. The average CV of all the experiments under study was 17.97%. Based on average CV and average size of experiments (48 degrees of freedom), the upper fiducial limit was worked out as 21.78% at 95% confidence. The difference between average fiducial limit and fiducial limit based on average size of experiments was less than 5%. These yardsticks were reasonable and reliable. The average fiducial limit of CV (22.54%) covered the region related to average size (21.78%) also. Therefore, 22.54% (rounded value 23%) was considered as the yardstick for CV% of pulse experiments conducted in RCBD, for accepting or rejecting the results.

The ratio of number of experiments not having significant F-test to those significant was worked for each class of frequency distribution (Table 2).

The results revealed that the ratio consistently increased with the increase in CV of the experiments. They also indicated that the efficiency (of detecting difference in treatment means) of F-test decreased with the increase in CV of experiments. The average ratio was observed to be 0.31. The ratio for the class 15–18% was almost equal to the average ratio. This class included 17.97%, the mean CV of all experiments. The upper fiducial limit of CV (based on average degrees of freedom) was 21.78%. This value was covered in the class 21–24% (including 23%, the yardstick). It can be seen from Table 2 that more than 50% of the experiments having 21% and more CV failed to detect difference in treatment means. Results clearly showed that when the coefficient of variation in pulse experiments in RCBD exceeds 23%, the experimental findings should not be considered for any purpose. This upper fiducial limit (23% CV) is based on truncated t-distribution and the theory of probability and is therefore, recommended for further use as a yardstick for CV of field experiments, to be conducted in RCBD on pulse crops, in Gujarat. For generating an acceptable yardstick for CV of agricultural field experiments, it is suggested that the large number of experiments (> 10,000) conducted on different crops be evaluated for this purpose.

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