Statistical methods applied to analysis of electric power production costs

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This paper gives several examples of the application of probability models and statistical analysis in assessing the cost of production of electricity. A typical power-generating system usually consists of a diverse mix of generators with varying capacities, reliability characteristics and costs. Because electricity cannot be stored, the particular set of units used to supply power at any given time depends upon the magnitude of the demand and the availability of the generators, both random quantities. The cost of electric power production is thus a random variable. Computation of the basic parameters of its distribution is difficult because of the large state space associated with a typical power-generating system. The paper mentions these difficulties and outlines several approximation schemes for estimating these quantities. A Monte Carlo simulation study carried out to explore whether an accurate temperature forecast will improve the accuracy in the estimation of production costs is summarized. Finally, the paper describes how the production costing models can be used to estimate marginal costs which are essential in predicting market clearing prices when electricity is deregulated.

ONE of the characteristics of large-scale electric power production is that it cannot be stored. Thus at every moment there should be sufficient production to meet the demand (or load). If there is a sudden spurt of demand or a unit that produces electricity cheaply fails, more expensive units need to be pressed into service. The demand for electric power and the failure–repair cycles of electric power-generating units are often characterized by stochastic processes. The cost of electric power production during a given time interval is a random variable because of its dependence on these two factors. Probability models and statistical analysis play an important role in assessing the cost of electric power production. This paper is an attempt to review the author's work in this area. One of the special features of this particular application of stochastic modelling is that the state space associated with a typical electric power-generation system can be very large. This makes the computation of seemingly ordinary quantities such as the mean and variance of the production costs quite difficult.

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Applications of statistics in industry in areas such as quality control and design of experiments are well known. But it appears that both the statistics and engineering communities may not be fully aware of the potential usefulness of statistical models in the electric power industry. Electricity is a crucial commodity and the cost of electric power production is of great interest to the public. The descriptions given in this paper are representative of the electric utility industry in the United States, although here the picture is changing at a very fast pace. Until recently the electric power industry in this country was regulated. Under regulation, the electric utility companies are obligated to supply power to their customers at a price that is fixed by the public regulatory commissions. This price is based on the projected cost of the utility company to which is added a guaranteed rate of return. It has thus been of great interest to these companies to forecast accurate estimates of production cost. Presently the electric power industry is in the process of being deregulated. This implies that both buyers and sellers of electricity will be able to trade electricity at the price prevailing in the market. As we will see, this change to deregulation will not make the production costing models that were developed in the context of regulation irrelevant in the upcoming market-oriented environment.

We begin this paper with a short overview of an electric utility company in the current regulated set-up. The role and development of production costing models is given next. Analytical expressions for the mean and variance of production cost and several approximate methods for their computation are then outlined. The next section briefly describes a case study with actual data in which the effect of temperature on the production costs was analysed. The last section describes one possible application of these models in a deregulated environment.

The electric utility companies solve the problem of providing power from electricity-generating units to widely scattered demand points through a three-tiered system. Elements of this system are: power generation system, transmission system and distribution system. In the power generation system, electric power is produced from a number of different types of generating units of varying capacities and sizes. Transmission systems carry large amounts of power for a long distance at a high voltage level. From the transmission sources, distribution systems carry the load to a service area by forming a fine network.

Under the regulated set-up, a given electric utility company that is responsible for supplying power over a specified geographical area and has direct relationships with customers usually owns these three subsystems. Under deregulation electricity will be traded like any other commodity and the producers and the consumers will have the option to buy and sell power in a marketplace created to provide competition. A given company may now own only a part of the power system. In this paper we will confine ourselves to a discussion of the role of the production costing models only insofar as the power generation system is concerned.

Production costing models

In describing a power generation system one needs to consider both the demand which it needs to meet and the supply which is obtained from the generating units that constitute the system. Electric power consumption varies with the time of the day, the day of the week, and the season. In order to run the electric power generation system economically so as to meet the demand, it is thus necessary to turn the machines on and off at appropriate times. But the generating units cannot be turned on and off in a haphazard manner. Besides the start-up and shut-down costs, one also needs to consider certain operating constraints that dictate how frequently and in what manner the units can be shut-off or switched on. They are, for example: minimum capacity, maximum capacity, minimum up-time, minimum down-time, ramping rate, etc.[1]. Given a profile for the electricity demand during a specified time interval, the optimization problem whose solution gives the sequence in which the units belonging to a utility company should be turned on and off so as to meet the demand and not violate any of the operating constraints is known as the unit commitment problem. It is solved in conjunction with the economic dispatch problem that determines the quantity of power that each committed unit should produce so as to minimize production costs. In the traditional formulation of the unit commitment and economic dispatch problems the demand is regarded as deterministic and the generators are assumed to be available with certainty.

A discussion of the unit commitment and the economic dispatch problems is relevant for estimating the production costs of electricity, because the solution of these two problems yields the optimal production costs for a generating system which needs to generate enough power to meet the demand of its customers. The unit commitment problem is considered to be a particularly difficult combinatorial optimization problem. Solution procedures for this problem when the load and the availability of the generators are considered as stochastic quantities are not yet well developed. A second model for production costs into which it is relatively easy to incorporate the stochastic aspects of the load and the generator availabilities is often used in the electric power industry. In this model the unit commitment constraints that incorporate the history of operation of the generating units are ignored and it is postulated that a strict merit-order loading prevails, according to which the individual generators are dispatched to meet the demand. This order is often determined based on the variable costs (the direct cost to generate one unit of electricity) of the units. This implies that for a given level of demand, the unit with the cheapest operating cost is dispatched first to meet the prevailing load, followed by the next cheapest unit and so on. The model was first introduced by Baleriaux *et al.*[2]. Ryan and Mazumdar have enlarged its scope by introducing stochastic processes in its formulation. The subsequent discussions given in this section pertain to this model.

In this model it is assumed that the costs are being calculated for a power generation system consisting of N units during the interval $[0, T]$. The following assumptions are also made:

1. The generators are dispatched in a fixed, predetermined loading order and the actual set of units being used at any given time depend upon the magnitude of the load, the loading order and the set of available units.
2. The jth unit in the loading order has capacity c_j, variable energy cost d_j (\$/MWH). The unit operates between two states, up and down, in accordance with an alternating renewal process $Y_j(t)$ which is in steady state:

$$Y_j(t) = \begin{cases} 1 & \text{if the unit } j \text{ is up at time } t, \\ 0 & \text{if the unit } j \text{ is down at time } t. \end{cases}$$

The steady state unavailability index for this unit, i.e. $\lim_{t\to\infty} P[Y_j(t) = 0]$ is denoted by p_j. The relation $E[Y_j(t)] = 1 - p_j$ holds for all values of t in the study interval.
3. The load at time t, which is denoted by $u(t)$, is a stochastic process also in the steady state. The load has a certain amount of predictable variation depending on the time of the day, season, etc. but on top of it, there is random variation.
4. For all values of $i \neq j$, $Y_i(t)$ and $Y_j(s)$ are statistically independent for all values of t and s. Also, each $Y_j(t)$ is independent of $u(t)$ for all values of t.
5. The up and down process of a generating unit continues whether or not it is in use. This assumption is made to ensure that the steady state assumption for the generating units holds.

Let $e_i(t)\mathrm{d}t$ and $Z_i(T)$ denote, respectively, the energy produced by unit i during the time intervals $[t, t + \mathrm{d}t]$, and $[0, T]$, respectively. From the above assumptions it follows that

$$e_1(t) = \min[u(t), c_1]Y_1(t),$$

$$e_i(t) = \min\left[\max\left(u(t) - \sum_{k=1}^{i-1} c_k Y_k(t), 0\right), c_i\right] Y_i(t),$$
$$i = 2, 3, \ldots, N,$$

$$Z_i(T) = \int_0^T e_i(t)\mathrm{d}t. \tag{1}$$

Let finally $K(T)$ denote the cost of production for the generation system. Then,

$$K(T) = \sum_{i=1}^{N} d_i Z_i(T). \tag{2}$$

In many applications the cost resulting from being unable to serve the demand (cost of unserved energy) is added to this expression. This is not considered here.

Mean and variance of production costs

From eq. (1) we see that when unit 1 is considered, we may write

$$E[Z_1(T)] = E\left[\int_0^T e_1(t)\mathrm{d}t\right] = \int_0^T E[e_1(t)]\mathrm{d}t$$

$$= \int_0^T E[\min(u(t), c_1)Y_1(t)]\mathrm{d}t.$$

Now define

$$I(t; x) = \begin{cases} 1 \text{ if } u(t) \geq x, \\ 0 \text{ if } u(t) < x. \end{cases}$$

Thus we can express

$$\min(u(t), c_1) = \int_0^{c_1} I(t; x)\mathrm{d}x.$$

Because of the assumed independence of $Y_1(t)$ and $u(t)$,

$$E[Z_1(t)] = \int_0^T\int_0^{c_1} E[I(t; x)Y_1(t)]\mathrm{d}x\mathrm{d}t$$

$$= (1 - p_1)\int_0^{c_1}\int_0^T P\{u(t) \geq x\}\mathrm{d}t\mathrm{d}x \tag{3}$$

$$= T(1 - p_1)\int_0^{c_1} G_T(x)\mathrm{d}x,$$

where $G_T(x)$ is the average probability that the load is greater than x, the average being taken over the interval $[0, T]$, i.e.

$$G_T(x) = \frac{1}{T}\int_0^T P\{u(t) \geq x\}\mathrm{d}t.$$

When the load $u(t)$ is a deterministic time-varying function, $P\{u(t) \geq x\} = I(t; x)$, i.e. $G_T(x)$ now measures the proportion of the time that the load exceeds or equals x in the time interval $[0, T]$. This quantity is known as the load-duration curve in the power systems literature[1]. Similarly, proceeding in the same fashion[4], it can be shown that

$$E[Z_i(T)] = T(1 - p_i)\int_0^{c_i} E\left[G_T\left(x + \sum_{j=1}^{i-1} c_j Y_j\right)\right]\mathrm{d}x, \tag{4}$$

where the Y_js are independently distributed Bernoulli random variables with $P\{Y_j = 0\} = p_j$ and $P\{Y_j = 1\} = 1 - p_j$. The integrand in eq. (4) is equivalent to

$$\frac{1}{T}\int_0^T P\left\{u(t) - \sum_{j=1}^{i-1} c_j Y_j \geq x\right\}\mathrm{d}t.$$

If we now define a random variable U whose survivor function is given by $G_T(\bullet)$, then eq. (4) can also be expressed as

$$E[Z_i(T)] = T(1 - p_i)\int_0^{c_i} P\left\{U - \sum_{j=1}^{i-1} c_j Y_j \geq x\right\}\mathrm{d}x. \tag{5}$$

The expected cost of production is then given by

$$E[K(T)] = \sum_{i=1}^{n} d_i E[Z_i(T)].$$

An important question centres on how best to compute eq. (5) when the distribution function of U and the p_js is specified. Examining eq. (5) we observe that there is little hope for obtaining a closed form expression for the value of the integrand. For a given value of x, when the c_is and p_is are all different, a large number of arithmetic operations will be necessary for an exact computation of eq. (5). The number of such operations grows exponentially with i. For a large system, the amount of computation may be prohibitive. Mazumdar[5] has compared the accuracy obtained from applying two approximation formulas to evaluating eq. (5) and determining the expected production costs for a given large system. The results show that both the Edgeworth and Esscher's large-deviation formulas are capable of providing accurate estimates of eq. (5) in an acceptable amount of computer time.

The computation of the variance of $K[T]$ is a much more difficult proposition. In order to appreciate the sources of difficulty, consider eq. (2). The variance of the production cost is given by

$$\mathrm{Var}[K(T)] = \sum_{i=1}^{N}\sum_{j=1}^{N} d_i d_j \operatorname{cov}(Z_i(T), Z_j(T)).$$

However, the computation of the above covariance terms is not an easy proposition. To appreciate the reasons, consider a typical covariance term. From the definition of $Z_i(T)$

$$\mathrm{cov}(Z_i(T), Z_j(T)) = \int_0^T \int_0^T \mathrm{cov}(e_i(t), e_j(s)\mathrm{d}s\mathrm{d}t), \tag{6}$$

where

$$e_i(t) = \min\left[\max\left(u(t) - \sum_{k=1}^{i-1} c_k Y_k(t), 0\right), c_i\right] Y_i(t),$$

$$e_j(s) = \min\left[\max\left(u(s) - \sum_{k=1}^{j-1} c_k Y_k(s), 0\right), c_j\right] Y_j(s).$$

Although $Y_k(t)$ and $Y_j(s)$ are assumed to be independent for $k \neq j$, it will not be correct to assume that $Y_k(t)$ and $Y_k(s)$, the random variables that refer to the operating status of the generating unit k at times t and s, will be uncorrelated. (Intuitively, if a unit is up at a certain time, we will expect it to remain up a short interval thereafter.) In the expressions for $e_i(t)$ and $e_j(s)$ above, suppose that $i < j$. Then in the expression for $e_i(t)$ we have terms involving $Y_1(t), Y_2(t), \ldots, Y_{i-1}(t)$ and in the expression for $e_j(s)$, we have $Y_1(s), \ldots, Y_2(s), \ldots, Y_{i-1}(s), \ldots, Y_{j-1}(s)$. Thus for computing the variance of $K[T]$, the covariance of $e_i(t)$ and $e_j(s)$ needs to be evaluated for each t and s for each pair of units i and j, by taking into account the contribution of each common covariance term $\mathrm{cov}(Y_k(t), Y_k(s))$. This appears to be a particularly daunting task.

Several procedures for computing the variance have been outlined in the recent literature. In these papers it has been assumed that each $Y_i(t)$ follows a continuous time Markov chain with known transition rates. With load being considered deterministic, Mazumdar *et al.*[6] provided a systematic scheme for computing the variance. This procedure can be illustrated by considering the term $\mathrm{Var}[Z_2(T)]$. From the definition of $e_j(t)$ it is seen that

$$\begin{aligned} e_2(t) &= \min(\max(u(t) - c_1, 0), c_2) Y_1(t) Y_2(t) \\ &\quad + \min(u(t), c_2)(1 - Y_1(t)) Y_2(t), \\ e_2(s) &= \min(\max(u(s) - c_1, 0), c_2) Y_1(s) Y_2(s) \\ &\quad + \min(u(t), c_2)(1 - Y_1(s)) Y_2(s). \end{aligned} \tag{7}$$

When the transition rates of the two-state Markov chain for $Y_k(t)$ are given by λ_k and μ_k, it is well known[7] that

$$\mathrm{cov}(Y_k(t), Y_k(s)) = p_k q_k \mathrm{e}^{-(\lambda_k + \mu_k)|s-t|}, \tag{8}$$

where

$$p_k = \frac{\lambda_k}{\lambda_k + \mu_k} = 1 - q_k. \tag{9}$$

When λ_k refers to the transition from the up to the down state (failure rate) and μ_k refers to the repair rate, p_k is the steady state unavailability index for the kth unit. Thus the task now is to compute $\mathrm{cov}(e_2(t), e_2(s))$ based on eq. (8). For a given k, the number of terms in the expression for $e_k(t)$ will be 2^{k-1}. The computational effort for the $\mathrm{cov}(e_k(t), e_j(s))$ terms thus rises at an exponential rate with increased j and k. Also after these covariance terms have been computed, they need to be entered into the double integral expression in eq. (6) for evaluating $\mathrm{cov}(Z_k(T), Z_j(T))$. The entire process can become very time-consuming indeed.

Because of the lengthiness of the exact computation for the variance, several approximation schemes have been advanced for reducing the computational effort. Ryan and Mazumdar[3] have shown how the double integral can be approximated by a single integral when the transition rates are large. Lee *et al.*[8] have simplified the computations first by breaking the interval $[0, T]$ into T subintervals and replacing the integrals by sums. This can be justified since most of the data on demand and supply are reported in hourly intervals. They have developed a recursive scheme for carrying out the computations based on the assumption that the capacities of the individual units have a largest common divisor greater than one. This work appears quite useful for routine computations by the industry. The formulas given by the authors however have one error that has been pointed out and corrected by Kapoor and Mazumdar[9], who have also proposed another approximation scheme for the computation of the variance using the bivariate Gram–Charlier series formulas. Shih *et al.*[10] have recently proposed a recursive scheme for this computation. This procedure uses the properties of the fundamental matrix of a discrete time, finite Markov chain and uses the asymptotic formulas given by Kemeny and Snell[11] for the mean and variance of a function defined on the state space of a Markov chain.

Monte Carlo computation of production costs

In view of the difficulties associated with the analytical computations, Monte Carlo simulation becomes an attractive alternative for estimating the quantities of interest in electric power production costing models. Writing the simulation code based on the model given in the section on 'Production costing models' is reasonably straightforward. Numerical examples giving the results of such simulation can be found in Mazumdar and Kapoor[12]. Two other recent papers have explored the possibility of developing variance reduction procedures, whereby the simulation estimates can be obtained at a given level of precision with fewer total number of Monte Carlo runs, than that which will be necessary with the direct Monte Carlo method. One proposed procedure for variance reduction can be explained with reference to eq. (5). Note that the expected value of the energy produced by the ith unit depends on the marginal probability distributions of Y_i and U. It is not necessary to use the stochastic processes associated with the unit up and down times to estimate the expected value of the production costs. Mazumdar[13] and Mazumdar and Kapoor[14] have shown that by introducing two-state continuous time Markov chains with suitably chosen transition rates λ_i and μ_i to capture the transitions between the up and down states for each unit i, we can reduce the variance of the Monte Carlo estimate of eq. (5). These transition rates need to be chosen such that eq. (9) holds. Mazumdar and Kapoor[14] have also described a variance reduction method using the control variate technique for estimating the variance.

A recent paper[15] has combined statistical analysis of two years' hourly load data with Monte Carlo simulation to answer several questions related to the variance of production costs. It is clear from the discussion given above that there are two major components for the variance of production cost: load and generator failures. A natural question is which of these two components is a more important contributor? Also it is well known that in the United States one of the variables on which the load depends is the ambient temperature. Thus a question that arises in this connection is as follows: if an accurate temperature forecast is available, can it be used to make the corresponding forecast of the production cost more accurate? There is not much hope for an answer to these questions being obtained by using analytical formulas. Therefore Monte Carlo simulation is used. Because the load for different seasons demonstrates different patterns and also there is much difference between the magnitude of loads between weekdays and weekends, we select the data pertaining to weekdays for the summer season only. When the hourly load data were analysed, it is found that correcting for periodicities, an AR(3) model fits the data well. The fitted model, an ARIMA $(3, 0, 0) \times (0, 1, 0)_{120}$ is:

$$\begin{aligned} u(t) = &-2.49 + 1.103\,[u(t-1) - u(t-121)] \\ &+ 0.00007024[u(t-2) - u(t-122)] \\ &- 0.1412\,[u(t-3) - u(t-123)] \\ &+ u(t-120) + z(t). \end{aligned} \tag{10}$$

Here, $z(t)$ is a Gaussian white noise process with mean 0 and variance $\hat{\sigma}_z^2 = 990.04$. Next for the same data set, the values of hourly temperatures are considered, in addition to the hourly load values. The plots of the load $u(t)$ vs temperature τ_t (°F) for each hour t show that the following regression equation (the coefficients are given in ref. 15) is appropriate for representing these data:

$$u(t) = \hat{\beta}_{0,t} + \hat{\beta}_{1,t}\tau_t + \hat{\beta}_{2,t}(\tau_t - 65)\delta(\tau_t) + x(t).$$

where $\delta(\tau)$ is defined as:

$$\delta(\tau_t) = \begin{cases} 0 & \text{if } \tau_t \le 65, \\ 1 & \text{if } \tau_t > 65, \end{cases},$$

and

$$x(t) = x(t-120) + 0.879[x(t-1) - x(t-121)] + z(t), \tag{11}$$

where similar to eq. (10), $z(t)$ is a Gaussian white noise with mean 0 and estimated variance $\hat{\sigma}_z^2 = 2032.55$. This equation captures the feature that the electricity demand is high at low as well as high temperatures and relatively low at moderate temperatures. It also shows that when the influence of temperature is taken out from the load data, the time series for the particular data set could be represented by an AR(1) process.

Having first considered the demand side of the power-generating system the supply side is next considered. A representative system consisting of 17 generators compatible with the load is chosen. The characteristics of these units listed in their loading order are given in Table 1.

The design of the simulation is as follows. A 24-h period for which an accurate temperature forecast for each hour is considered available. Then a random sample of the hourly load for this period is separately generated using eqs (10) and (11). This operation was repeated L times. For each such 24-hourly load sequences generated, Q sequences of successive up-times and down-times for each unit for the 24-h period from the information given in Table 1 are next generated from their respective exponential distributions. Finally, using the loading order and matching the hourly load with the hourly available capacity of each generator, the energy produced by each unit during the 24-h period and the corresponding production cost are computed We have in hand a total of $L \times Q$ observations in all, Q values of cost nested within each 24-h load sample. The data were analysed using a one-factor random effects model in which the treatment corresponds to the load and within load variation is the result of variability of the generator up- and down-times during the 24 h period. Specifically we assumed that the cost C_{ij} is given as

$$C_{ij} = \mu + l_i + g_{ij} \quad (i = 1, 2, \ldots, L;\ j = 1, 2, \ldots, Q), \tag{12}$$

where μ is a constant, l_i are independent and identically distributed i.i.d. normal random variables with mean zero and variance σ_l^2, and g_{ij} are i.i.d. normal random variables

Table 1. A generation system

Units	Capacity (MW)	Mean time to failure $1/\lambda$	Mean time to repair $1/\mu$	Energy cost $/MWH
1–2	400	1100	150	6.00
3	350	1150	100	11.40
4–7	150	960	40	11.40
8–9	150	1960	40	14.40
10–12	200	950	50	22.08
13–15	100	1200	50	23.00
16	50	2940	60	27.60
17	100	450	50	43.50

with mean zero and variance σ_g^2. It is assumed that the random variables l_i and g_{ij} are all independent. We denote the mean square between loads and the mean square within loads by MSL and MSG, respectively. It is well known[16] that

$$E[\text{MSL}]=\sigma_g^2+Q\sigma_l^2;\; E[\text{MSG}]=\sigma_g^2.$$

Thus the estimates of σ_g^2 and σ_l^2 are

$$S_g^2=\text{MSG};\; S_l^2=\frac{\text{MSL}-\text{MSG}}{Q};\; S^2=S_l^2+S_g^2.$$

S^2 is an estimate of σ^2, the variance of the production cost. It has two components, S_g^2 and S_l^2. The former measures the variability of the production cost due to the generator up- and down-times and the latter, due to load. The results of simulation using $L = 300$ and $Q = 300$ for a total of 90,000 observations has been reported in Valenzuela and Mazumdar[15] using the load models of both eqs (10) and (11) and the generation system given in Table 1. The results of this experiment showed that the load component amounted to about 80% of the total variance of production costs. Also, eq. (11) leads to a considerably lower variance in the estimated value of S^2, suggesting that inclusion of temperature in the equation would result in much better estimates of production costs compared to that obtained from eq. (10). If these conclusions are borne out by confirmatory studies, then an obvious conclusion will be that more effort should be made towards a better prediction of hourly loads by considering a more refined regression equation for load vs temperature and additional factors beyond temperature, that are known to influence loads.

Estimation of marginal costs

The production costing models can also be used to estimate marginal cost at any given hour. In the context of electricity production the marginal cost, i.e. the cost of producing an additional unit of electricity for a generation system, can be regarded to be the variable cost of the last unit used to supply power to meet the demand. Thus the marginal cost varies from hour to hour because of the variation in the hourly demand. It also depends on the mix of generators used to supply the demand at any given hour. It is a random variable because of reasons similar to those for the production cost. Shih and Mazumdar[17] have used this formulation to provide a procedure for computing the mean and variance of the average marginal cost during a certain time period. The estimation of the mean and variance of the marginal costs will be of much importance in the near future in the United States because of the upcoming deregulation of electric power. In addition to producing power from its generating units, each electric power-generating company then will have the option of buying from or selling power to pools or markets specifically created to provide open competition. It is assumed that the producer will be a price taker, i.e. its actions will not affect the market price. According to economic theory the market-clearing price at any given hour will be equivalent to the marginal cost.

Using this model for market-clearing prices, Valenzuela and Mazumdar[15] have recently provided a solution to the unit commitment problem (see section on 'Production costing models') for the deregulated environment. The objective function is now to maximize profit for each generator. They have first shown that in order to solve this problem, it is enough to consider each generating unit individually, one at a time. To model the market-clearing price, they assume that the generators participating in the market are brought into operation in an economic merit-order of loading similar to the model given in the section on 'Production costing models'. The ith unit in the loading order has a capacity c_i (MW), variable energy cost d_i (\$/MWH), and a forced outage rate q_i. The market-clearing price at a specific hour t, m_t, is equal to the operating cost (\$/MWH) of the last unit used to meet the load prevailing at this hour. This unit is called the marginal unit and is denoted by $J(t)$. Thus m_t is equal to $d_{J(t)}$. A different formulation in which the information on m_t is obtained from the time series analysis of load and market price is given by Allan and Ilic[18]. The values of $J(t)$ and $d_{J(t)}$ depend on the prevailing aggregate load and the operating states of the generating units in the loading order. One needs to consider two decision variables, P_t and ν_t, for determining an optimum commitment schedule. The first variable denotes the amount of power to be generated by the unit under question at time t, and the latter is a control variable whose value is 1, if the generating unit is committed at time t and 0, if otherwise. The optimization problem in this situation in the deterministic case can be written as

$$\begin{matrix}\text{Maximum}\\ \text{profit}\end{matrix} = \operatorname*{Max}_{\nu_t, P_t} \sum_{t=1}^{T} \{P_t d_{J(t)} - \text{CF}(P_t) - S(x_t)[1-\nu_{t-1}]\}\nu_t, \tag{13}$$

subject to the operating constraints for the particular unit: capacity limits, minimum up-time, and minimum down-time. Here, $CF(p)$ is a known cost function that gives the cost incurred for producing p units of power. Usually, this is assumed to be a quadratic function of the form: $CF(p) = a + bp + cp^2$. The variable x_t gives the consecutive time that the unit has been on (+) or off (–) at the end of hour t and $S(x_t)$ gives the start-up cost function that depends on the value of x_t.

The maximum profit over the period T eq. (13) is a random variable because the marginal unit $J(t)$ at an hour t is a random variable. It is assumed that at the time of the decision, hour zero, the marginal unit and the load for all the hours before hour zero are known. Denoting the marginal unit at time zero by j_0, the sub-problem is solved by maximizing the conditional expected profit over the period T. The optimization problem is now expressed as

$$\text{Max } E[\text{profit} \mid j_0] = \underset{v_t, P_t}{\text{Max}} \sum_{t=1}^{T} \{P_t E[d_{j(t)} \mid j_0] - CF(P_t) - S(x_t)(1 - v_{t-1})\} v_t, \quad (14)$$

subject to the same operating constraints for the unit. The solution to this problem has been given by Valenzuela and Mazumdar[19] using probabilistic dynamic programming. In order to calculate the conditional expectation in eq. (14), the conditional probability distribution of $J(t)$ given $J(0)$ needs to be calculated. Because

$$\Pr[J(t) = j \mid J(0) = j_0] = \frac{\Pr[J(t) = j \text{ and } J(0) = j_0]}{\Pr[J(0) = j_0]}, \quad (15)$$

this implies that the probability distribution of $J(t)$ and $J(0)$ as well as the marginal probability distribution of $J(0)$ are required to be evaluated in these computations. For the calculation of these probabilities the authors have used the production costing model given in the section on 'Production costing models', by considering the overall market to be a system consisting of N generating units under the supervision of an Independent System Operator (ISO) which controls the dispatch of generation. They have argued that this model captures the fundamental stochastic characteristics of the market.

In order to evaluate the probability expressions given in eq. (15) they have pointed out that the events $J(t) > j$ and

$$u(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0,$$

are equivalent. Thus,

$$\Pr[J(t) > j] = \Pr\left[u(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0\right]. \quad (16)$$

The marginal probability distribution of $J(t)$ can be evaluated by computing the probability expression given in the right hand side of eq. (16), which is seen to be very similar to eq. (5). As regards the numerator of eq. (15), they have noted that

$$\begin{aligned} \Pr[J(r) = m \text{ and } J(t) = n] &= \Pr[J(r) > m-1 \text{ and } J(t) > n-1] \\ &\quad - \Pr[J(r) > m \text{ and } J(t) > n-1] \\ &\quad - \Pr[J(r) > m-1 \text{ and } J(t) > n] \\ &\quad + \Pr[J(r) > m \text{ and } J(t) > n]. \end{aligned}$$

Since the two events

$$\left\{u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0 \text{ and } u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0\right\}$$

and $\{J(r) > m \text{ and } J(t) > n\}$, are equivalent, the following equality holds:

$$\Pr[J(r) > m \text{ and } J(t) > n] = \Pr\left[u(r)\sum_{i=1}^{m} c_i Y_i(r) > 0 \text{ and } u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0\right]. \quad (17)$$

Computational procedures for approximating the probability expressions given in eqs (16) and (17) have been investigated at length by Valenzuela[20], who has based his investigation on the earlier work by Mazumdar[5], Mazumdar and Kapoor[12], and Iyengar and Mazumdar[21]. The three analytical approximation schemes considered by him are: normal approximation, Edgeworth series expansion, and the large-deviation approximation. He has compared these approximations with an extensive Monte Carlo simulation which he has used as a benchmark. His conclusions have been that for a generating system consisting of a large number of generating units with typical characteristics, the normal and Edgeworth approximations provide reasonably accurate results. The large-deviation approximation which gives surprisingly accurate results for small systems becomes computationally very time-consuming for large systems and is thus not competitive with the other two methods.

In the numerical example given by Valenzuela and Mazumdar[19], they have considered a system of 150 units representing the market for which information similar to that given in Table 1 was available. The model for the load is similar to that given in eq. (11), and it is assumed that the temperature forecast for the 24-h period for which the unit commitment decision is being made is accurate. They consider solving the commitment problem for a single unit by solving the optimization problem given in eq. (14). For this purpose, they compute the conditional probabilities using the normal and Edgeworth approximations, and Monte Carlo simulation using 200,000 replicates. The optimum schedule is found to be the same for each method with comparable estimates of maximum profit. From the standpoint of computational effort, normal approximation takes the least time, followed by Edgeworth approximation and Monte Carlo simulation.

One potential opportunity for application of statistical methods in the deregulated environment is to devise strategies for hedging risks for both buyers and sellers of electricity, by entering into special contracts or buying and selling specific derivative financial instruments. Some discussion on this topic is given in Pilipovic[22]. In this work an estimate of the statistical distribution of marginal costs will play an important role.

Conclusions

The above discussion has shown that statistical methods and probability models play a major role in the computation of electric power production costs. The challenges arise from the largeness of the state space associated with typical electric power generation systems. The example of the last section shows that there is an opportunity for interaction of optimization techniques with probability computations for solving very real practical problems in electric power system planning.

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