Cepheid distance estimation for Virgo cluster

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A measurement of distance to the Virgo cluster and a few of its member galaxies by direct method is clearly important for a reliable determination of the Hubble constant as well as for studying the dynamics of a nearby rich galaxy cluster. Cepheid variables in a few galaxies in the Virgo cluster were observed with the Hubble Space Telescope (HST) over the last few years. This work is a reanalysis of the HST data following our study of the Galactic and Magellanic Cloud Cepheids. The log (period) vs $V$-magnitude relation is re-calibrated using the Galactic, LMC as well as the HST observations. The number density of Cepheid variables as a function of their period is used to determine the role of flux-limited incompleteness and a prescription is given to correct for this bias in the sample. The extinction correction is carried out using period vs mean $(V-I)_{0}$ colour and $V$-amplitude vs $(V-I)$ colour at the brightest phase relations. The distance and error estimation is based on $L_{1}$ minimization. The mean distance to Virgo cluster is estimated to be $20.5 \pm 1.8$ (random) $\pm 2.5$ (systematic) Mpc.

A natural scale length for the Universe is provided by the Hubble constant ($H_{0}$) and undoubtedly a reliable determination of its value is one of the central problems of cosmology. Over the years, there has been a lively debate about the value of $H_{0}$ and the present estimates range from less than 50 km s$^{-1}$ Mpc$^{-1}$ to over 80 km s$^{-1}$ Mpc$^{-1}$. The major reason for the discrepancy is primarily due to the conventional distance ladder method involving multiple steps. Its main drawback is that an analysis of the systematic errors becomes difficult when the calibrating local sample and the observed sample at the next step of the ladder are not identical. Consequently, it is believed that an accurate measurement of the distance to a galaxy cluster which is located at around $\sim 20-30$ Mpc, without involving intermediate steps, will lead to a reliable direct estimate of the value of $H_{0}$, provided the recession velocity of the cluster is independently known. The Virgo cluster, which is the nearest cluster of galaxies, is fairly rich in terms of galaxy population, and an average of the distances to the individual galaxies by different methods should provide a good estimate to its mean distance. One of the key projects of the Hubble Space Telescope (HST) was devoted to a calibration of the extragalactic distance scale, mainly by using the Cepheid variables.

The classical Cepheid variables are known to provide an important standard candle to measure distances to galaxies up to $\sim 30$ Mpc. The Cepheid distance scale based on the period–luminosity relation is considered to be among the most reliable methods of distance calibration because the physics of Cepheid pulsation is reasonably well-understood and the relation between the pulsation period and luminosity of the star is a well-established observational quantity. Cepheid variables are radially pulsating giants and supergiants, having pulsation periods in the range of less than a day to upwards of 100 days. Their pulsation is very stable and the amplitude of light variation in the $V$ (Johnson) band may be up to nearly 2 magnitudes, although most of the Cepheids have amplitudes between 0.6 and 1.3 magnitude. Cepheids are among the most luminous stars, having a narrow range of surface temperatures. The intrinsic scatter in their period–luminosity relation is believed to be less than 0.3 mag. However, the Cepheid distance scale cannot be directly calibrated from the observation of nearby stars and consequently, several systematic effects could undermine its effectiveness as a standard primary candle to determine extragalactic distances beyond a few Mpc. A major problem concerning the calibration of the Cepheid distance scale is the following: Is a single period–luminosity relation applicable to the entire instability strip? Are the preferential pulsation modes of Cepheids period-dependent?

Theoretically, it is widely accepted that at shorter periods, a good fraction of the Cepheid variables should be first overtone pulsators, while at longer periods almost all of them are likely to be fundamental mode pulsators. The crucial question is: where does the transition period lie? There is no agreement between the theoreticians on this question which is extremely important while determining the slope of the period–luminosity relation. A mixture of fundamental mode and first overtone at shorter periods and pure fundamental mode at longer periods will have shallower slope compared to a sample containing only fundamental modes at all periods. Another aspect which is not taken into serious consideration is their evolutionary status: Most of the Cepheid variables are in their second or third crossing of the instability strip in the Hertzsprung–Russell diagram during the core helium burning phase. However, at periods less than around 15 days, the contribution to their number density could arise from stars at

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other phases of evolution, depending on the metal content of the star. Consequently, treating the short and long period Cepheids as one group, even if they are fundamental mode pulsators, could affect the period–luminosity relation.

We have reanalysed the available HST data for Cepheids in many of the galaxies close to or within the Virgo cluster. Our approach to the calibration of the Cepheid period–luminosity relation and the distance estimation to distant galaxies is based on the following five considerations:

- We compare the observed number density of Cepheids as a function of their pulsation period with the stellar evolutionary models for three local galaxies: Milky Way, Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC). Its relevance for both the determination of modes of pulsation and possible systematic errors due to incompleteness of the data in a target galaxy is discussed in the next section.
- We determine the slope and zero point of the period–luminosity relation for the long period Cepheids (period $\geq$ 15 days).
- The Cepheid variables in similar evolutionary phase and mode of pulsation obey a tight period–colour relation as well as amplitude of pulsation as function of the colour at the brightest phase. The use of these relations is then discussed.
- The present observations of Cepheids in the Virgo cluster galaxies are invariably affected by the faintness of stars. Correction to offset the consequent systematically shorter distance is estimated.
- The Cepheid variables of the distant galaxies follow a skew distribution in the period–luminosity diagram, essentially due to causes originating from their faintness. To minimize the errors due to higher weightage to the discordant points, we follow the $L_1$ minimization rather than the conventional method based on $\chi^2$. Our results for six galaxies in the Virgo cluster, observed through the HST by two groups are discussed. The distance to the Virgo cluster centre is estimated using these galaxies and the Hubble constant is determined.

Finally, we examine the significance of our distance estimate for the structure of the Virgo cluster and discuss future prospects for more robust determination of Cepheid distances.

**Number density distribution of Cepheids**

Recent microlensing projects, particularly MACHO, EROS and OGLE, have provided us with very large databases of Cepheid variables in the Magellanic Clouds. Ideally, these should serve as a testing ground for the reliability of the theory of stellar pulsation as well as calibration of the Cepheid distance scale. We can find the number density of Cepheid variables as a function of period from these as well as some other smaller catalogues. The relative number density at a given period is determined by three effects, namely, the fraction of stars which have the correct range of mass, the amount of time they spend in the instability strip, as well as the modes of pulsation. The following inputs were used in our models:

(a) The number density of stars as a function of their initial mass, was computed from the Salpeter mass function.

(b) The evolutionary models with overshoot for various metallicities computed by the Geneva group (Schaller et al.\textsuperscript{1} for $Z = 0.02$ (Galaxy), Schaerer et al.\textsuperscript{2} for $Z = 0.008$ (LMC) and Charbonnel et al.\textsuperscript{3} for $Z = 0.004$ (SMC)) along with the instability strip calculated by Alibert et al.\textsuperscript{4} were used to estimate the time spent by a star inside the instability strip.

(c) The observed number density from the catalogues of Payne–Gaposchkin\textsuperscript{5}, Kholopov et al.\textsuperscript{6}, Udalski et al.\textsuperscript{7,8}, Beauchieu et al.\textsuperscript{9}, Alfonso et al.\textsuperscript{10} and Alcock et al.\textsuperscript{11} was used to determine the transition period beyond which most of the Cepheids are likely to be fundamental mode pulsators in their core helium burning phase. The main features of the number density diagram used for the diagnostics are (i) the position of the main peak which should correspond to the first overtone period of the lowest mass star that occupies the instability strip during its core helium burning, (ii) the full width at half maximum of the main peak which is determined mainly by the width of the Cepheid instability strip, and (iii) the position and height of a secondary peak or plateau near 12-day period, which signifies the transition to purely fundamental mode pulsation. Unfortunately, the catalogues do not agree with each other on the numerical value of all these three features, which is a main limitation of our work. Nevertheless, we have obtained a reasonable match between the observed number density diagram and the stellar evolution models, by assuming a smooth transition between fundamental mode and first overtone. A detailed description of the models and their comparison with observed number density distributions has been given by Mazumdar and Narasimha\textsuperscript{12}. A typical result for the LMC is displayed in Figure 1. The following main results relevant to the distance calibration emerge from our analysis of Cepheid number densities:

- For the LMC, the stars of period longer than about 11 days are fundamental mode pulsators at second and third crossing of the instability strip.
- For lower metallicity, the period of transition progressively decreases, and the fraction of first overtone pulsators increases.
- The main peak in the diagram should have a width of approximately 0.45 in log $P$, which is compatible with a typical width of the instability strip of $\delta$ log $T_{\text{eff}}$ ~ 0.06.

Consequently, by choosing 15 days as the shorter cut off for the period in the calibrating as well as the target
Cepheid samples, we expect to avoid most of the overtone pulsators. By comparing the observed number distribution in the target galaxy with that of the standard galaxy, like the Milky Way or LMC, we can estimate the extent of incompleteness in the sample.

Slope and zero point of the period–luminosity relation

We recall, the period–luminosity relation is the backbone of the Cepheid distance scale. The linear relation between Cepheid magnitude and logarithm of the pulsation period is a direct consequence of Cepheid pulsation theory. This relationship has an intrinsic scatter due to the finite range of temperatures over which Cepheid pulsation is sustained in a star during its post-main sequence phase. The zero point of this relation has to be determined by independent methods so as to enable us to compare the apparent magnitude of a star with its intrinsic brightness. However, none of these parameters of the period–luminosity relation can be definitively estimated from theoretical considerations.

![Figure 1](image)

Figure 1. (Top panel) Theoretical model for number density of LMC Cepheids (solid line). Also plotted are the number density distributions of various observational surveys of Cepheids (various dotted lines). (Faint dotted lines correspond to the parts of the distributions which have not been used in the fitting procedure); (Bottom panel) Distribution of fractional abundance of fundamental mode (solid line) and first overtone (dotted line) Cepheids as a function of log P for chemical composition of (Y = 0.25, Z = 0.008) as obtained from the theoretical model.

Ideally, the slope of the period–magnitude relation of a sample of single-mode fundamental or first overtone Cepheids having same metallicity, is expected to be around – 3.33, since the dynamical time of the star varies as three-fourths power of the luminosity. This term dominates the effects associated with the changing surface temperature or mass of the star along the instability strip. However, the observed slope could be very different if (a) we mix the fundamental mode and overtone pulsators, (b) stars at different evolutionary phases having structural changes are present in the sample, (c) there is any systematic effect like saturation or flux-limited incompleteness. Consequently, the choice of the period range becomes important, and the sample of standard stars too should be chosen to avoid the systematic biases. Hence a calibration of the period–luminosity relation which is relevant for distant galaxies should be made in nearby galaxies only with Cepheids having periods greater than 15 days, avoiding contamination from short-period pulsators, which might have, on the average, a different slope of the period–luminosity relation. We should like to emphasize that such a partition is necessary for a reliable distance estimation, irrespective of the interpretation of the pulsation mode or evolutionary status of the Cepheid variables.

The Cepheids in the LMC are among the most popular calibrating candidates for the period–luminosity relation due to the availability of multi-wavelength data collected over many years. The extinction towards the LMC is also estimated to be small and is, therefore, unlikely to affect the slope of the period–luminosity relation. The value of the slope is fairly robust at – 2.77, if we use all the Cepheids in the period range of 2 to 50 days. However, in such cases the slope is heavily weighted by the shorter-period Cepheids, which is not desirable in the calibration of the period–luminosity relation as a distance indicator. Here we examine the value of the slope of the period–luminosity relation as obtained from linear best fits to LMC Cepheids from different sources of data.

The large number of Cepheids present in the OGLE and EROS databases should provide a robust calibration of the slope of the period–luminosity relation. However, both the projects were conceived primarily to detect microlens events and as such, they are tuned to respond to variability in the faint stars. Consequently, the brighter stars like the Cepheid variables are subject to saturation effects at higher luminosities. As a result, the two catalogues do not match well, and the slope of the period–luminosity relation depends strongly on the period range selected. In spite of these severe limitations, it turns out that the classical Cepheids in the OGLE catalogue for LMC in the period range of 15 to 30 days have a mean slope of around – 3.1, which is significantly steeper than the conventional value of – 2.77. We have also derived the slope of the period–luminosity relation from simple
linear best fits to $V$-magnitude vs log $P$ data available from several sources in the literature. The general behaviour of the slope for each sample is very similar. The full sample, consisting of Cepheids with periods between 2 and 50 days, has a slope close to $-2.75$ for the $V$ vs log $P$ diagram, on the average. But beyond a period of 10 days, the slope is much steeper, lying between $-3.15$ and $-3.45$. The slope in the $I$ band also follows a similar trend. The most obvious explanation of these results is that a sample with period as short as 2 days is likely to be populated by numerous overtone Cepheids. As discussed at the beginning of this section, this would always make the slope shallower. We are able to exclude these overtone Cepheids having periods above 10 days, and the resulting slope is typically higher! Clearly, the importance of classification of Cepheids at short periods and the choice of a proper period range to avoid contamination from overtone pulsators cannot be underestimated in the context of the distance scale.

The result is more pronounced for external galaxies. We have analysed a few of the galaxies from the HST Key Project. If we take the full period range, the data is subject to incompleteness corrections and the slope shows a huge range from 0 to $-3$. However, if we restrict to Cepheids in the range of 30 to 60 days, where we believe that the incompleteness or other biases should not affect the slope appreciably, we get a value generally in agreement with each other, in the range of $-2.8$ to $-3.6$, with a mean slope of $-3.15$.

The zero point of the Cepheid period–luminosity relation is another contentious issue. Conventionally, it is calibrated by assuming a distance modulus to LMC. Recently, the zero point has been determined using the trigonometric parallaxes and proper motions of nearby stars measured by the HIPPARCOS satellite. Overall there appears to be general consensus among the HIPPARCOS results that the absolute magnitude of a 10-day Cepheid variable ($M_V^{10}$) is $-4.24 \pm 0.13$ mag in the $V$-band. Several other independent zero point calibrations are available. For example, Gieren et al.\textsuperscript{15} derived a value ($M_V^{10} = -4.06 \pm 0.03$) slightly fainter than the HIPPARCOS zero point from Galactic and LMC Cepheids using the infrared Barnes–Evans surface brightness technique. Madore and Freedman\textsuperscript{13} observed a number of Cepheids in LMC using multiple bands. From these multiwavelength period–luminosity relations and assuming LMC distance modulus to be 18.50 mag, they derived a value of $M_V^{10} = -4.16 \pm 0.05$. Considering a suitable average of all the different methods of determination discussed above, we have adopted a value of $-4.16 \pm 0.20$ mag as the zero point of the Cepheid period–luminosity relation in the $V$ band. Incidentally, this value is identical to that estimated by Madore and Freedman\textsuperscript{13}, which has been accepted in many distance scale programmes, including the HST Key Project\textsuperscript{16} and the Supernovae Ia peak brightness calibration project\textsuperscript{17}. Incorporating the slope adopted above, we have arrived at the following period–luminosity relation for Cepheid variables of period greater than 15 days:

$$M_V = (-3.15 \pm 0.25) (\log P - 1) - (4.16 \pm 0.20).$$

To use this calibration of the Cepheid period–luminosity relation for distance estimation, we still require a reliable method to correct for attenuation of starlight by the intervening matter.

### Extinction correction

The progenitors of Cepheid variables are believed to be stars of intermediate mass and consequently, they are generally seen near gas-rich environments. The extinction correction to take care of the absorption of starlight by the intervening matter is, therefore, important. The position of the Cepheids in the period–colour diagram, where it occupies a very narrow strip, is suggestive of a method to correct for the extinction statistically. However, it would be necessary to derive the position of the local Cepheids in the Milky Way and LMC in the period–colour diagram accurately.

For most types of absorbers dominated by silicate grains, the extinction of the incident radiation is inversely proportional to the wavelength of the photon at optical frequencies\textsuperscript{18}. Consequently, absorption of light is accompanied by a characteristic reddening. We use the four-band observations of Galactic and LMC Cepheids to correct for the absorption by utilizing the extinction law due to Cardelli et al.\textsuperscript{18}. The de-reddened colours of the long-period Cepheids are used to map the Cepheid instability strip in the period–colour diagram.

By analysing the de-reddened colour and amplitude of pulsation, we find that the colour at the brightest phase of variability is related to the amplitude. This can be understood by appealing to the driving mechanisms for the Cepheid pulsation. The radiative opacity in the partial ionization zones of the hydrogen, helium and metals increases with temperature and this provides a kind of ‘heat engine’, energized by the so-called ‘x-mechanism’. But when the surface temperature of the star increases, the mechanism reaches saturation, which could be a reason for the tight relation between colour at the brightest phase and the amplitude for Cepheids having similar evolutionary phase and same mode of pulsation.

The derived period–colour–amplitude relations are given below. The quantities in the brackets denote the standard deviations of the best fits.

- \(\langle B - V \rangle_0 = 0.21 \log P + 0.60 (\pm 0.02)\),
- \(\langle V - R \rangle_0 = 0.13 \log P + 0.67 (\pm 0.01)\),
- \(\langle V - R \rangle_{\text{max}} = -0.28 \Delta V + 0.88 (\pm 0.02)\).
Using the above relations, we are in a position to compute the extinction corrected flux for the Cepheids observed in two photometric bands ($V$ and $I$). In practice, for HST observations of Cepheids in far-off galaxies, decent light curves are available in visual wavelengths ($V$ band) only, while the near-infrared ($I$ band) data suffer from poor phase-sampling. It turns out that, from the $V$ band light curve and observed $I$ band fluxes, a good estimate of the mean $V$ magnitude as well as $\langle V-I \rangle$ colour is possible. However, extinction correction from the data will be reliable only if the observed data points populate the instability strip in the Milky Way and the target galaxy in a similar fashion. This can be ensured only if there is no systematic incompleteness in the data.

Incompleteness correction

A crucial aspect of our analysis is the correction for incompleteness of the Cepheid sample, which is important whenever the standard candle has an intrinsic scatter and we have to work with a flux-limited sample. A major task in any extragalactic distance measurement is to isolate signal from the noise near the limiting magnitude at which a precise determination of the stellar brightness is barely feasible. Notice that the problem is to determine whether the star in question is a classical Cepheid, based on observations at a few fixed epochs. As the period decreases the number of photons collected drops. This rapid change in the signal-to-noise ratio causes faint stars to be systematically missed in the sample, while the brighter stars preferentially detected at a fixed period produce an increase in the average brightness of the stars at that period, if the scatter in the period–$V$-magnitude diagram is large. The resulting overestimation of the brightness of the observed stars turns out to be proportional to logarithm of the rate at which faint stars are missed due to poor signal, and varies as the square of the width of the period–luminosity relation. For instance, the available HST data before extinction correction have a scatter in the $V$-magnitude of order 0.45 mag at a fixed period for almost all the galaxies in the Virgo cluster. This results in a systematic underestimation of the mean brightness of intermediate period Cepheids by 0.2 to 0.4 magnitudes, while those having period less than 10 days are almost entirely missed. The consequent decrease in the slope of the period–luminosity relation for the extragalactic samples has already been discussed in the previous section. We compensate for this systematic underestimation of the brightness of the Cepheids at the shorter end of our adopted period–luminosity relation by appealing to the number density diagram for the target galaxy and the local galaxy having similar star formation history. For the various samples of galaxies in the Virgo cluster we have analysed, the correction for flux-limited incompleteness can be given by the following interpolation formula.

\[
V_{\text{complete}} = \begin{cases} 
V_{\text{incomplete}} + \sigma^2_{\text{eff}} \frac{\gamma}{\alpha} \left[ \frac{\log P_2 - \log P}{\log P_2 - \log P_1} \right] & \text{for } P \leq P_1, \\
V_{\text{incomplete}} & \text{for } P_1 < P \leq P_2, \\
V_{\text{incomplete}} & \text{for } P > P_2.
\end{cases}
\]

The value of $\gamma$ depends on the detector characteristics, and cannot be determined theoretically. However, we have used the relative number density of observed Cepheids to extract its value. The value of $\alpha$ is supplied by the adopted slope of the period–luminosity relation (see the section of period–luminosity relation) and is equal to 3.15. The periods $P_1$ and $P_2$ are determined from the appropriate number density graphs. The analysis of the incompleteness problem and numerical simulations of the same have been described in detail by Narasimha and Mazumdar\(^1\).

We should stress that our scheme for correcting the incompleteness bias is entirely a statistical one, where instead of increasing the mean magnitude at a fixed period by the specified correction term, we increase the magnitude of each observed star in that period range.

Results

Extinction correction

Extinction correction is important for distance calibration even for the face-on spirals. However, in the absence of multi-colour photometry or at least well-sampled light curves in two bands, the extinction correction carried out would be at most statistical in nature and would not take into account the differential extinction with respect to period, nor the possibility of only part of the instability strip being sampled. In the absence of better alternatives we have adopted three relations, namely $\langle V-D_0 \rangle$ vs $\log P$, $\langle V-D_0 \rangle_{\text{max}}$ vs $\Delta V$ and $\langle V \rangle_0$ vs $\log P$, for distance calibration as well as extinction correction. Due to the small number of data points for which we have a reliable measure of $\langle V-D_0 \rangle_{\text{max}}$ we have primarily used the linear relations between $\langle V-D_0 \rangle$ vs $\log P$ and $\langle V \rangle_0$ vs $\log P$ only. The third relation (connecting $\langle V-D_0 \rangle_{\text{max}}$ with $\Delta V$) has been used only as an additional check. We minimize the absolute deviation $\chi_1$ defined by

\[
\chi_1 = \sum_i \left( \alpha \left( \log P_i - \log \bar{P} \right) + \beta \left( \log P_i - y_1 \right) \right),
\]

where $\log \bar{P}$ is the averaged $\log P$ of all the data points in the relevant period range.

Ideally, the weights $a_1$ and $a_2$ should be determined from the error estimates in the photometry as well as from
the scatter in the two relations. We have chosen the two weights to be equal such that the scatter in the log $P$–$V$ diagram is comparable to the expected value of 0.3 mag and the scatter in the best fit line for $(V − R_0)$ is comparable to the error in the observed colours in our data. The deviation $\chi_i$ can be computed for a specified set of parameters $a$, $b_1$, $p$ and $y_1$ by choosing the reddening $E(V − I)$ and extinction $A_V$ for each star. Following Cardelli et al.\textsuperscript{18}, we have chosen a constant ratio $A_V/E(V − I) = 2.44$. We should again like to stress that such a procedure automatically assigns less weightage to the few data points that lie far from the line, either due to large errors or due to the star being in a different stage of evolution.

**Distances to spiral galaxies in the Virgo cluster**

Our main results as well as the error contributions from various sources that were analysed are summarized in Table 1. We have used the Cepheid data for three galaxies (NGC 4321 (ref. 16), NGC 4535 (ref. 20) and NGC 4548 (ref. 21)) observed by the HST Key Project, and three more (NGC 4496 (ref. 22), NGC 4536 (ref. 17) and NGC 4639 (ref. 23)) observed by the Supernova calibration team. All the galaxies are considered to be part of the Virgo cluster, though not necessarily among the core members. The final results for the Cepheid variables in the six galaxies in the Virgo cluster, after corrections for extinction and incompleteness of the sample, are shown in Table 2. The corresponding period–$V$-magnitude relation for one of the galaxies, NGC 4321 (M100) is displayed in Figure 2.

All the coordinates and velocities in this table are calculated relative to the massive elliptical galaxy M87 which is generally believed to be very close to the centre of the Virgo cluster\textsuperscript{24}. The following remarkable results emerge from our analysis:

- The three galaxies, NGC 4496A, NGC 4535 and NGC 4536 appear to be associated, as seen from their distance and velocity measurements.
- All the three galaxies appear to be falling towards the centre of Virgo cluster with a velocity of the order 800 km/s.
- The two spiral galaxies NGC 4321 and NGC 4639, which appear to be positioned at mirror image locations with respect to M87 also have opposite apparent velocities with respect to M87.

*All the six spiral galaxies seem to be dominated by an infall velocity component rather than the random velocity that we would have expected for a virialized system.*

We are, therefore, tempted to conclude that the spiral galaxies in the Virgo cluster, are located typically at 3 to 5 Mpc distance from the Virgo centre and are falling towards the core of the cluster. Any inference on the Hubble constant based on a single galaxy would consequently be misleading.

The distance to the Virgo cluster has been a matter of debate for many years. Estimates ranging from 14 to 25 Mpc are found in the literature. We have used the two galaxies, NGC 4321 and NGC 4639, which have similar infall velocity towards the core and are located approximately 4 Mpc on either side in the opposite directions of M87, to determine the distance to the Virgo cluster. Our present estimate for the distance to the Virgo centre is 20.5 ± 1.8 (random) ± 2.5 (systematic) Mpc.

It turns out that, on the average, our distance estimates to the three HST Key Project Virgo galaxies are higher by

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**Table 1. Results and error budget**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Mean value</th>
<th>Random error</th>
<th>Systematic error</th>
<th>Comment</th>
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<tbody>
<tr>
<td>Calibration of the period–luminosity relation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Slope at log $P = 1$</td>
<td>mag</td>
<td>$-3.15$</td>
<td>$0.25$</td>
<td>Range of values in calibrating samples</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>mag</td>
<td>$-4.16$</td>
<td>$0.20$</td>
<td>Distance to nearby Cepheids</td>
<td></td>
</tr>
</tbody>
</table>

| Period–$V$-magnitude relation for target galaxies: | | | | | |
| Slope | | $-3.13$ | $0.10$ | $0.05$ | Some Cepheids of $P > 55$ days could appear at 48–55 days |
| Intercept at mean log $P$ | mag | $0.10$ | $0.10$ | Extinction and incompleteness corrections not independent |
| Mean extinction correction | mag | $0.31$ | $0.08$ | $0.08$ | Error in $(V − I)$ large; Reddening due to unresolved stars; Recession of galaxy–K correction; Galactic period–colour–amplitude relations not well-determined |
| Mean incompleteness correction | mag | $0.20$ | $0.12$ | Model for the efficiency of detection not known; Error in periods |
| Zero point calibration of the detector | mag | | | $0.08$ | Observation problem |

**Distance to Virgo** | Mpc | $20.50$ | $1.80$ | $2.5$ | Average of galaxies at mirror image positions |
| Recession velocity of Virgo | km/s | $1179$ | $100$ | Infall to Virgo centre of Local Group not same as velocity component towards Virgo |
| Hubble constant | km/s/Mpc | $58$ | $6$ | $9$ | |

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366 CURRENT SCIENCE, VOL. 80, NO. 3, 10 FEBRUARY 2001
Table 2. Results on Virgo cluster galaxies observed with HST

<table>
<thead>
<tr>
<th>Galaxy name</th>
<th>Data source</th>
<th>Gal. long. (deg.)</th>
<th>Gal. lat. (deg.)</th>
<th>Velocity wrt M87 (km/s)</th>
<th>No. of Cepheids</th>
<th>Distance from us (Mpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 4321</td>
<td>Ferrarese et al.</td>
<td>-12.64</td>
<td>+2.41</td>
<td>+328</td>
<td>59</td>
<td>18.9</td>
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<td>Saha et al.</td>
<td>+6.79</td>
<td>-8.16</td>
<td>+432</td>
<td>85</td>
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<td>Macri et al.</td>
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<td>-3.85</td>
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<td>NGC 4536</td>
<td>Saha et al.</td>
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<td>-9.76</td>
<td>+510</td>
<td>54</td>
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<td>NGC 4548</td>
<td>Graham et al.</td>
<td>+1.92</td>
<td>+2.34</td>
<td>-770</td>
<td>24</td>
<td>17.5</td>
</tr>
<tr>
<td>NGC 4639</td>
<td>Saha et al.</td>
<td>-10.52</td>
<td>+1.50</td>
<td>-276</td>
<td>18</td>
<td>22.0</td>
</tr>
</tbody>
</table>

Figure 2. Final period–luminosity diagram for M100 Cepheids along with the plot for the best fit period–luminosity relation.

\[ V_{\text{peculiar}} = \frac{2 \pi a}{R^2} \tau \approx 75 \text{ km s}^{-1}, \]  

(7)

where \( R \) is the distance to Virgo centre and \( \tau \) is the age since the formation of the Virgo cluster. Using the six galaxies we analysed, the typical infall velocity towards M87 is of the order of 800 km s\(^{-1}\) at a distance of 5 Mpc, extrapolates to an infall velocity of 50 km/s for the Local Group. We take the recession velocity of Virgo to be 1179 km s\(^{-1}\), as given by Jerjen and Tammann and estimate the Hubble constant to be:

\[ H_0 = 58 \pm 6 \text{ (random)} \pm 9 \text{ (systematic)} \text{ km s}^{-1} \text{ Mpc}^{-1}. \]  

(8)

On the other hand, if we adopt the velocity given by Rowan-Robinson et al. for the infall velocity of the Local Group towards Virgo to be 362 km s\(^{-1}\), \( H_0 \) will accordingly increase to 67 km s\(^{-1}\) Mpc\(^{-1}\).

Recession velocity of Virgo centre and Hubble constant

The present work does not address the problem of recession velocity of the Virgo centre with respect to the Local Group. We would, however, point out that if one takes a central line-of-sight velocity dispersion, \( \sigma_v \), of the order of 800 km s\(^{-1}\) and structural length, \( a \) of 1.5 Mpc for the Virgo cluster, the velocity of the Local Group towards Virgo produced by the mass centered at Virgo cluster would be of the order of

Summary and prospects

Our strategy to investigate the calibration of Cepheids based on extragalactic distance scale is two-fold:

- Compilation of a reasonably well-tested local complete sample of the parent population of Cepheid variables and quantification of some of their characteristics for using them as benchmarks for a determination of distance to far-off galaxies.
- Carrying out tests on a set of homogeneous data of good quality for an external galaxy and devising a
method to extract the calibration characteristics without getting unduly distorted by the noise.

- Analysing the velocities as well as distances of a few galaxies located around the Virgo cluster to estimate the distance and recession velocity of the Virgo core.

For the local Cepheid variables, we were guided by the light curves, number density as function of period and amplitude, and by the theory of stellar pulsation. The models based on stellar pulsation are limited by the size of the helium core and the boundary conditions in the outer envelope where convection is supersonic, apart from the more fundamental problem of coupling between convection and pulsation. Still, as a working model, the period–colour and the period–colour–amplitude relations for the instability strip that we have obtained should be useful for the calibration of the Cepheid distance scale.

We have used Cepheid data for nearby galaxies as well as HST data on Cepheids for galaxies at intermediate distances to determine the slope of the period–$P$–magnitude relation for the population that would be targeted for the measurement of distances to farther galaxies. The slope of the relation is found to be steeper than the commonly adopted values and appears to be consistent between various galaxies. We have compared the values of the zero point of the Cepheid period–luminosity relation determined by independent methods. Discrepancies in the zero point are expected to be resolved when the different distance calibration for LMC are harmonized and direct distance estimates to more number of local Cepheid variables become available. Equally important is the intrinsic scatter in the period–luminosity relation if we wish to provide a trustworthy error analysis of our distance estimations. But we do not have enough data yet to determine the extinction corrected scatter.

We have demonstrated the presence of substantial flux–limited incompleteness bias by using a diagram of the relative number density of Cepheids in the Virgo cluster members as function of the period. We attempt to provide a prescription for correction to offset this bias when a volume–limited test sample of a similar population is available. We have also carried out numerical simulations of the incompleteness problem using a toy model for the distribution function of the population and the efficiency of the detector to check our formalism. We have used $L_1$ minimization for the determination of distance modulus in order to marginalize the effects of a few deviant data points.

We have obtained a distance of $20.5 \pm 1.8$ (random) ± 2.5 (systematic) Mpc to the Virgo cluster. We arrived at this value by correlating the positions, velocities and Cepheid distances of six galaxies in the Virgo cluster. The estimation of the Hubble constant is clearly affected as much by the infall velocity as the distance. Our estimate of the Hubble constant, based on the analysis of spiral galaxies towards the Virgo cluster is $58 \pm 6$ (random) ± 9 (systematic) km s$^{-1}$ Mpc$^{-1}$.

The systematic errors associated with the Cepheid distance scale can be reduced to a large extent through multiwavelength observations of a selected sample of Cepheid variables in nearby galaxies. It is indicated from our analysis that a reliable estimate of the distance to galaxies situated within 30 Mpc is well within the capability of the HST, provided the observing strategy addresses some of the problems specific to Cepheids which we have attempted to highlight in the present work.


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