

# Studies of Coriolis force-induced transition in internal flows in a rotating system

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**The subject matter of this paper is transition of the flow in a channel on a rotating system due to the effect of Coriolis force. The phenomenon is studied through flow visualization, the occurrence of transition showing up as regularly spaced rolls of longitudinal vortices. The effects of channel aspect ratio and of non-monotonic changes of channel cross-sectional area on observed transition are studied. The observations are set against a simple linear theory based on following the evolution of infinitesimally small disturbances under the influence of Coriolis force in a channel of uniform cross section. The comparison shows the scope and limitations of the linear theory.**

## 1. Introduction

Transition is one of the very fascinating phenomena of common occurrence in fluid flows. Fluids are known to flow in a wide diversity of patterns, and transition of one form or another is observable in almost all of them, in flows in nature as frequently as in engineering applications. Transition generally manifests itself through significant changes in the qualitative characteristics or scaling behaviour of crucial physical quantities which is often the feature by which its occurrence is recognized in an experiment, and, more recently, in direct numerical simulations (DNS) of fluid flow too. The physical quantities in which the changes are readily observable may be either local or global in nature, such as, e.g. the velocity distribution, or the forces and moments exerted by the flow on its surroundings. Transition in a flow is also often accompanied by a dramatic change in the topological pattern of the streamlines and/or pathlines in the flow, and this is the feature often most readily accessible to the experimental technique of flow visualization. The inherent scientific interest and engineering significance of transition have made its study a focal point of interest in fluid mechanics research for several decades. Despite this incessant effort, there are many questions yet unanswered and the subject continues to occupy a key position in fluid mechanics research as of this day.

From a point of view associated with following the dynamics of fluid flow, transition may be regarded as

the process taking the flow pattern from one vortical state to another. Generally, a flow undergoes transition on the parameter characterizing the state of the flow crossing a certain threshold. Crossing the threshold upsets the delicate balance between the set of forces that is keeping the flow in its initial state. Commonly, and in particular in flows met with in engineering applications, the set of governing forces are primarily of inertial, pressure and viscous origin. In these flows, therefore, destabilization of the balance between the governing forces, and hence transition, sets in on the Reynolds number, the parameter measuring the relative dominance of the inertial and the viscous forces, crossing a certain threshold value. However, alternative mechanisms of transition may come into play when other kinds of forces are involved, striking examples of which are encountered when body forces are acting on the flow. Two fundamentally different kinds of body force come into play when the system is rotating, viz. the centrifugal force and the Coriolis force. The process of flow transition may then be activated on the parameter characterizing the influence of either of these body forces crossing a threshold of its own.

A general account of flows under the influence of the body forces caused by rotation may be found in standard works<sup>1-3</sup>. A cursory glance through current literature would show that, among the two kinds of body force caused by system rotation, transition caused by the centrifugal force has been more extensively studied and documented<sup>4</sup>. With the objective of complementing this work the first author initiated a few years ago at the Ruhr University in Bochum, Germany, a set of studies on transition induced by the Coriolis force. The present paper draws from this set of studies, some parts of which have already appeared in journals, some others have been reported only at conferences and the rest hitherto unpublished. The studies have been undertaken with theory and experiment proceeding in parallel, but the account given in this paper places a slightly stronger emphasis on the experimental side. The authors' justification for this choice of theirs for the present paper, if one is needed, is that even a casual survey of literature in the past two decades on this subject shows gains in insight acquired through numerical studies largely outnumbering those from experiments<sup>5-7</sup>. This trend lies of

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We dedicate this paper to Prof. Satish Dhawan on the occasion of his completing 80 years.

course in the natural course of development of the subject, enabled by the giant leaps in computer technology going hand in hand with major developments in algorithms, and it is welcomed by the authors too. However, for a sound scientific understanding of the complex fluid flow phenomena of transition support of experiment seems to be still indispensable, and it is in this spirit that the authors wish their present paper to be read. It is their hope that the experiments presented herein serve to correct the tilted balance, however marginal that may be, and draw attention to some of the still unanswered questions and hitherto unaddressed problems in this area. Despite the stated emphasis on the experiments, since a proper interpretation of the outcome of the experiments cannot be envisaged without setting them against a suitable theoretical background, a brief description of the effects of rotation on the flow expectable from theoretical considerations precedes the account of the experiments. This is in §2. An account of the experiments themselves then follows in §3. These have been chosen to show both the scope and limitations of the simple theory outlined in §2. A brief discussion of the experimental outcome and conclusions therefrom then follow in §4 and 5.

## 2. Effects of system rotation on an internal flow

The effect of system rotation on a flow is a subject dealt with extensively in several standard text-books and review articles, so we restrict ourselves herein to sketching them briefly only to ease reading of this paper.

From a physical point of view, a rotation of the system in which the flow is taking place is associated with two kinds of body force, viz. the centrifugal force and the Coriolis force. There are similarities and differences between the effects of these two on the flow, which may be readily seen on writing the dynamic equation of motion for the flow in the rotating system. This, after division by the density  $\rho$ , is as follows:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - 2\boldsymbol{\Omega} \times \mathbf{u} + \nu \nabla^2 \mathbf{u},$$

where  $\boldsymbol{\Omega}$  is the rotation vector and  $\mathbf{r}$  the radius vector from the axis of rotation. The two terms following the pressure gradient are, in that order, the centrifugal force and the Coriolis force. cursory inspection of the terms then shows that the Coriolis force  $2\rho\boldsymbol{\Omega} \times \mathbf{u}$  is present only when the flow velocity vector  $\mathbf{u}$  in the rotating system has a component perpendicular to the vector of system rotation, whereas the centrifugal force is active even when there is no flow as such in the rotating system. It is also straightforward to verify that the centrifugal force field, being irrotational, can be written as

a gradient of a scalar, hence it permits absorption of its effect on the flow through a suitable redefinition of pressure. In contrast, the Coriolis force field is rotational. Besides these outstanding differences, the parameter characterizing centrifugal force is seen to depend upon the product of the square of the angular velocity of system rotation and a characteristic length, whereas the parameter characterizing the effect of Coriolis force is dependent upon the angular velocity of system rotation itself, i.e. linearly, and contains no characteristic length.

Both the kinds of force caused by system rotation, viz. the centrifugal force and the Coriolis force, may trigger the flow to undergo transition through mechanisms peculiar to them. When the flow is wall bounded, it has been observed that in both cases the first stage of transition takes the flow to a vortical state characterized by fluid particles describing helical paths around a cylindrical surface whose axis is parallel to the main flow direction. Such a vortical state is denoted as one with steady longitudinal vortices. This vortical state is so steady and well ordered that turbulence is not considered the appropriate word for its description. When the force inducing transition is the centrifugal force the longitudinal vortices that form are familiar under the name of Taylor or Görtler vortices. Physical reasons also indicate that in a flow in later stages of transition with a more complex vortical state for which the label turbulent is more appropriate, the fluctuating motion, and hence the transport processes resulting therefrom, do depend upon the body force acting, which is the centrifugal and/or the Coriolis force. Insofar as the effect of the Coriolis force is concerned, a crucial role is played by the relative orientation between the vectors of system rotation and of the local rotation of the fluid particle<sup>8</sup>. We recall that the latter is defined through the antisymmetric part of the tensor formed by the spatial gradient of the velocity field,  $\nabla \mathbf{u}$ .

### 2.1 Effects of Coriolis force

Physical insight into some gross effects of Coriolis force on the flow in a rotating system may be obtained from examining the equations of motion for flow in the relatively simple geometrical configuration of a channel passage formed by the gap between two plane impermeable rigid walls of infinite extent kept parallel to each other and perpendicular to the plane of rotation (see Figure 1). The overall flow pattern expectable in this configuration is such that for an observer rotating with the system, fluid particles flow in planes parallel to the plane of rotation. Therefore, in a frame of reference rotating with the walls, Coriolis force is active and is directed from one wall to another. This flow geometry is an idealization closely related to the geometry of the

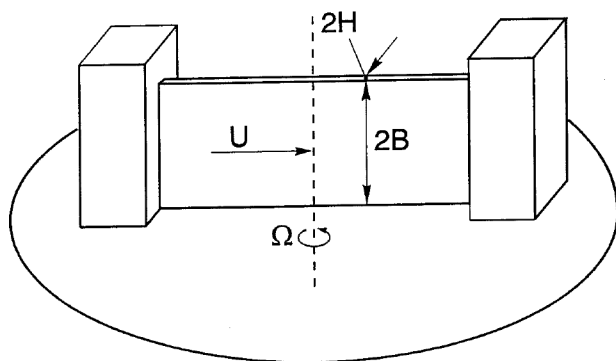


Figure 1. Sketch of flow geometry.

experiments to be described in §3. The equations of motion may then be solved in very simple terms to yield the velocity profile and the pressure. On solution the velocity profile turns out to be fully developed, i.e. it exhibits no changes in the streamwise direction. Furthermore, it possesses the same shape as in the case of no system rotation, i.e. of a parabola. For this highly idealized geometrical configuration, the effect of system rotation is felt only in the pressure field. In a rotating system there is a difference in pressure present at the opposite walls at any streamwise location. This pressure difference would be zero if the system were not in rotation. It is worthy of note that the streamlines/pathlines of this solution remain straight and parallel.

While the equations of motion as such do admit in the above idealized situation the simple solution of a parabolic velocity profile over the entire range of the relevant flow parameters, viz. the Reynolds number ( $Re$ ) and the rotation number ( $Ro$ ), a closer examination of the dynamics of the flow that goes under the heading stability investigations indicates that this solution is not expected to be actually observable in experiment when the parameters cross some critical values. The thought behind this reasoning is based on the conjecture that incidental disturbances, however small they may be, are inevitably present in any experiment, so only such states of flow are observable in an experiment that are stable with respect to disturbances. For the class of flow problems in question here, the stability investigation shows that the solution with the parabolic velocity profile is not stable with respect to infinitesimally small disturbances when the flow parameter values lie in a range we designate supercritical. Such a finding hints at transition imminent in the flow.

The study of the stability characteristics of solutions with respect to infinitesimal disturbances is a vast research area in fluid mechanics with a rich tradition of its own. It attempts to trace the evolution of the entire class of possible disturbances that could arise. The method is based on following the dynamics of

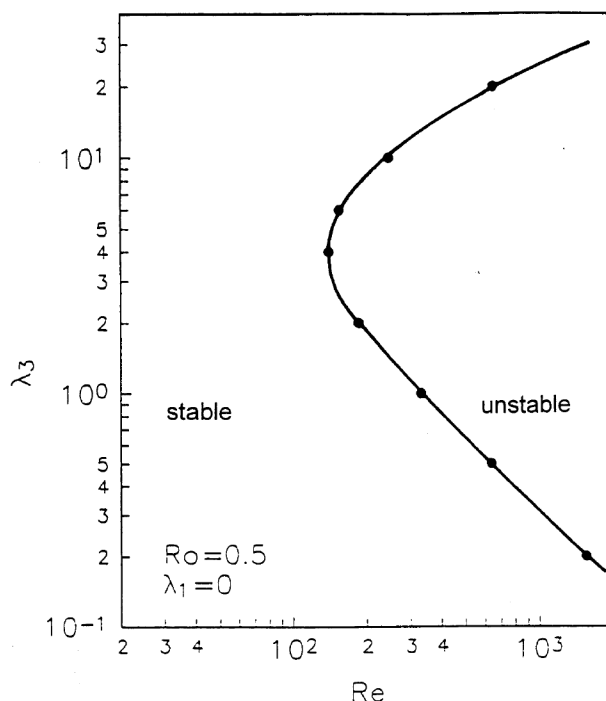


Figure 2. Plot showing disturbance behaviour vs Reynolds number for the channel flow in a rotating system.  $\lambda_3$  is the spanwise wave number of incidental disturbances.  $\lambda_1 = 0$  denotes the class of disturbances associable with the formation of longitudinal vortices.

infinitesimally small disturbances through appropriate linearization of the equations of motion of fluid flow. The extensive body of research work in this area stretches over several decades, includes contributions from leading fluid dynamicists from all over the world, and continues as of this day to hold a pivotal position in fluid mechanics research. The studies hitherto have resulted in the development of mathematical methods tailored to handle the wide variety of stability problems that arise in fluid mechanics. Accounts of these may be found in standard reference works of today<sup>9</sup>. Applied to the stability of the idealized flow problem currently in question, these investigations lead to a diagram known as the stability diagram. An important step in obtaining a stability diagram is getting a plot of disturbance behaviour with respect to the parameters characterizing the flow and the disturbance. Such a plot shows, for a given set of the flow parameters  $Re$  and  $Ro$ , which disturbances, characterisable through their wave number set  $\lambda_1$  and  $\lambda_3$ , grow or decay. An example of such a plot is given in Figure 2 in which for purposes of clarity one of the flow parameters  $Ro$  and one of the wavenumbers  $\lambda_1$  are held fixed at the values given on the figure. The plot therefore shows, at any value of the  $Re$  on the horizontal axis whether a disturbance of a certain wave number  $\lambda_3$  on the vertical axis grows or decays. This leads to a classification of the flow at this value of the flow parameter as being unstable or stable in that order

with respect to a disturbance of the particular wave number. Stability investigation involves preparation of such plots of disturbance behaviour for the entire range of flow parameters  $Re$ ,  $Ro$ , and disturbance wave numbers  $\lambda_1$ ,  $\lambda_3$  in question. These put together lead to a final stability diagram in a hyperspace which then provides a framework well adapted for viewing the outcome of experiments to be described in §3. A detailed discussion of the meaning and implications of the stability diagram is beyond the scope of the present paper. Here it suffices to note two points of importance therein. Firstly, the stability diagram delineates in the space of the two flow parameters, viz. Reynolds number  $Re$  and the rotation number  $Ro$ , regions in which the solution with the parabolic velocity profile is stable or otherwise with respect to infinitesimally small disturbances. Secondly, Coriolis force induces transition at a Reynolds number that is two powers of ten lower than if rotation were absent. This facilitates realization in experiment of the flow undergoing transition predominately under the influence of Coriolis force alone, relegating effects of other mechanisms into the background. For a more detailed account of these studies the reader is referred to the original literature<sup>10-12</sup>.

When the theory outlined above is used as a setting to interpret the outcome of experiments to be shortly described, certain points of importance have to be observed. Among these, the effects of the finite aspect ratio, i.e. the ratio of the span of the channel  $2B$  to the gap height  $2H$ , of streamwise changes in the channel width  $2B$ , and of small but finite, i.e. not infinitesimally small, disturbances are of special significance. All these are at the cutting edge of research in fluid mechanics today. The first, viz. finite aspect ratio, is inevitable in any laboratory experiment. Comparison of the outcome of the experiment with a theoretical result for infinite aspect ratio is justifiable only as long as the aspect ratio in the experiment is sufficiently large. But how large is sufficiently large? This question calls for a careful assessment of the end-wall effects on transition. The second, viz. the effect of changes in cross section, is relevant not only in many applications. It requires a basic understanding of the propagation and evolution characteristics of disturbances in surroundings that themselves change. The answer to the third point is but a step towards obtaining such solutions of the equations of motion that are closer in their topological character to observation. This is particularly of interest when the flow parameters lie in the supercritical regime, in which, in anticipation of the experiments to be described, we state here in advance the finding that the streamlines/pathlines remain no longer straight and parallel but describe helical paths. This dramatic change in the topological property of the flow on crossing the critical set of parameters is a feature the theory has to aim to capture in order to qualify for its setting against the experiment.

### 3. Visualization studies of the flow in transition

#### 3.1 The apparatus

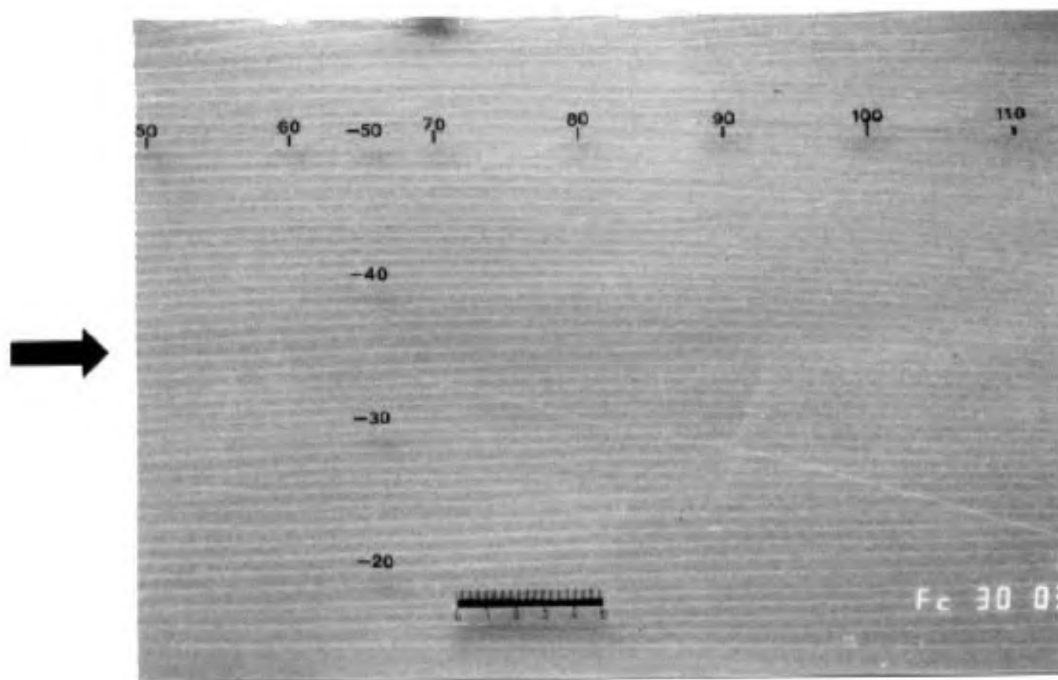
In order to be able to capture and study characteristics of the flow in transition under the influence of Coriolis force it is necessary to realize experimentally a flow in which effects of other possible sources of transition are either eliminated or reduced to a low level. Reaching this goal in the laboratory crucially depends upon the design concept of the apparatus for the experiments which sets its working fluid, dimensions and experimental parameters. The apparatus that was designed and built at the Ruhr University in Bochum, Germany, has been described in ref. 12. A photograph of this facility is shown in Figure 3. The facility is essentially a rotating table of 1200 mm diameter on which were mounted the pair of plexiglass plates forming the channel walls. The gap width (channel height) is 5 mm. The span of the channel, and thus its aspect ratio, could be varied in steps by introducing false side walls. Mounting of replaceable end blocks of suitable shape was the means to change the cross-sectional area of the channel.

Water is the working fluid of the facility. The flow through the channel is driven by the pressure difference between the two settling chambers, also mounted on the rotating table. The pressure difference required is maintained by a pump in the circulating system. The channel walls and the settling chambers are so arranged that the spanwise direction of the channel coincided with the rotation axis. Therefore, if the flow remains in planes parallel to the rotating table, which may be expected to

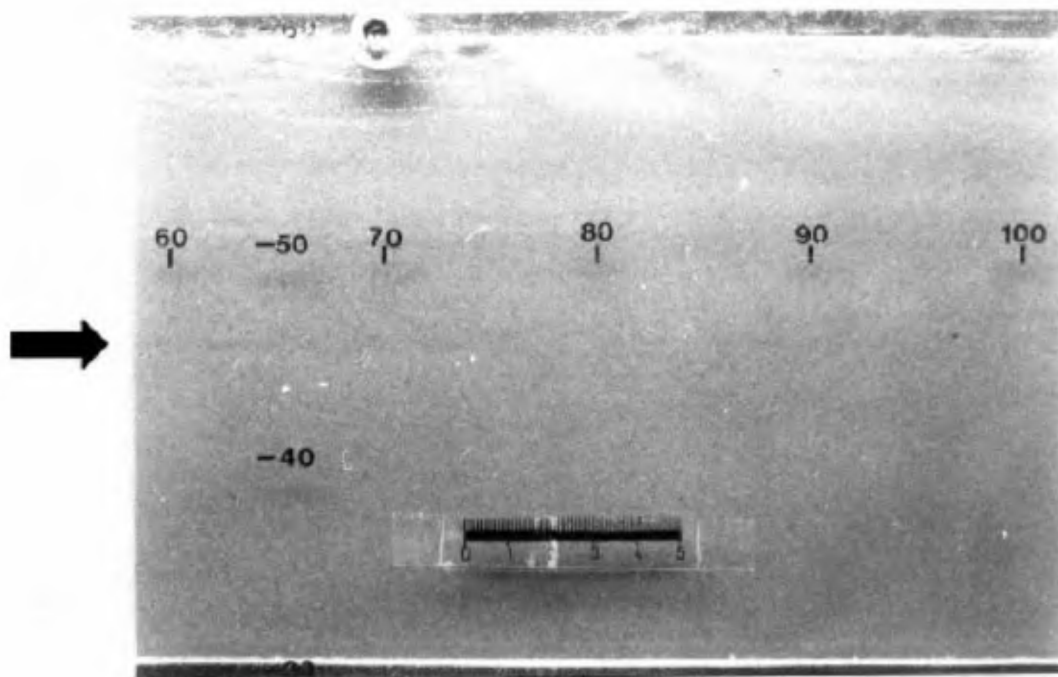


Figure 3. The experimental apparatus with the rotating table.





**Figure 4.** The visualized flow pattern in the uniform channel of aspect ratio 50:1. Flow parameters:  $Re = 67.9$ ;  $Ro = 0.33$ . The stray lines across the rolls are not relevant to the flow pattern. They were caused by optical reflections in the apparatus.

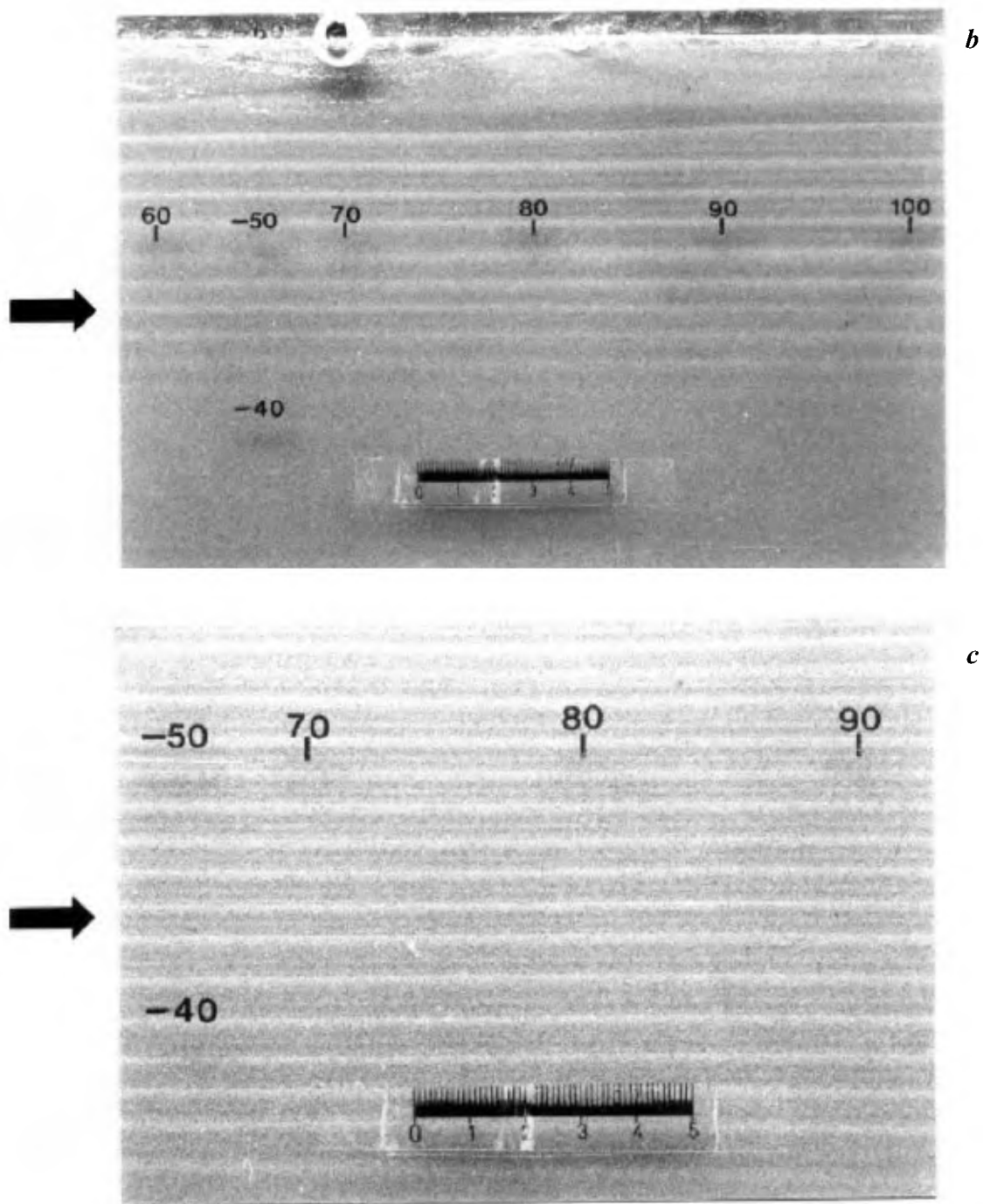


**Figure 5a.** The visualized flow pattern in the uniform channel of aspect ratio 30:1. Flow parameters:  $Re = 42.3$ ;  $Ro = 0.09$ .

be the case when the flow is in the subcritical range, the Coriolis force is directed from one wall to the other.

The technique for identifying transition in our experiments was flow visualization. Seeding of the flowing water with small-sized particles of suitable shape

and light-reflecting properties together with proper illumination renders the gross topological features of the flow streamlines/pathlines visible, thus permitting detection of transition when it occurs. For details of the seeding and illumination in our apparatus we refer to



**Figure 5 b, c,** The visualized flow pattern in the uniform channel of aspect ratio 30:1. (b) Flow parameters:  $Re = 42.3$ ;  $Ro = 0.15$ ; (c) Flow parameters:  $Re = 52.9$ ;  $Ro = 0.19$ .

ref. 12. A feature of our apparatus that enables a more direct setting of the flow transition characteristics gained from the apparatus against the theory of §2 is that the camera and illumination are mounted on the rotating table, and therefore the topological characteristics of the flow visualized are in a frame of reference rotating with the table.

### 3.2 The experiments and their results

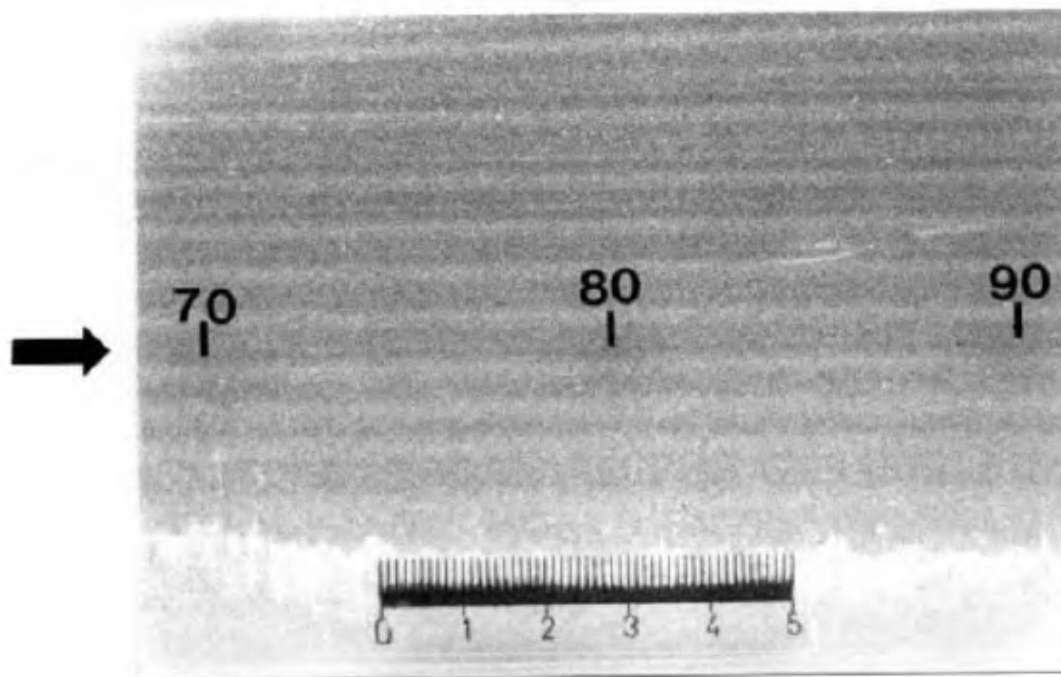
We have conducted with our apparatus two sets of experiments of the flow in transition, one studying the effects of finite aspect ratio of the channel of constant cross section and the other those of changes of channel sectional area. The flow parameters adjustable in the

experiments are the volume flow rate  $Q$  and the angular velocity of the rotating table  $\Omega$ . But it is more meaningful to view the results in terms of the dimensionless flow parameters, the Reynolds number  $Re$  and the rotation number  $Ro$ , since, with the support of the theoretical foundation outlined in §2, inferences can then be drawn from the experimental observations that are of wider applicability. The conversion from the dimensional experimental parameters to their dimensionless counterparts is straightforward according to the definitions listed at the head of the paper. In cases when uncertainty could arise in conversion, as it would for the channel with changing cross sectional area, the procedure adopted is indicated.

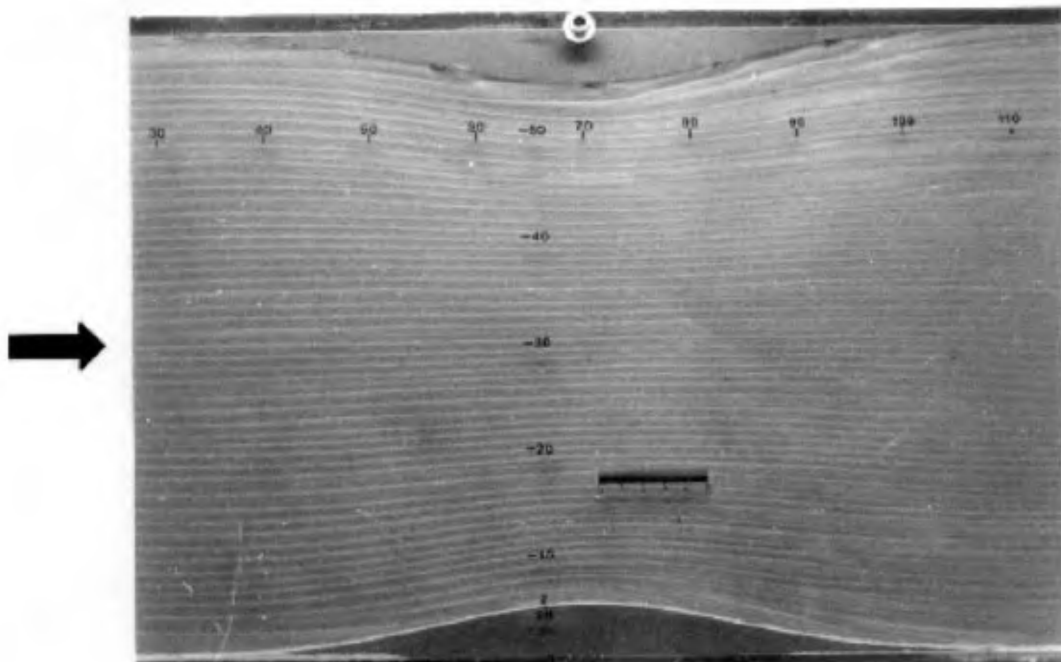
In all these experiments the criterion for classifying the flow according to its vortical state was the topological feature of the pattern of flow as made visible. It was designated subcritical when no spanwise patterns were visible, and supercritical when light reflections from the seeded particles grouped themselves into spanwise rolls. The former is evidence of streamlines/pathlines remaining straight whereas the latter is indicative of the formation of longitudinal vortices in the flow with streamlines/pathlines describing helical paths. Some of the results have already been published, see ref. 12. In this paper we present results that supplement those published in ref. 12 and have hitherto not appeared in print. In all the photographs of the visualized flow patterns the general flow direction is from left to right, as indicated by the arrow. The scale in the photographs indicates distances in centimeters and the numbers, distances in terms of the semi-channel height  $H$ .

**3.2.1 Effects of channel aspect ratio:** Figures 4, 5 a–c and 6 show photographs of the visualized flow patterns in the uniform channel of aspect ratio 50:1, 30:1 and 20:1 respectively. The flow parameters at which the flow pattern is realized are given in the captions for the figures. Figures 4 and 6, which are for channel aspect ratios of 50:1 and 20:1 respectively, are visualizations of the supercritical state of the flow. They clearly show the formation of roll patterns typical of longitudinal vortices that are periodic spanwise. Figure 5 a and c show for the channel of aspect ratio 30:1 subcritical and supercritical states of flow whereas Figure 5 b shows a state in which the flow is subcritical in a spanwise part of the channel and supercritical in the other.

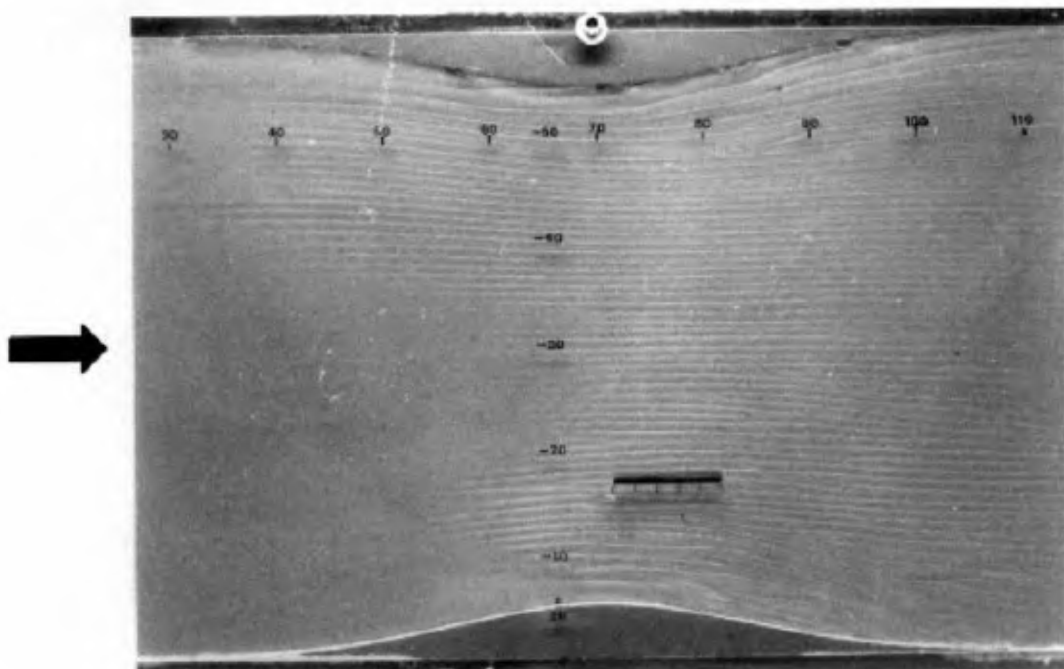
An overall impression of the effect of channel aspect ratio on transition may be gained on comparing the visualized flow patterns in the photographs (Figures 4–6). The interested reader may also compare these with photographs of the visualized flow in ref. 12 which are for a channel aspect ratio of 60:1. The photographs of the flow show that for an aspect ratio smaller than 30 longitudinal vortices appear in the channel at generally lower values of the rotation number  $Ro$  than otherwise. In channels of aspect ratio greater than 30, the values of the parameter set  $Re$ – $Ro$  at which the flow assumes a supercritical state is seen to be essentially independent of the aspect ratio. This observation is indicative of transition in the channel being influenced by the side walls at aspect ratios of 30 and lower.



**Figure 6.** The visualized flow pattern in the uniform channel of aspect ratio 20:1. Flow parameters:  $Re = 34.6$ ;  $Ro = 0.25$ .



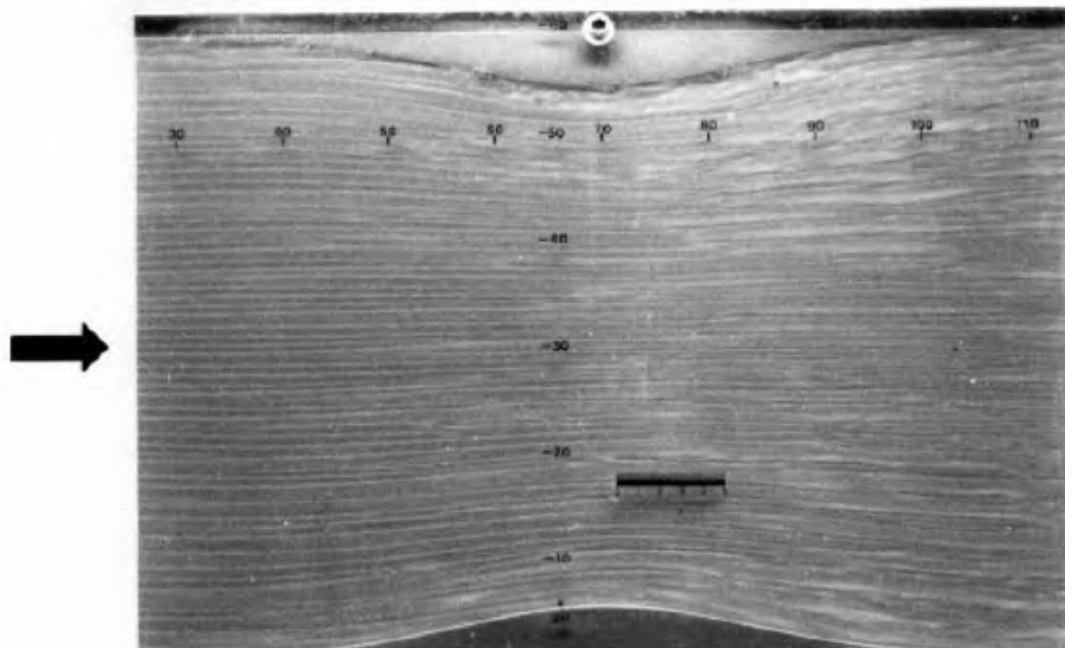
**Figure 7.** The visualized flow pattern in the channel with a bell-shaped change of its span width. Flow parameters at channel constriction:  $Re = 90.0$ ;  $Ro = 0.17$ .



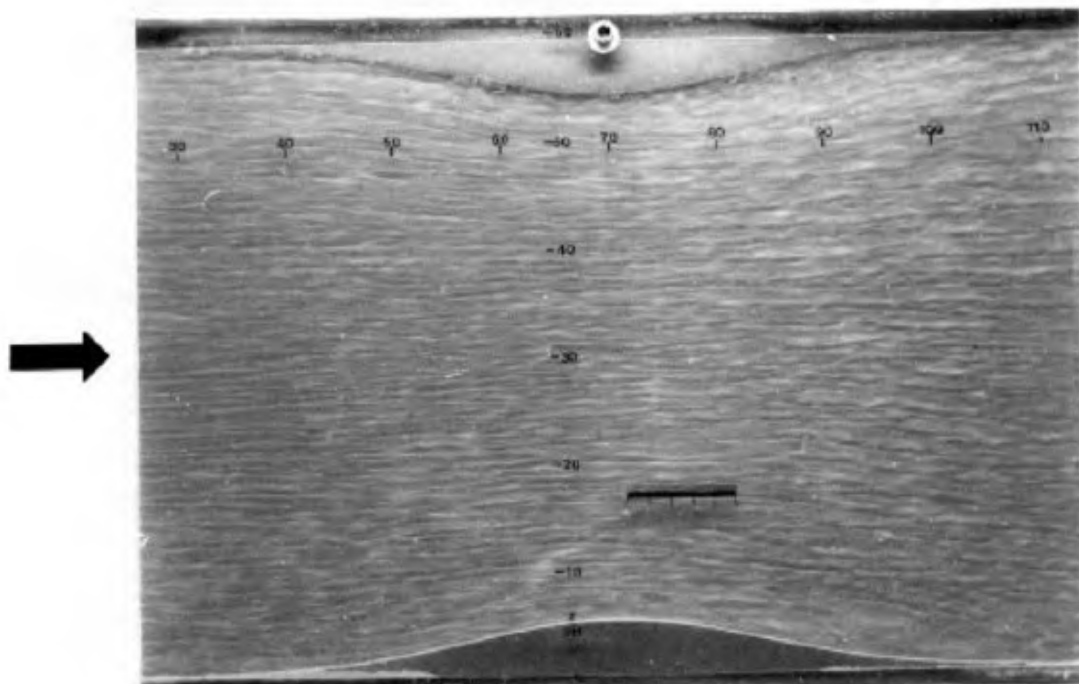
**Figure 8.** The visualized flow pattern in the channel with a bell-shaped change of its span width. Flow parameters at channel constriction:  $Re = 90.0$ ;  $Ro = 0.29$ .

**3.2.2 Effects of change in channel cross sectional area:** The effects on Coriolis force-induced transition in flows in three classes of channel with a change of cross sectional area were studied. These are: the cross-sectional area increasing or decreasing monotonically as the viewer proceeds downstream, and the cross-sectional

area changing according to a bell-shaped curve, i.e. a decrease of area first followed by an increase. Examples of visualizations of the flow in channels of either increasing or decreasing cross-sectional area were presented in ref. 12. In this paper we present examples in which the channel cross-sectional area follows the



**Figure 9.** The visualized flow pattern in the channel with a bell-shaped change of its span width. Flow parameters at channel constriction:  $Re = 175.0$ ;  $Ro = 0.09$ .



**Figure 10.** The visualized flow pattern in the channel with a bell-shaped change of its span width. Flow parameters at channel constriction:  $Re = 480.0$ ;  $Ro = 0.03$ .

bell-shaped curve. Photographs of the visualized flow in this class of geometry are given in Figures 7–10. The difference between these figures is in the set of flow parameters at which the flow patterns are realized in the experiment. Values of the dimensionless parameters given in the captions for these figures have been evalu-

ated at the constriction of the channel. A comparison of the visualizations shows that, with such changes of channel cross-sectional area, a variety of vortical states of the flow can be realized. The flow pattern can be supercritical everywhere or only in parts. Also, the rolls are wavy at certain values of the parameter set which is

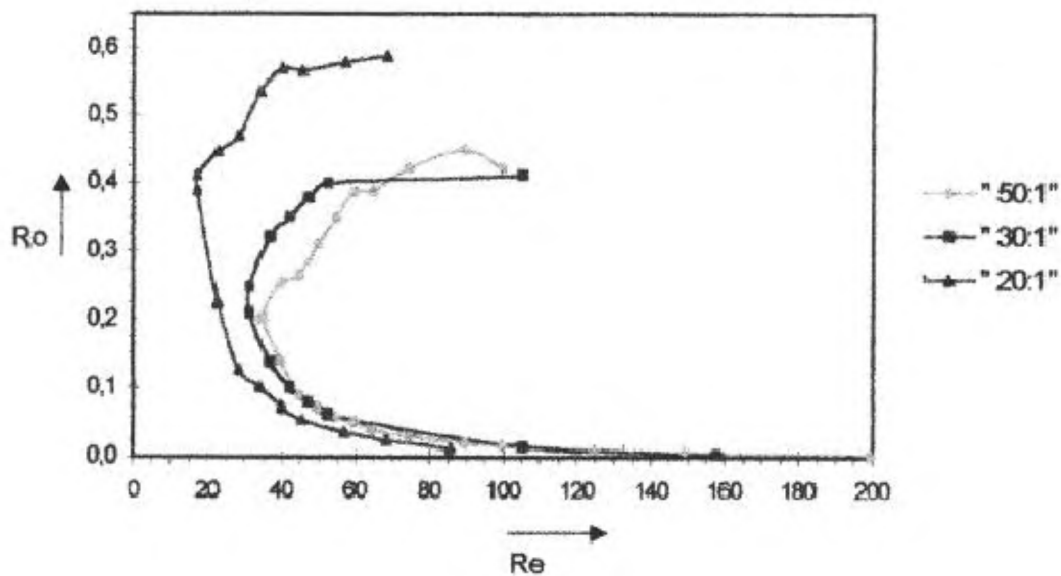


Figure 11. Experimentally realized boundaries between subcritical and supercritical states of flow.

indicative of a possible unsteadiness in the flow when the flow parameters assume these values.

#### 4. Discussion

To gain an understanding of the experimental outcome it is meaningful to view them against the results of the linear stability theory outlined in §2. The boundary between the supercritical and subcritical states of flow as realized in the experiments in the uniform channel of different aspect ratio are therefore plotted in terms of the dimensionless parameters of the problem in Figure 11. A comparison of Figures 2 and 11 then shows that the linear stability does indeed capture the boundary between subcritical and supercritical regimes of flow qualitatively properly for the channel of uniform cross section, provided the aspect ratio is large. The quantitative differences, particularly for the larger aspect ratio channel, are possibly attributable to effects of finite amplitudes of disturbances causing transition. For the channels of lower aspect ratio, as would be expected, end-wall effects on transition gain importance.

Marliani *et al.*<sup>12</sup> report that transition of the flow in the channel with monotonic decrease of area is also captured reasonably by the simple theory when the theory is interpreted to be applicable locally. With the channel cross-sectional area decreasing, the flow evolves from a subcritical state without vortices into a supercritical state with vortices. However, for the opposite case, with the channel cross-sectional area increasing streamwise, when longitudinal vortices already formed upstream

vanish downstream, the boundary between the subcritical and supercritical states is not captured by the simple theory. This is understandable since the theory investigates the growth/decay of disturbances from an initial state of the flow with no vortices. It would be more appropriate for this case to study disturbance evolution characteristics in a flow in which the vortices already exist. The situation is essentially the same for the channel with the bell-shaped change in cross-sectional area.

#### 5. Conclusions

Channel flow under the influence of Coriolis force due to system rotation undergoes transition at substantially lower Reynolds numbers than that in a non-rotating system. The first stage of transition is to a vortical state characterized by steady longitudinal vortices that are spaced in a regular periodic manner in the spanwise direction. Linear stability theory along classical lines does describe the onset of transition satisfactorily in uniform channels of large aspect ratio but it is unable to do so when the aspect ratio is lower. The effects of the end walls in channels of lower aspect ratio and of changes in cross-sectional area of the channel on transition are major issues that are still outstanding and in which there is need for further research.

#### Notation

$2H$  the gap between the plane parallel walls, referred to as channel height, see Figure 1

## INSTABILITIES, TRANSITIONS AND TURBULENCE

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$2B$	span of channel, referred to as channel width, see Figure 1
$A$	cross-sectional area of channel, $A = 4BH$
$Q$	volume flow rate through channel
$U_{\text{ref}}$	reference velocity, defined through $Q = \frac{2}{3}AU_{\text{ref}}$
$\vec{\Omega}$	angular velocity vector of the rotating system
$\Omega$	magnitude of the angular velocity vector $\vec{\Omega}$
$\mathbf{u}$	flow velocity vector in the rotating system
$\mathbf{r}$	radius vector, measured from axis of rotation
$\nu$	kinematic viscosity of the flowing fluid
$Ro$	Rotation number, definition: $Ro = \Omega \cdot H / U_{\text{ref}}$
$Re$	Reynolds number, definition: $Re = U_{\text{ref}}H/\nu$

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