Quantum-no-deleting principle

In a recent scientific correspondence[1] Bhagwat *et al.* fielded baseless arguments that the quantum-no-deleting principle[2] is untenable. We refute and reject their criticism and remind interested readers that the quantum no-deleting principle is a *fundamental* consequence of linearity of quantum theory. At the outset, let us point out that if one could delete an unknown state against a copy (as Bhagwat *et al.* would have us believe) then one could send superluminal signals using non-local resources[3], thereby violating one of the basic principles of special theory of relativity[4].

Quantum information theory explores the potential vastness of information contained in an unknown quantum state. What we can do and what we cannot do with the largely inaccessible quantum information contained in an unknown state is a topic of fundamental importance. An unknown qubit (two-state system), for example, contains doubly infinite bits of information[5] because its specification requires two real numbers on the Bloch sphere. The knowledge of a quantum state plays a crucial role in information processing. For example, if we *know* a state we can copy it perfectly but an *unknown state* cannot be copied[6]. If one could copy an arbitrary state, then by using EPR entangled states one can send signals faster than light[7]. Recently, we have asked: if some one (who knows the state) prepares two (or more) copies of a qubit and gives us (who do not know) to delete a copy keeping the other intact, can we do so? The answer turns out to be 'no' – a fundamental consequence of linearity of quantum theory. However, if we know a state or if we have copies of qubits in orthogonal states then we can delete a copy perfectly because orthogonal states carry classical information.

The quantum deleting machine defined in ref. 2 is a *linear operator* (not necessarily unitary) that acts jointly on two copies of a qubit and ancilla and transforms the composite state

$$|\psi\rangle|\psi\rangle|A\rangle \rightarrow |\psi\rangle|\Sigma\rangle|A_\psi\rangle, \qquad (1)$$

where $|\Sigma\rangle$ is the blank state, $|A\rangle$ and $|A_\psi\rangle$ are initial and final states of the ancilla. Here, ancilla can be the deleting machine itself or any other quantum system external to the input copies of the qubit. In our paper we have excluded swapping as a proper deletion of quantum information. Here, when we say swapping we mean not only ordinary swapping which takes $|\psi_1\rangle|\psi_2\rangle \rightarrow |\psi_2\rangle|\psi_1\rangle$ but also generalized swapping such as swapping up to a unitary operation which takes $|\psi_1\rangle|\psi_2\rangle \rightarrow |\psi_2\rangle U|\psi_1\rangle$. Since swapping is excluded in (1) the final state of ancilla $|A_\psi\rangle$ should not have $|\psi\rangle$ hidden in a subspace of its available Hilbert space. We have shown that though one can delete a copy of qubit in orthogonal states, one cannot delete an unknown copy. In other words, there is no linear operator which can perform deletion for arbitrary states with final state of ancilla being independent of the input state. Since our proof uses only linear operator and *not* unitary or Schrödinger evolution the result is valid for reversible as well as irreversible operations.

Here we refute the doubts raised by Bhagwat *et al.* In ref. 1 the authors say that the sought after transformation[1] is indeed possible. Recall as we have stated, the transformation[1] for unknown states should only exclude moving the copy wholly into a subspace of the ancilla. (After all, if a house maid hides the dirt beneath the carpet instead of cleaning it, this operation cannot be considered as an act of cleaning.)

Further, these authors say that 'there is a sudden concern about swapping and Pati and Braunstein (PB) identify swapping with erasure'. This is not a fair representation. We clearly say that swapping a qubit with a standard state and then *dumping* it into environment can be regarded as erasure of the last qubit. Thus swapping in conjuction with dumping is an irreversible operation though swapping itself is a reversible operation. Contrary to what these authors have attributed, our paper *never identified swapping alone with erasure*.

In para 6, these authors argue that orthogonality of the final state of ancilla does not imply swapping. The example they cite is also a *swapping up to a unitary operator*. Since Hilbert space is invariant under local unitary operations, this property must be utilized in considering generalized swapping. When the authors of ref. 1 say that they have constructed quantum deleting machines, what they have actually done is constructed only *swapping machines*! That is, up to a unitary transformation on the ancilla subspace alone, they perform a simple swapping operation. No deleting has occurred, merely one copy of the original state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, has been mapped into the final state of the ancilla as $|A_\psi\rangle = \alpha|A_0\rangle + \beta|A_1\rangle$. Except for a re-identification of the basis states (otherwise known as a unitary transformation) the state is wholly moved to the ancilla subspace.

In para 7, these authors say that the quantum-no-deletion principle has not been proved for irreversible operations, in spite of their (PB's) restricting to uncopying through Schrödinger evolution! Surprisingly, nowhere in our original paper[2] did we mention that we restrict to *Schrödinger evolution*. Bhagwat *et al.* make comments on unspelt issues of our paper! As we have stated in the beginning, our proof only assumes linearity which is broader than mere unitary Schrödinger. Further, these authors saying that our discussion is 'high-sounding' is pure rhetoric on their part and should be seen as such by thoughtful readers.

Finally, in para 8, these authors conclude with an erroneous statement that 'reversible as well as irreversible deletion of known or unknown quantum state should always be possible'. Quantum deletion is indeed possible for known state. But the understanding of deletion of an unknown state only points at the nature of conceptual difficulties in comprehending quantum information theory. If one can delete an unknown state then many consequences will go heywire in quantum world (like faster than light communication). There is a very lucid explanation about our quantum no-deletion principle of unknown states by Zurek[8] using only unitary operations. As pointed out in ref. 8 quantum no-cloning and no-deleting principles really touch the heart of quantum mechanics.

1. Bhagwat, K. V. *et al.*, *Curr. Sci.*, 2000, **79**, 1161.
2. Pati, A. K. and Braunstein, S. L., *Nature*, 2000, **404**, 164.
3. Pati, A. K. and Braunstein, S. L., LANL arXiv, quant-ph/0007121.
4. Svetlichny, G., LANL arXive, quant-ph/9511002.
5. Jozsa, R., *Geometric Issues in Foundations of Science* (eds Huggett, S. *et al.*), Oxford Univ. Press, Oxford, 1997.
6. Wooters, W. K. and Zurek, W. H., *Nature*, 1982, **299**, 802.
7. Dieks, D., *Phys. Lett.*, 1982, **A92**, 271.
8. Zurek, W. H., *Nature*, 2000, **404**, 130.

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MEETINGS/SYMPOSIA/SEMINARS

Workshop on Recent Trends in Landslide Assessment and Monitoring

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