Direct numerical simulations of spots

Joseph Mathew[†] and Arup Das*

Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560 012, India
*Present address: Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana IL 61801, USA

Although many properties of turbulent spots have been determined by laboratory measurements and flow visualization, there still remain unanswered questions about the mechanisms involved in the spreading of spots, the role of instability waves, and the structure of the turbulent flow within and the non-turbulent but modified flow outside spots. Direct numerical simulations offer detailed, well-resolved data which could reveal these mechanisms. A few such simulations have been done in flat plate boundary layers, in channel flows (Poisueille and Couette flows) and in pipe flows. Although one study has examined the growth and breakdown of instability waves at the edges and another has computed particle trajectories to determine where fluid is entrained, most of these studies have only reached the stage of establishing the correctness of the simulations by comparisons with features observed in laboratory flows. However, it is a reasonable expectation that in the near future, simulation studies will begin to make definitive statements about mechanisms of spot growth and propagation.

Introduction

Spots are isolated regions of strong fluctuations within a laminar shear flow. These fluctuations have properties which are close to those of the corresponding turbulent flow. They travel downstream, grow in size, and coalesce with neighbours to form extended regions of turbulence. Thus, spots are an essential feature of transitions to turbulence in several common situations such as flows past bodies.

Spots were first reported by Emmons[1] who observed them in a thin layer of water flowing down an inclined plate. He anticipated correctly that they would appear in other wall boundary layer transitions also. In fact, spots appear in channel flows and even in pipes where they have been called puffs. Spots may be created readily, and many properties such as propagation speeds, spreading rates, average shapes and mean velocity fields have been measured. And, an economical model due to Narasimha[2] based only on the appearance and propagation of spots has provided accurate answers to the practical question of predicting surface shear stresses in the transition zone. This intermittency model

for the transition zone, supported by experiments at IISc and elsewhere appears in the definitive work of Dhawan and Narasimha[3]. It is also, perhaps, the best known and earliest contribution arising from the transition studies begun by Dhawan at IISc.

Several intriguing questions regarding essential processes in spots – growth, spreading and interaction with the surrounding laminar flow still remain unanswered. Lateral spreading rates are strikingly different from those for spreading of turbulent–nonturbulent interfaces in common turbulent flows. Spots are also peculiar because fluid can traverse, in turn, a laminar region, enter the turbulent region comprising the spot and exit into another laminar region. This makes the turbulence in the spot a compact, wave phenomenon (perhaps, partially). Usually, fluid which has been entrained into a turbulent region from nonturbulent surroundings, such as in a burning gas jet, becomes turbulent and remains within the turbulent region. Relaminarization is a slow process. An attempt to understand these issues, and perhaps some aspects of turbulence at interfaces, has led us to conduct numerical simulations of these islands of turbulence.

In what follows, a review of numerical simulations of turbulent spots is offered. There have been several transition computations which have examined the initial growth of special disturbances. Such studies which have examined only the early stages are excluded from this review. Only those that deal with the dynamics of mature spots will be considered. A review of numerical simulations of transition in wall-bounded flows has been available[4]. There is very little overlap here because most spot simulations have appeared later.

Numerical methods for spot simulation

Numerical solutions of turbulent flows are difficult to obtain because of the wide range of length and time scales that are always present. Turbulent flows are also strictly three-dimensional. Spot simulations present the same difficulties and, so, require the special methods that have been developed recently for turbulence simulations called Direct Numerical Simulation (DNS). DNS has the connotation that a turbulent flow is being simulated without any modelling of turbulence, unlike all other calculations of turbulent flows. So turbulence itself can be studied. The objective then is not to obtain quantities such as mean velocities for complicated

---

[†]For correspondence. (e-mail: joseph@aero.iisc.ernet.in)

flows, which can also be obtained from experiments, or often predicted from models, but to understand the turbulent flow (see ref. 5 for a recent review of DNS). In every case the Navier–Stokes equations are integrated from appropriate initial conditions. The flow undergoes transition to the required turbulent flow, or relaxes from a disturbed non-physical flow to turbulence.

The range of scales of motion that must be resolved in a DNS increases with the Reynolds number $Re = UL/v$ where U is the flow speed, L is a relevant length and v is the fluid's kinematic viscosity. For given computational power, this places a limit on the highest Reynolds number flow that can be simulated. So, most simulations have been at Reynolds numbers which are well below those of industrial flows. But the intent is still insight, and DNS results, even at low Reynolds numbers, have been illuminating. It turns out that some of the limitations of DNS such as low Reynolds numbers and small spatial extents are acceptable characteristics of flows with spots.

The essential requirement for DNS is a method which provides adequate spatial resolution and maintains accuracy over a large number of time steps. Most DNS have used Fourier-spectral methods because the increased accuracy (truncation error falls exponentially fast with increasing resolution) does not come with significantly larger computation costs; fast transform methods are available. Fourier methods are especially suited because differentiation is *exact* for all resolved wave numbers. Special difficulties such as aliasing and resulting numerical instability can be handled easily, but there is an implied periodicity in the solutions. The attendant concerns about interactions between image flows have to be relieved by, at least, empirical demonstrations of insensitivity to period in the solutions. Recently, finite difference/finite volume schemes have also been used for DNS of non-periodic or spatially developing flows, or for compressible flows.

Spots in flat plate boundary layers

Many characteristics of spots have been studied in the boundary layer flow over laboratory flat plates. In planform, the spot occupies a region shaped like an arrowhead pointing downstream, and the leading edge is lifted up (has an overhang). Its height is more than the boundary layer thickness and, so, a portion lies in the outer freestream flow. The leading and trailing edges propagate at about 90% and 50% of the free-stream speed, respectively. The difference implies a growth, and the lateral spreading is at a half-angle of about 10°. This lateral growth rate is larger by an order of magnitude compared to the growth rate in the direction normal to the wall which, like turbulent boundary layers, is about 1°. So the mechanism ought to be different. The velocity profile in the spot wake resembles that of thin-

ner boundary layers that obtain in the more stable accelerating flows. So, the region traversed by the spot turns out to be more stable and has been termed a 'calmed region'[6]. Although this is consistent – if the wake did not suppress fluctuations, the spot would be a turbulent front rather than a compact region – the mechanism involved is not known. The boundary layer spot is particularly interesting because an upper portion lies in the free-stream where the surrounding flow is irrotational, whereas the lower region grows into rotational flow. It is therefore more difficult to construct a physical model which matches the two different growth mechanisms.

Analyses of spot simulations have been aimed at clarifying growth mechanisms, especially lateral spreading mechanisms, and the role of Tollmien-Schlichting (TS) waves which are the linear instability waves of boundary layer flows. Indications that lateral spreading occurs differently were obtained in the experiments of Gad-el-Hak *et al.*[7] who generated spots over a plate towed in a water tank. Visualization was done by releasing fluorescent dyes (of different colours) near the plate leading edge and in the initial spot and illuminating with a laser sheet. Lateral spreading was shown to occur independently of, and faster than, the diffusion of the initial spot. Following ref. 8, this mechanism was called 'growth by destabilization' of the surrounding nonturbulent but rotational flow which is different from the classical mechanism of entrainment which converts nonturbulent, irrotational flow into turbulent, rotational flow. The role of TS waves also needs to be established. Wygnanski *et al.*[9] had observed oscillations near the spot wingtips (upstream trailing edges) in hot-wire traces of stream-wise velocity and supposed them to be oblique TS waves which break down resulting in spot growth. Gad-el-Hak *et al.*[7] observed dye oscillations down-stream of the spot leading edge, but did not attribute these to TS waves because the oscillation wavelengths were smaller than required. In principle, it would seem that such issues could be addressed directly by simulations because complete field data, well-resolved in time and space, are generated. In practice, the techniques required to obtain unambiguous answers, such as, for example, simultaneous pathline tracing, adds considerably to the computational effort.

There are very few simulations of boundary layer spots, even though it was one of the earliest attempted. The first DNS of spots that were taken far enough in time to make comparisons with experiments were by Henningson *et al.*[10] with a fully spectral code. The computational domain was 120 and 40 times the initial boundary layer thickness in the streamwise and spanwise directions, respectively, and 170 Fourier modes were used in each coordinate. The normal direction was resolved using 33 modes. The initial disturbance was introduced at a Reynolds number of 250 based on displacement thickness (about one-third of the boundary

layer thickness) and free-stream speed, and the simulation proceeds till the Reynolds number (thickness) grows to 800. According to linear theory, boundary layers are stable for $Re < 520$, but the large perturbation grew even at $Re = 250$. An arrow-head shaped region developed, exhibited the well-known overhang, and propagated at speeds close to those measured in experiments. Tollmien-Schlichting waves were not observed. Although a comparison simulation of spots in plane channel flow was followed by extensive analyses, analyses of these boundary layer simulations have not been reported.

Singer[11] simulated the initial development of the flow in a flat plate boundary layer into which a small quantity of fluid had been injected. This is a frequently used method for producing spots in laboratory flows. The computations reveal the early stages of evolution of the disturbance into a spot. The injected fluid did not have streamwise momentum initially and so acted as a blockage giving rise to a high pressure region upstream followed by a low pressure region. A strong hairpin vortex appeared with its head just downstream of the high pressure region and the two legs trailed upstream. Additional hairpin vortex heads developed from the primary vortex. Vortices helped span new vortices by ejections of fluid resulting from an unsteady separation process in regions of adverse pressure gradient. Eventually, the disturbed region evolved into an arrowhead shape with a leading edge overhang. But, the averaged velocity profile did not yet resemble that of a turbulent boundary layer. Later, Singer[12] extended these simulations to a later time and obtained profiles that were approaching that of fully developed turbulent flow. Skin friction levels were at turbulent levels. There was still no evidence of any Tollmien-Schlichting waves though they may have been precluded by the low Reynolds number and the small extent of the spot which was only about twice a TS wavelength. However, Singer[11] notes that spanwise vorticity carried near the spot leading edge induces strong velocity perturbations which may have caused dye oscillations in experiments[7]. A direct demonstration from the simulations of the growth by destabilization (or any other physical model) has not yet appeared. So, although the questions remain and some of the tools for numerical studies have been developed, suitable simulations, analyses and definitive answers are awaited.

Spots in plane channels

The first study of spots in a Poiseuille flow (fully developed flow between parallel plates) was undertaken by Carlson et al.[13]. Titanium dioxide-coated mica platelets were dispersed in the working fluid (water). Propagating turbulent spots provided regions of high normal stress and strain rate which caused these platelets to align themselves and provided effective visualization.

The Reynolds number based on channel half-height and centerline velocity was just over 1000. A similar visualization was done by Alavyoon et al.[14] over a range of Reynolds numbers from 1100 to 2200. From an initial, nearly elliptical region, the leading edge spread laterally forming a crescent while the trailing edge became pointed. So these spots also appeared like an arrowhead but were pointed upstream. Thin streamwise-aligned streaks were prominent throughout the spot. Over the forward half these streaks were nearly uniform and comprised the crescent. At the crescent tips there were oblique waves. The spreading rate (half-angle) increased linearly from 6° at $Re = 1000$ to 12° at $Re = 2200$.

The only simulations of spots in a Poiseuille flow were also reported in Henningson et al.[10]. The Reynolds number was 1500. The computational domain of size $35 \pi \times 2 \times 25 \pi$ was resolved with $256 \times 33 \times 256$ spectral modes in the stream-wise, plate-normal and span-wise directions. A large localized perturbation of the channel flow evolved into a crescent-shaped region which was very similar to that observed in laboratory flows. But these regions did not show the nearly uniform stream-wise aligned streaks which were seen clearly in the laboratory. Near the channel centerline, the spot consisted of random, fine-scale structure. Flow close to the walls exhibited the low and high speed streaks usually found in turbulent flows near walls. After the initial disturbance evolved into a mature spot, different parts of the spot travelled at constant speeds. For example, the leading edge (crescent tip) travelled at 0.80 and the trailing edge at 0.54 times the centerline velocity of the Poiseuille flow. The overall shape remained similar at different times. Oblique waves at the crescent tips were quite clear, and Henningson[15] proceeded to analyse these waves using kinematic wave theory. These waves could grow by a wave energy focusing mechanism, but cross-flow instability engendered by the formation of inflexional profiles of the span-wise velocity component could support a larger growth rate and was supposed to be the more dominant mechanism. Again, a clear and direct demonstration of the spot growth mechanism is awaited.

It has been separately reported[16] that the mean and fluctuating parts of velocity distributions within the spot are quite close to those of the corresponding fully developed turbulent channel flow when the spot is subdivided into a turbulent central region and a wave region at the crescent tips.

Couette flows

The flow between two plates which slide past each other at a constant velocity is a plane Couette flow. Fluid velocity then varies linearly between the values at the two plates. One-half of this velocity range and the

channel half-height are used to define the Reynolds number. Plane Couette flow is stable to infinitesimal disturbances at all Reynolds numbers, but finite amplitude two-dimensional waves are unstable to three-dimensional perturbations resulting in transition at $Re \sim O(1000)$ (ref. 17). In experiments, transition can occur at lower Reynolds numbers depending on the disturbance levels. Couette flow experiments are not easy. So it is not surprising that the spot simulations of Lundbladh and Johansson[18] preceded the experiments of Tillmark and Alfredsson[19] and Dauchot and Daviaud[20].

Lundbladh and Johansson[18] simulated spots in plane Couette flows at $Re = 375$, 750, 1500. At $Re = 350$, they found that perturbations decayed. A spectral method with Fourier expansion in the stream-wise and span-wise coordinates and Chebyshev polynomials in the plate-normal coordinate were used. The spot occupies an elliptical region. Elongation in the sense of the shear produces an overhang near the walls. As the spots grew, the stream-wise growth rate fell, but the lateral growth rate remained constant. So, the ellipse aspect ratio fell towards unity. It was suggested that spreading mechanisms may have attained full strength after the spot grew to a certain size because the lateral growth rate showed a distinct change after a certain time, but remained nearly constant afterwards. Spots evolving at $Re = 375$ exhibited stream-wise streaks (in the midplane), but such streaks were not observed at higher Reynolds numbers. So mechanisms active close to the critical Reynolds number that sustains spot growth could be different from those at higher levels.

Spatially filtered velocity fields showed that the plate-normal velocity component vanished outside the spot, but velocity components parallel to the plate extended well outside. So the surrounding laminar flow profiles were altered considerably and growth by destabilization is plausible. Analyses to demonstrate that such mechanisms occur were not reported. Fluctuation intensities were highest at the edges of the spot rather like the increased levels observed in the transition zones.

Experiments[19] confirmed that Couette flow spots are elliptical and that the ellipse aspect ratio decreases towards unity as the spot grows. Waves were observed at the outer edges with crests aligned in the mean flow direction. Waves were clear at low Re and were overtaken by the spreading spot. From later experiments[20], the growth rate of these wingtip waves were calculated to be large enough to model break down by wave growth alone, unlike in Poiseuille flow. But they concluded that 'the question of which mechanism is really involved is still open.'

Couette flows with pressure gradients

Das[21] simulated spots in several Couette flows by varying the imposed mean pressure gradient. The study is at

a preliminary stage, but new kinds of spots have been simulated. The stream-wise velocity component of the undisturbed laminar flow

$$u(y) = y + P(1 - y^2)$$

depends on the wall-normal coordinate only. Different levels of stream-wise pressure gradient are imposed by selecting $P = -(Re/2)\mathrm{d}p/\mathrm{d}x$ which results in different background vorticity distributions for the flow in which the spot grows. When $P = 0.5$, the vorticity goes to zero at the upper plate (zero shear), and when $P = 1$, the vorticity in the upper quarter of the flow has the opposite sense from that in the fluid below.

To these flows an $O(1)$ localized perturbation was added to the velocity components. Both a coherent perturbation defined by an analytic cross-flow streamfunction, and a pseudo-random distribution generated by a random number generator at runtime, were used. The simulation results from integrating Navier–Stokes equations for incompressible flow using a third-order Runge–Kutta method. All spatial derivatives were approximated using finite difference formulae – upwind biased for convection terms, and central difference for all others. The solution was required to be periodic in the streamwise and spanwise coordinates, but sufficiently large periods were chosen to limit interactions among image flows. The Reynolds number was 375.

Spots shown in Figure 1 have evolved in Couette flows with different applied pressure gradients at time $t = 40$. Those in Figures 1 a–c were initiated from the same symmetric, analytical cross-flow stream-function and were obtained with pressure gradients $P = 0$, 0.5, 1.0. The spot Figure 1 d has evolved from a random
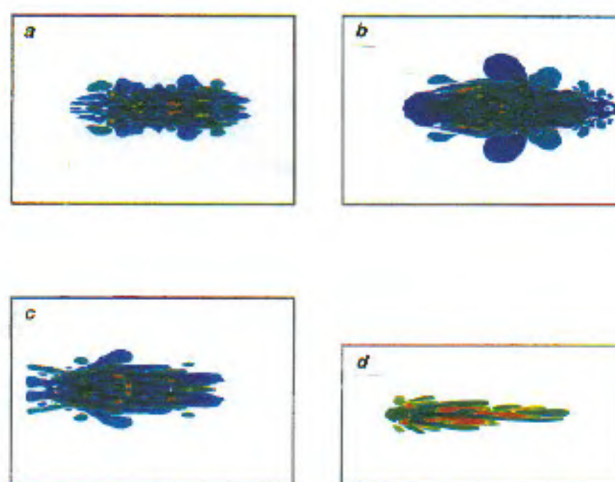

Figure 1. Spots in plane Couette flows. Vorticity magnitude contours on midplane parallel to channel walls. Levels increase from blue to red. a–c, Analytic initial condition $P = 0$, 0.5, 1.0; d, Random initial condition $P = 1$.

disturbance for the same background flow as in Figure 1 c. Vorticity magnitude has been used to identify the spot. Beyond $t \approx 10$, the spot grew larger, but the general structure of the mature spot changed little. The distinctions arising due to different surrounding flows had already become clear. With the stronger imposed pressure gradient, the spot appears as extending loops of vorticity anchored at the rear (Figure 1 c and d). This study has not gone beyond establishing a method to do the simulations, and obtaining simulations for a few cases. The approach to self-preserving forms of mature spots and propagation rates were also measured.

Puffs and slugs in pipe flow

Puffs and slugs denote compact regions of fluctuations in pipe flows. They are analogous to channel flow spots. Wygnanski and Champagne[22] observed that puffs occur when the Reynolds number is low (below 2700, based on pipe diameter and mean velocity) and that slugs occur in flows at higher Reynolds numbers. In puffs axial velocities fall smoothly from the leading edge towards the trailing edge, then rise sharply, whereas in slugs the velocity changes sharply at both edges.

Recently, Shan et al.[23] conducted DNS of transitional pipe flows at $Re = 2200$ and 5000 to obtain a puff and a slug from large disturbances like the spot simulations discussed above. In addition to obtaining velocity distributions and their statistics, the trajectories of several particles released from several cross-sectional planes were computed. From pathline statistics, it was concluded that a puff entrains fluid from the leading edge and detrains through the trailing edge. The slug entrains from both edges. So the slug can be characterized as a material property which travels with the flow, whereas the puff is more like a wave which propagates through the fluid. Without a doubt similar computations should be done for spots in channels and wall boundary layers.

Conclusions

A decade after the first DNS of spots was done, there are still very few that have followed. Analyses and definitive results not known from experiments are fewer still. But the correctness of DNS is only just becoming established and most DNS studies have considered little other than homogeneous turbulence and plane mixing layers anyway. Some of the interesting issues about spot dynamics may not follow from analysing a set of velocity fields alone. Incorporating procedures for analyses of evolving fields along with the simulation demands significantly greater computing effort. So, for example, the recent path-line analyses for puffs and slugs[22] were still done as post-processing using every 100th timestep. Nonetheless, since the methodology for simulations has become established, and interesting aspects remain unexplained, it is reasonable to expect that simulation studies will begin to make definitive statements in the near future.

1. Emmons, H. W., *J. Aero.*, 1951, **18**, 490–498.
2. Narasimha, R., *J. Aero.*, 1957, **24**, 711–712.
3. Dhawan, S. and Narasimha, R., *J. Fluid Mech.*, 1958, **3**, 418–436.
4. Kleiser, L. and Zang, T. A., *Annu. Rev. Fluid Mech.*, 1991, **23**, 495–537.
5. Moin, P. and Mahesh, K., *Annu. Rev. Fluid Mech.*, 1998, **30**, 539–578.
6. Schubauer, G. B. and Klebanoff, P. S., NACA Rep. 1289, 1956.
7. Gad-el-Hak, M., Blackwelder, R. F. and Riley, J. J., *J. Fluid Mech.*, 1981, **110**, 73–95.
8. Corrsin, S. and Kistler, A. L., NACA. Rep. 1244, 1955.
9. Wygnanski, I. J., Haritonidis, J. H. and Kaplan, R. E., *J. Fluid Mech.*, 1979, **92**, 505–528.
10. Henningson, D., Spalart, P. and Kim, J., *Phys. Fluids*, 1987, **30**, 2914–2917.
11. Singer, B. A., *Phys. Fluids*, 1994, **6**, 3724–3736.
12. Singer, B. A., *Phys. Fluids*, 1994, **8**, 509–521.
13. Carlson, D. R., Widnall, S. E. and Peeters, M. F., *J. Fluid Mech.*, 1982, **121**, 487–505.
14. Alavyoon, F., Henningson, D. S. and Alfredsson, P. H., *Phys. Fluids*, 1986, **29**, 1328–1331.
15. Henningson, D., *Phys. Fluids A*, 1989, **1**, 1876–1882.
16. Henningson, D. and Kim, J., *J. Fluid Mech.*, 1991, **228**, 183–205.
17. Orszag, S. A. and Kells, L. C., *J. Fluid Mech.*, 1980, **96**, 159.
18. Lundbladh, A. and Johansson, A. V., *J. Fluid Mech.*, 1991, **229**, 499–516.
19. Tillmark, N. and Alfredsson, P. H., *J. Fluid Mech.*, 1992, **235**, 89–102.
20. Dauchot, O. and Daviaud, F., *Phys. Fluids*, 1995, **7**, 335–343.
21. Das, A., M Sc (Engg.) Thesis, IISc, Bangalore, 1998.
22. Wygnanski, I. J. and Champagne, F. H., *J. Fluid Mech.*, 1973, **59**, 281–335.
23. Shan, H., Ma, B., Zhang, Z. and Nieuwstadt, F. T. M., *J. Fluid Mech.*, 1999, **387**, 39–60.