On the problem of turbulence

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A central theme in the history of the turbulence problem is about the method of 'closure' in the models and 'theories' which have been proposed. Closure has invariably been by empirical calibration with experimental data. In this note we draw attention to a paper by Morris, Giridharan and Lilley, in which for the first time empiricism is obviated. For the turbulent mixing layer, this is accomplished by including in its description the mechanism for production of turbulent shear stress (i.e. turbulent momentum transfer), by large-scale instability waves. Some implications for the theory of turbulent shear flows are discussed.

IN the list of labels that attempt to describe the century just ending, one might include 'The Century of the Problem of Turbulence'. The 'problem' had been identified by the end of the 19th century. In Horace Lamb's classic monograph, *Hydrodynamics* (1906, Third Edition), he introduces Article 365, on Turbulent Motion, with the sentence, 'It remains to call attention to the chief outstanding difficulty of our subject'. That statement is appropriate a century later and this note is to call attention to some progress. But first some background.

As a practical matter, the 'difficulty' in the latter half of the 19th century was found mostly in hydraulic engineering design. Pressure-flow relations in hydraulic pipes and conduits were quantitatively quite different from those predicted by theory, e.g. the Hagen–Poiseuille relation. Hagen in 1839 noted the appearance of 'peculiarities' as flow magnitude increased. To make such formulae work for them, engineers introduced 'apparent' coefficients of viscosity having values different (higher) than the actual coefficient of viscosity of water. Increasing with pipe size and flow velocity, such coefficients were simply empirical factors that allowed experience to be organized for further use. Even today one might find an investigator, attempting to model flows at atmospheric scales, e.g. a tornado, introduce for 'eddy viscosity' a value of 10^2 m^2/s, which is about 10^6 times the value of the physical kinematic viscosity of air.

By the end of the 19th century it had been clearly recognized that the problem was connected with the fact that the theoretical result is for smooth, *laminar* flow while the flow in large-scale applications is *turbulent*. The phenomenon had already been named in the 16th century by Leonardo da Vinci, whose sketches and notes on it are well known. The *transition* from laminar to turbulent flow in a pipe was demonstrated by Osborne Reynolds, in 1883, in a paper in which he also introduced the dimensionless parameter *ñUD/ì* as the similarity parameter, which allowed experiments at different scales to be correlated. It is now called the Reynolds Number, *Re*.

But even before Reynolds' experiment, it was recognized that turbulent flow results from *instability*, and scientists such as Helmholtz, Kelvin and Rayleigh initiated the discipline of flow stability theory. Starting with the stability of parallel flows, the theory and experiments have been extended to a variety of configurations. As noted in the book by Betchov and Criminale, the 'manifestations of instability' could be grouped, roughly, into three categories: (i) oscillations in nearly parallel flows, such as pipe flow, boundary layers, jets and wakes; (ii) flows with curved streamlines, as in Couette flow between concentric rotating cylinders, in which cellular as well as unsteady motions develop; and (iii) flows in which the reference, mean velocity is zero, as in convection of heat between two surfaces at different temperatures, where instability also results in cellular motion. An important 'manifestation' of instability is the turbulence that develops when the value of *Re* continues to be increased beyond the critical value.

The *transition* from initial instability to turbulence (as some parameter, usually *Re*, is varied) appears the most difficult to understand and describe. The corollary of that viewpoint is that the 'fully developed turbulent' motion which follows is simpler, in some sense. The developing instability is abandoned and the turbulence is now viewed as a *state*, which may be simpler to define and model. That view has been dominant in the two main trends that characterize most of turbulence research during the century. One is the search for models of the Reynolds Averaged Navier–Stokes (RANS) equations first derived by Reynolds, which address the mean-flow quantities and to which we will come presently, and the other is the so-called statistical theory of turbulence which seeks statistical descriptions of the turbulence itself, mainly through correlations of fluctuating velocity, in anticipation, of course, of ultimately contributing to the complete problem.

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This article is dedicated to Satish Dhawan on his eightieth birthday. I have long admired him for his contributions and dedication to science, teaching, technology, statesmanship and humanity.

It is useful to examine several aspects of the 'problem of turbulence' and what one might expect of a 'solution'. For a start, it may be noted that the turbulent flow is described by the unsteady form of the Navier–Stokes equations, which accurately apply to turbulent motion of Newtonian fluids such as water and air. But the complete detail provided is excessive. For many applications one needs only the *mean* flow field which, of course, will exhibit the effects of enhanced transport by turbulence in terms of growth rate, entrainment, etc. To describe this is the main goal in RANS modeling, indeed of turbulence theory.

Of course, it is also desirable and, in many cases, important to also describe the fluctuating motion. In fact one can argue that it is not possible to model the mean flow field without somehow incorporating the turbulent motion itself. That statement must certainly be true for a rigorous model but if any empiricism is allowed (even one constant!) then considerable organization of some turbulent flows can be achieved by *similarity* and *scaling* arguments alone. For example, for the canonical class of turbulent-free shear flows, which are assumed to have developed an equilibrium, self-similar state, a fitted constant for each flow does quite a good job of correlating the growth rate with the entrainment rate and the mean shear stress. One way to accomplish this scaling is to apply it to the eddy viscosity, í_T. Identifying a velocity scale, U, and a thickness scale ä, the velocity distribution is assumed to have a similarity form $u(y)/U = fn(y/\text{ä})$ and the eddy viscosity $\text{í}_T = \text{const}U\text{ä}$ then carries the constant which has to be fitted. Similarly, for turbulent pipe flows and boundary layers, 'mixing length theory' led to the logarithmic law, which does a good job of fitting the region close to the wall in both flows. Although the term 'mixing length' is reminiscent of turbulence, the model is based on scaling arguments, helped by experimental data, and it contains two constants that are fitted to the data. Its apparent universality gives it an appeal, which typically characterizes hopes for modelling of the RANS equations. However, rationalization of 'universal' but empirical constants like the Kármán constant $\hat{e}$, in terms of the underlying mechanics, is elusive.

The other direction in turbulence research, sometimes called the 'statistical theory', describes properties of the fluctuating flow field. Introduced in the 1930s by G. I. Taylor, it seeks equations and relations for correlations of the velocity fluctuations, i.e. a statistical description of the turbulence. A notable success was Kolmogorov's concept of a 'universal inertial range' for a limited range of the correlations at small scales. Again, this makes use of scaling and similarity arguments and introduces the 'fundamental' Kolmogorov constant. These and other more recent efforts, such as the theory of dynamical systems, have provided many insights into turbulence. But the original 'problem of turbulence', to account in a satisfying, nonempirical theory for the enhanced transport in turbulent shear flows, had remained out of reach.

The main purpose of this note is to bring attention to a paper by Morris *et al.*[1] (henceforth MGL) which unites the two tasks outlined above. It describes, without fitted constants, the mean flow properties of turbulent-free shear layers as well as the energy containing scales of the turbulence. This is accomplished by explicitly incorporating the instability mechanism, which produces and continually renews the turbulence. While the mixing layer is probably the simplest one for a successful application of the idea, because of its strong underlying Kelvin–Helmholtz instability, the result is very significant. The much noted problem of *closure* has been accomplished, for the first time, by introducing into the RANS scenario the physical mechanism which drives the flow!

The stage for the emergence of this model was set during the past quarter century with the realization that instability waves, and resulting 'coherent structures', are dominant features of mixing layers even beyond the initial transition region, i.e. at values of Reynolds number for which the flow is 'fully developed turbulent' (Figure 1). Various investigators had noted a connection between growth rates and instability amplification rates from basic linear stability theory, including nonlinear effects from instability cutoff, but it was left to MGL to ingeniously incorporate these ideas into a model that requires no calibration. For this class of flows it is a theory, without empiricism, for the first time. The principle can perhaps be extended to a broader class of turbulent shear flows.

Before briefly outlining the essence of the MGL theory, which was developed for plane mixing layers, we review the generally well-known equations that are applicable.

The equations

The RANS equations for nearly parallel flow, with mean velocity field (U, V) in (x, y), but admitting three-dimensional velocity fluctuation (u', v', w'), are the continuity equation

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{1}$$

and the momentum equation

$$\frac{\partial}{\partial x}(\rho U^2) + \frac{\partial}{\partial y}(\rho V U) = \frac{\partial}{\partial y}\left(\mu \frac{\partial U}{\partial y} - \rho \overline{u'v'}\right). \tag{2}$$

Figure 1. Mixing layer with uniform density, $U_2/U_1 = 0.38$ and $R_e = 0.5 \times 10^6$, based on $(U_2 - U_1)$ and length visible in the picture.

Apart from some refinements which include the correlation $\overline{v'^2}$, this is the basic set of equations relating the mean flow to the turbulent velocity correlation $(-\rho\overline{u'v'})$, which is also called the 'Reynolds stress' because it acts in the momentum equation in the same way as the Newtonian stress τ = ì$\mathrm{d}U/\mathrm{d}y$, i.e. to transport momentum across the flow. But, unlike τ, it cannot be rigorously related to the local velocity gradient, or to any other local property of the mean velocity field. It is, basically, the problem of *closure* for the RANS equations. In free turbulent shear flows, the Newtonian stress is negligible for high values of Reynolds number; those flows are therefore independent of viscosity, and considerable simplifications result. For a mixing layer (Figure 2) for which initial conditions are also negligible, it implies that the mean values and the correlations are functions only of the similarity variable $\eta = y/x$; the shear-layer growth $\delta(x)$ is linear; and the Reynolds stress $\rho(-\overline{u'v'})$ has its maximum and constant value on the zero streamline $\psi^* = (0)$, defined by y^*/x = const. (All other streamlines entering the mixing layer from either side are not straight. For suitable choice of the far-field boundary conditions, $y^* = 0$.)

As shown by von Kármán for a boundary layer next to a wall, integration of the momentum equation over the layer, in the y-direction, and taking into account the continuity relation, results in the integral relations,

$$\rho(-\overline{u'v'})^* = \frac{\mathrm{d}}{\mathrm{d}x}\int_{y^*}^{\infty} \rho U(U_1 - U)\mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{y^*} \rho U(U - U_2)\mathrm{d}y. \tag{3}$$

The statement here is that the stress $\rho(-\overline{u'v'})^*$ on the zero or 'dividing' streamline is equal to the rate of loss of momentum flux above $\psi^* = 0$ and to the rate of gain of momentum flux below $\psi^* = 0$. Momentum is exchanged between the faster, upper flow and the slower, lower flow by the action of the Reynolds stress. The integral relation can be put in dimensionless form by introducing the similarity variable $U/U_1 = f(y/\delta)$, where $\delta(x)$ is some measure of the scale (thickness) of the mixing layer. Simplifying for uniform density, the result is

$$\frac{(-\overline{u'v'})}{U_1^2} = \frac{\mathrm{d}\delta}{\mathrm{d}x}\int_{\eta^*}^{\infty} f(1-f)\mathrm{d}\eta = \frac{\mathrm{d}\delta}{\mathrm{d}x}\int_{-\infty}^{\eta^*} f(f - U_2/U_1)\mathrm{d}\eta. \tag{4}$$

This exhibits the direct relation between Reynolds stress and growth rate. The two integrals have the same numerical value I_{M}, which depends on the shape of the velocity profile.

In addition to the basic RANS equation for momentum, one can obtain two important energy equations, for the mean kinetic energy $K = \frac{1}{2}(U^2 + V^2)$, per unit mass, and for the turbulent kinetic energy $k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$. An integral of the mean kinetic energy equation, analogous to that for the momentum, is

$$\dot{E} = \frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{y^*} \rho U\left(\frac{1}{2}U^2 - \frac{1}{2}U_2^2\right)\mathrm{d}y + \frac{\mathrm{d}}{\mathrm{d}x}\int_{y^*}^{\infty} \rho U\left(\frac{1}{2}U^2 - \frac{1}{2}U_1^2\right)\mathrm{d}y$$

$$= -\frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty} \frac{1}{2}\rho U(U_1 - U)(U - U_2)\mathrm{d}y < 0. \tag{5}$$

The first two integrals represent gain of kinetic energy in the lower and upper parts of the layer, respectively. The first one is evidently positive and the second one, negative. But unlike the case for momentum, the two

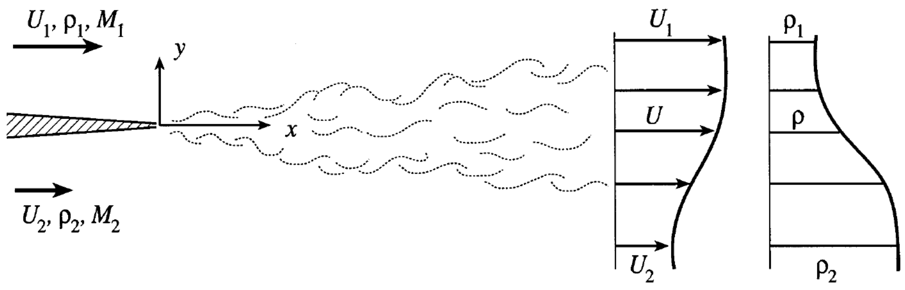

Figure 2. Plane mixing layer.

integrals do not balance each other; the mean kinetic energy is not conserved but is decreasing, as confirmed by the second equation, which is obtained after some rearranging and making use of the integral momentum equations. An alternative derivation gives

$$\dot{E} = -\int_{-\infty}^{\infty} \rho(-\overline{u'v'})\frac{\partial U}{\partial y}\mathrm{d}y - \int_{-\infty}^{\infty} \mu\left(\frac{\partial U}{\partial y}\right)^2 \mathrm{d}y. \tag{6}$$

The loss of mean kinetic energy from the incoming flow is generated from the product of Reynolds stress and mean velocity gradient in the shear layer, analogous to that from the viscous dissipation term,

$$\left(\mu\frac{\partial U}{\partial y}\right)\frac{\partial U}{\partial y}.$$

As in the momentum equation, the viscous contribution is negligible at high values of *Re*. The Reynolds-stress term also dissipates mean-flow energy, but does not (immediately) convert it into heat, but rather into turbulent kinetic energy, as becomes evident from the following equation for k.

The integral relation for the rate of increase of turbulent kinetic energy is

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}(\rho U k + \overline{p'v'})\mathrm{d}y$$
$$= \int_{-\infty}^{\infty} \rho(-\overline{u'v'})\frac{\partial U}{\partial y}\mathrm{d}y - \int_{-\infty}^{\infty} \mu\,\overline{\frac{\partial u_i'}{\partial x_j}}\,\overline{\frac{\partial u_i'}{\partial x_j}}\,\mathrm{d}y. \tag{7}$$

(Convection terms involving correlations like $\overline{u'k'}$ have been omitted.) The first term on the right hand side is the same as in (eq. (6)) but with changed sign; it now contributes to increase of turbulent kinetic energy k and is, appropriately, called the '*turbulence production*' term. The last term, contributing to decrease of k, accounts for viscous dissipation by the turbulent, fluctuating velocities and is not negligible, because the gradients may be very large. In fact, it is thought that at high Reynolds number the gradients increase as ì decreases so as to keep the dissipation term constant and thus independent of *Re*; this accommodates the message from the momentum equation and from experience, that free turbulent shear flows are independent of ì at high *Re*. This viscous dissipation term is an important component in approaches to traditional RANS modelling, e.g. in the popular *k–å* model.

Like the momentum equation, the energy equations can be written in non-dimensional forms. In these the growth rate appears as coefficient of integrals whose numerical values can be defined by choosing a shape for the velocity profile $U/U_1 = f(y/ä)$. That is, the growth rate is proportional to various other quantities that are proportional to the turbulence intensity, in particular the Reynolds stress, the rate of dissipation of mean kinetic energy and the rate of production of turbulent kinetic energy. Entrainment rates can be similarly correlated. Thus the various effects of turbulence in this self similar flow are obtained from the fitting of a single constant. But in that constant is the essential mechanics of the turbulence, which so far has not been put in. That closure is what the MGL model accomplishes, by recognizing that the development of the flow is dominated by large-scale structures like those in Figure 1, which can be 'described by a superposition of instability waves'. As had been noted by various investigators, the local properties of these structures are described well by linear stability theory and the parametric dependence of spreading rate correlates well with maximum amplification rates from linear theory. Those properties are used by MGL to model the turbulence production term that appears in both energy equations. Non-linearity comes from the effects of shear-layer growth.

The MGL model

The paper by Morris *et al.*[1] is comprehensive, including effects of non-uniform density as well as compressibility. To simplify and shorten the presentation here, we restrict it to homogeneous, incompressible flow and make use of the integrals from the preceding section, some of which differ in form from those used by MGL.

The only parameters remaining from Figure 2 are U_1 and $r = U_2/U_1$.

The velocity profile shape f(ç; r) in the integrals is assumed to be a tanh function. This is also the profile used to calculate local values, at ä(x), of the eigen values and eigenfunctions for excitation frequencies ù and â (i.e. spanwise modes are included). The amplitudes of the instability waves are calculated, simultaneously with ä(x) as the shear layer develops, from the equations that will be noted presently follow and from a linear, inviscid stability equation. A spectrum of excitation frequencies is made available.

In the turbulent energy eq. (7) the integrands contain squares or products of fluctuating velocities, which are proportional to the square of the amplitude, A, for each developing wave. Therefore the dimensionless, self similar form of the turbulent energy eq. (7) can be written

$$\frac{\mathrm{d}}{\mathrm{d}x}(\delta A^2 I_k) = A^2 I_{\mathrm{P}}, \tag{8}$$

where I_k is the flux integral and I_{P} is the production integral. Their numerical values, which depend on the eigen functions, do not in fact have to be evaluated!

A 'normalization' sets $I_k = 1$ and eq. (8) may be displayed as

$$\frac{\mathrm{d}A^2}{\mathrm{d}x} = \frac{A^2}{\delta} I_{\mathrm{P}} - A^2\frac{\mathrm{d}\delta}{\mathrm{d}x}. \tag{9}$$

For exactly parallel flow, dä/dx = 0, the equation is compared with the relation for growth of amplitude from linear stability theory,

$$\frac{\mathrm{d}A^2}{\mathrm{d}x} = A^2(-2\alpha_{\mathrm{i}}),$$

where á$_{\mathrm{i}}$ is the imaginary part of the wave number and is negative for positive growth. This is used to make the identification

$$I_{\mathrm{P}} = (-2\alpha_{\mathrm{i}})\delta \tag{10}$$

and then eq. (9) takes the form

$$\frac{\mathrm{d}A^2}{\mathrm{d}x} = A^2(-2\alpha_{\mathrm{i}}\delta) - A^2\frac{\mathrm{d}\delta}{\mathrm{d}x}. \tag{11}$$

A second equation relating $A^2(x)$ and ä(x) is obtained from the mean energy eq. (6), i.e.

$$I_E\frac{\mathrm{d}\delta}{\mathrm{d}x} = A^2 I_{\mathrm{P}},$$

where

$$I_{\mathrm{P}} = \frac{1}{2}\int_{-\infty}^{\infty} f(1-f)(f-r)\mathrm{d}\eta.$$

With I_{P} identified in eq. (10), the energy relation is finally reduced to

$$\frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{2}{I_E}A^2(-2\alpha_i\delta). \tag{12}$$

Equations (11) and (12) are a coupled set of equations for the simultaneous development of $A^2(x)$ and ä(x) for a given wave responding to frequencies (ù, â). The growth of the shear layer depends on energy transfer from all spectral components to which it is responding; the total effect is summed symbolically in the two equations (2.22 and 2.23 in MGL),

$$(\mathrm{d}/\mathrm{d}x)A^2(x;\,\omega,\,\beta) = -A^2(x;\,\omega,\,\beta) \times \{2\alpha_i(\omega,\,\beta) + \delta^{-1}(\mathrm{d}\delta/\mathrm{d}x)\},$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}x} = -\frac{4\delta}{(1-r)^2[rI_3 + (1-r)I_4]} \times \int_0^{\infty}\int_{-\infty}^{\infty}\alpha_i(\omega,\beta)A^2(x;\omega,\beta)\,\mathrm{d}\beta\ \mathrm{d}\omega.$$

but the actual calculations are made on a set of $N + 1$ equations for each set of N waves in the spectrum. (The integrals I_3 and I_4 are different from those in our simplified presentation.)

The set of equations and integrals developed by MGL accommodate non-uniform density as well as compressibility, hence are not quite as simple as shown here. To calculate a developing shear layer, for given parameters, U_2/U_1, ñ$_2$/ñ$_1$ and M_1, the linearized inviscid stability equations (Rayleigh; Lilley) are used to solve for the eigenvalues and eigenfunctions for a range of local frequencies ùä and âä, (i.e. scaled with local ä), thus describing a 'weakly nonlinear' development. The excitation spectrum at $x = 0$ is flat, with amplitude 0.01. From the eigen solutions, only the á$_i$ are needed for solving eqs (11) and (12). The eigenfunctions are used separately, for defining the spectrum of fluctuating velocities in (ù, â) coordinates. It is found that streamwise instabilities (ù) rapidly become dominant, i.e. the motion tends strongly to be two dimensional.

From the solutions for $A^2(x)$ and ä(x), results for the dependence of dä/dx on U_2/U_1, ñ$_2$/ñ$_1$ and M are obtained and found to agree well with experimental data. From the eigenfunction part of the solution, good agreement is found for the spectral distribution in the low-frequency part of the spectrum, which simply does not extend to higher frequencies than those that are selected by the developing equations. Comparisons of the distributions of turbulent correlation, including Reynolds stress, which could be obtained from the eigen solutions are not presented. The comparisons made are impressive, recalling once more that the model contains no empirical constants.

Observations

The results outlined above may seem astonishing, perhaps improbable, to many who are schooled in the traditional theories of turbulence. 'Where is the turbulence, the chaos, the small scales?' one might ask. The turbulence is in the large-scale wave packets, whose distribution is not deterministic, because of the broad spectral forcing, while chaos from three dimensionality and small structure is absent. The turbulence seems to be minimal. Indeed, the theory accords with a maxim attributed to Einstein, that 'everything should be as simple as possible but not too simple'. That is, it should not omit the essential, which here is the production of Reynolds stress by instability waves. This essential has been missing from models of turbulent shear flow; without it the problem of 'closure' seems bound to remain empirical. How the principle can be implemented in other shear flows is a separate (but not trivial) matter. Perhaps, quoting J. E. Broadwell, simplicity of the mixing layer makes it the 'hydrogen atom' of turbulent shear flows. For example, in plane jets and wakes spanwise instabilities are as strong as the streamwise ones, hence the resulting primary instability is three dimensional. Correspondingly, in axisymmetric jets and wakes the primary instability is helical, not axisymmetric, so the implementation will be more difficult in practice if not in principle. For boundary layers and other wall-bounded flows it will be necessary to include the coupling with the wall region, where viscosity and small scales do participate in the development of the layer.

It is interesting to contemplate the implications of the results and how they may relate to some of the axioms of turbulence research. One of these is the view that, although free turbulent flows 'are independent of Reynolds number', the viscous dissipation must be included in modelling them, as already mentioned above. This was not necessary for the MGL model, neither was it necessary to include small scales. MGL argue that the development of the large scales and the growth of the shear layer are not sensitive to the details of energy transfer to small scales and, ultimately, to heat, hence omission of the viscous term in eq. (7). They describe alternative, simpler procedures to account for energy that must be dissipated. In fact, the result conforms with the cascade description of spectral distribution of energy and its dissipation, first by small-scale turbulence and then by viscosity, and it indicates that this two-stage dissipation mechanism does not impact the large-scale momentum exchange. The view that models must explicitly include the dissipation scenario is influenced by the Eulerian point of view, in which *local* dissipation appears equally important with *production*. But a more appropriate view is a Lagrangian one: the momentum exchange is local but the dissipation of the energy which is lost in the process need not occur at the same place; it need not even appear in thermal motions but could remain in the layer as small-scale turbulent motion, as noted by Onsager many years ago. In the Eulerian, local energy balance, the viscous dissipation term accounts for energy lost in earlier momentum exchanges; if viscosity were identically zero it would not appear at all and then some terms in the energy balance would look different, with the dissipation term missing and others, e.g. convection of turbulent energy, changed to exhibit the Eulerian balance.

Another feature is that motions induced by spanwise instabilities are not important in the model, nor are other manifestations of three-dimensionality. The importance attributed to three-dimensionality, vortex stretching, etc. in turbulence may be relevant to the dissipation mechanism but not to the primary, stress producing components.

Still another implication to be considered relates to the coupling of the turbulence to the external forcing. Inviscidly unstable, the shear layer responds convectively to the smallest amplitudes of forcing, but that forcing must have a broad spectrum (ideally continuous and flat) if the layer is to grow linearly as postulated in dimensional/similarity analysis and as observed in the laboratory. If the broad spectrum is not available or if its amplitude is too small the layer will respond to available tones or narrow bands, provided they override the background. This effect and the resulting nonuniform development of $d\ddot{a}/dx$ was first demonstrated by Oster and Wygnanski. An implication of all this is that turbulent theories which seek a kind of local, constitutive description, universal and independent of the particular flow, are not likely to be successful. The MGL model provides an alternative *methodology*.

Concluding remark

The MGL theory brings together the two trends in turbulent research mentioned earlier, i.e. to account for the greatly enhanced transport and to describe the turbulence. The first is not separable from the second but, for free turbulent shear flows, only the large-scale energetic part of the turbulent structure is needed.

1. Morris, P. J., Giridharan, M. G. and Lilley, G. M., *Proc. R. Soc. London*, 1990, **A431**, 219–243.

ACKNOWLEDGEMENTS. This article is not a review and explicit references are not provided except for the paper on which the discussion is focused. During the last quarter century many investigators have made significant contributions to the ideas that have developed toward that focus. This writing has benefited from many discussions with J. E. Broadwell.