**An Introduction to the Mechanics of Fluids.** C. Truesdell and K. R. Rajagopal. Birkhäuser Verlag AG, P. O. Box 133, CH-4010, Basel, Switzerland. 2000. 296 pp. Price. SFr 128/DM 148.

The publication of a new book, even co-authored, by Clifford Truesdell, savant, aesthete, trenchant critic of modern scientific mores and co-founder and leader of the modern continuum mechanics movement, is always an occasion. There was therefore no hesitation on my part in replying positively when I was given the opportunity to review the above book. However, when I began to read it my enthusiasm gave way to a mild dejection: even the most talented amongst us cannot afford to be self-indulgent or be forgetful of the high standards that we have set others and ourselves. The book is supposed to be an introduction to fluid mechanics for students who are mathematically minded, although the authors emphasize that the requisite mathematical knowledge is not beyond that taught to undergraduates in mathematics departments. The problem is not the mathematics; it is the motivation, which even mathematicians require. But first things first.

The stage is set by the ominous first paragraph of the book:

‘A body  is a set that has a topological structure and a measure structure. It is assumed to be a σ-finite measure space with a non-negative measure μ( ) defined over a σ-ring of subsets of  called subparts of the body. The open sets of  are assumed to be the σ-ring of sets. The members of the smallest σ-ring containing the open sets are called Borel sets of .’

With an appeal to the Radon–Nikodym (R–N) theorem, the mass of  is defined as a Lebesgue integral of the mass density over the volume. But after page 2 all talk of σ-rings, Borel sets, Lebesgue integration and the R–N theorem stops, and so why were they introduced in the first place? The motion of a fluid particle can be described in three ways: in terms of the *material description* $(X, t)$, the *referential description* $(\mathbf{X}, t)$ and the *spatial description* $(\mathbf{x}, t)$. In the first description we track individual particles labelled $X$, in the referential or ‘Lagrangian’ description $\mathbf{X}$ is the location in three-dimensional Euclidean space of $X$ when $t = 0$, while in the spatial or ‘Eulerian’ description $\mathbf{x} = \chi(X, t)$ where $\chi$ describes the motion of  . Naturally, all three descriptions are related. Since the referential description is conceptually simple it is preferred when general theorems are to be proven. But when practical problems involving boundaries are to be solved, the spatial description is preferred and this is what most practising fluid dynamicists use. The point being made here is that on page 3, Truesdell and Rajagopal (TR in future) abandon the material description in favour of the referential system for the rest of the book. If they had done this on page 1, the whole elaborate definition of a body given earlier could have been dispensed with.

The kinematics of fluid motion and the basic physical laws governing such motions are taken up in Chapter 2. Once the *deformation gradient* $\mathbf{F} := \nabla\chi(\mathbf{X}, t)$ is defined in terms of the *motion* $\chi$, kinematics is treated purely as an application of vector and tensor analysis to $\mathbf{F}(\mathbf{X}, t)$ and various fields derived from it. There is virtually no physical interpretation of the resulting fields; it is not accidental that there is not a single diagram or figure in the whole book. The fields that arise include the *stretch, rotation, stretching* and *spin* tensors. But the vorticity $\omega$, on which Truesdell has written a well-known monograph, is just defined, named in quotation marks and then dismissed in favour of the spin tensor $\mathbf{W}$. After a discussion of forces, moments and the important *Euler-Cauchy stress principle* regarding contact forces, TR derive *Cauchy's laws of continuum mechanics*. The first law relates the rate of change of momentum to the forces in play and the second asserts the symmetry of the stress tensor, $\mathbf{T}(\mathbf{x}, t)$.

Although all fluids have to satisfy the physical conservation laws, how any particular fluid will respond to a force field will be determined by the *constitutive equation* that it satisfies. From an operational point of view these equations relate the stress $\mathbf{T}(X, t)$ to the motion $\chi(X, t)$ and have to satisfy certain general principles. In this book, TR restrict themselves to a class of materials called *simple materials* with long-range memory which can exhibit stress relaxation, creep and fatigue. If one makes the further restriction that the material cannot support shear stress when in equilibrium, one gets a *simple fluid*. A simple fluid is by no means simple as it can display a wide variety of complex phenomena; fortunately this class includes the inviscid and Navier–Stokes fluids that we are familiar with. The simple fluid is complex enough that to make progress one has to limit oneself either to special types of flows or to special materials. In the former case, TR discuss in detail Noll's work on *monotonous motions* in which in spite of the ‘detailed and everlasting memory’ of the fluid, one is able to get simple, explicit solutions for the fields. In his article on viscometry in the *Annual Review of Fluid Mechanics* (1974, **6**), Truesdell explains why this happens as follows: ‘The reason is that these particular flows leave the fluid very little to remember, so the subtleties of its memory are given no chance to come into play’.

A sub-class of monotonous motions are the *viscometric flows*, treated in detail in Chapter 5. These flows are important because they are simple to analyse and consequently are frequently used by experimentalists in their attempts to determine material properties. For steady viscometric flows, the stress tensor is related to certain *Rivlin–Erickson tensors* by three viscometric functions $\tau(\kappa)$, $\sigma_1(\kappa)$ and $\sigma_2(\kappa)$ of the shear $\kappa$ alone. In general *normal stress* effects will be found, which are not seen in a Navier–Stokes (Newtonian) fluid. The authors show very effectively that one can be seriously wrong in drawing conclusions about the assumed viscometric functions from one or two viscometric experiments unless one understands clearly how they affect the viscometric flow field. More disturbingly, since the viscometric functions are meaningful only in steady viscometric flows, they tell us little about a general flow of the fluid. This is very different from the situation with Navier–Stokes fluids where a single, constant shear viscosity $\mu = \tau/\kappa$ determines all flows of the fluid.

We recall that the other alternative, if one wants tractable problems, is to specialize the fluid. This is done in Chapters 6 and 7 by restricting the fluid to have *infinitesimal memory*, i.e. to only having short-term memory. In a class of materials of *differential type*, the stress tensor $\mathbf{T}$ is a function of the local deformation gradient $\mathbf{F}$ and its derivatives alone. A fluid of this type which is isotropic and in which **T** depends only on the first two Rivlin–Erickson tensors is called a *Rivlin–Erickson fluid* of complexity 2. Even in this very special case the stress depends on 8 functions other than the pressure and these cannot be determined viscometrically. Only by further restriction of this fluid does one arrive at an incompressible *fluid of grade 2* for which the stress, apart from the pressure, depends on 3 *constants*, $\mu$, $\alpha_1$ and $\alpha_2$ alone. Finally we have a 'non-Newtonian' fluid which can be determined viscometrically! Various detailed examples are considered of the flows of Rivlin–Erickson fluids, at times restricted to grade 2, through straight pipes (including secondary flows), past and against flat plates, between rotating discs, etc. At times matters are a little unsatisfactory because the boundary conditions cannot be specified or cannot be satisfied or there is a multiplicity of solutions. More serious is the conscious policy decision of the authors to ignore thermodynamics in their treatment; a consequence is that there is no explicit equation for the balance of energy. It may be noted that on page 180 it is stated in connection with the weighted energy method '... here "energy" is a mere word, not even defined'. Under these circumstances it is not clear what is really happening when a flow is said to be 'unstable' as in the theorem on page 138.

Further simplification of the constitutive equation leads to the *fluid of grade 1* or the *linearly viscous fluid* or the *Navier–Stokes fluid* or what is commonly called the Newtonian fluid. This is familiar territory and TR nicely cover the Bernoullian theorems and the important issue of dynamical similarity. Exact solutions in respect, among others, of flows past infinite plates, between rotating plates and of idealized jets are discussed. The flow past a *semi-infinite* flat plate, so dear to the hearts of conventional fluid dynamicists, is neither considered nor referred to, presumably because it does not have an exact solution. Serrin's swirling vortex is taken up in great detail; but this important example is complicated enough that the student will have to refer to the original paper for more explanation. Both in this section and the following one on the *stability and uniqueness* of flows, TR have provided much good material for an inquiring student to get a feel and taste for the mathematical issues and techniques that can bear on fluid mechanics. An additional advantage is that citations in the text introduce the student to the modern literature on the mathematical aspects of the subject. If I am not wrong the only reference to turbulence in the book is on page 167 where, in dealing with Serrin's vortex, the phrases 'turbulent flow' and 'kinematic eddy viscosity' appear, with no comment or explanation.

Finally, the *fluid of grade 0* or the *elastic fluid* is the inviscid or ideal fluid of hydrodynamics. While there is a general discussion of the special properties of irrotational flows, the book relegates its single example of a potential flow to an exercise. Although there is a nice discussion of the relationship between the circulation around an aerofoil (not called as such) and the lift on it, I doubt that a person coming across this for the first time will understand its connection to the flight of birds and planes. Rotational Gerstner waves, the Stokes conjecture on waves of greatest height and some aspects of ring vortices are studied in some detail. The non consideration of thermodynamics forces TR, when having to deal with *compressible* ideal fluids, to restrict themselves to ones that are *barotropic*, i.e. ones for which $p = k\rho^{\gamma}$. Although they consider the flow in a converging–diverging channel, they cannot account for the possibility of *shocks*. It is therefore somewhat ironic that the book concludes with a short chapter on singular surfaces, ones across which jumps can occur.

The most positive aspect of this book is its brevity; a large number of topics are covered within the space of a little more than 250 pages. The paragraphs are brief, the sections short and the longest chapter, the one on Navier–Stokes fluids, is about 50 pages long; most others are less than 30 pages. Another positive feature are the exercises scattered throughout the book; solutions are outlined for those in Chapter 5 onwards. The details regarding mathematical derivations are uneven and I found that many were given for what looked to me like simple results and little for others that looked tough. But there is a subjective element in this and so perhaps this difficulty is unavoidable. To a conventional fluid dynamicist the choice of topics, based on exact tractability, may appear to be eccentric. The issue of physical understanding and relevance, on which the authors perforce take an ambiguous stand, is more serious. Even a mathematics student would not be harmed, and may even be benefitted, by being shown some pictures of streamlines, by being told the physical basis for the assumption of certain boundary conditions, and by being told the relationship of a mathematical result to what may be seen in the laboratory or in nature. The finest mathematicians have done this for themselves and their readers, so what need for these pretensions? The embarrassing ambiguity of the authors in these matters is shown, for example, by: (1) the fact that the phrase 'boundary layer', which first appears on page 116, appears in italics on page 132 and again in italics on page 144 and that the half-hearted explanation of the term is so unsatisfactory; (2) the remarkable statement on page 139: 'On the other hand, experimenters usually regard an unstable flow as something that does not occur in nature'; and (3) the strange statement on page 158: 'This class of solutions, if generously interpreted, includes various jets'. It is a pity that by taking the extreme position that fluid mechanics is a branch of mathematics alone, a position that they have not been able to consistently hold, TR have lost the opportunity to write a special, different book that could have positively influenced the rest of the fluid mechanics community. As it stands, I cannot see this book being a good introduction for any student; for a practitioner with patience, it can be an interesting introduction to the mathematical aspects of the subject.

There is a negative feature of the book that will irritate both the mature reader and the student alike. The book is plagued by errors; errors in the index, grammatical errors and plain typos. There is an error in the preface and one on the first page! I do not believe that I have ever seen a technical book with as many errors in it as this one. There are pages with as many as four errors; even the error function defined on page 189 is wrong in at least two ways. I wonder if the book has been proof read at all. Truesdell was a lover of books with a classical scholar's regard for perfection. What has happened here?

I cannot think of many better things one can do on a free evening than sit