The gravity–gauge theory correspondence

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At the end of 1997 a young Argentinian physicist conjectured that certain string theories are identical to certain other well-understood theories called gauge theories. This conjecture has been at the focus of attention of much of the string theory community over the last year and a half, and has generated over 1000 research papers. In this article I will attempt to explain the conjecture and why it has generated so much excitement.

1. The setting

String theorists have come to agree that almost all known quantum theories containing gravity are described by the fluctuations about different lowest energy states or vacua of a single theory called M theory. Since we know of infinitely many quantum theories containing gravity, M theory must possess very many inequivalent vacua.

In a nongravitational theory space and time constitute the unchangeable background that hosts the dynamics of particles. Space and time are themselves dynamical in M theory, so different vacua of the theory describe different background spacetime geometries. One particularly distinguished vacuum corresponds to 11 infinite flat spacetime dimensions, ten space and time. While the full quantum dynamics of fluctuations about this flat background is unknown, at low energies (note 1) it is governed by a well understood classical theory called 11-dimensional supergravity. Vacua in the 'neighborhood' of this one describe curved 11-dimensional spacetimes whose deviation from flatness (note 2) is small. Fluctuations about such vacua at low energies are governed by 11-dimensional supergravity on the appropriate background.

Another distinguished set of vacua of M theory have one spatial dimension curled up into a small circle (note 3) the remaining ten spacetime dimensions in such vacua are flat and infinite. Fluctuations about such vacua are described by a theory of strings propagating in 10 flat spacetime dimensions called IIA string theory. These strings interact with each other with an interaction strength measured by a continuous parameter, the string coupling constant, which is a function of the radius (note 4) of the spatial circle. Probabilities for the outcome of experiments involving few strings in such theories may be expanded in a Taylor series in the string coupling constant. Rules for the computation of all coefficients in this expansion are known. Unfortunately this Taylor series does not converge for any nonzero value of string coupling, so these rules do not completely specify the dynamics of fluctuations about IIA vacua except in the trivial special case of zero string coupling in which case one obtains a theory of free strings. At low energies and small string coupling IIA string theory reduces to a well understood classical theory called IIA supergravity. Vacua describing curved 10-dimensional spacetime plus a small 11th circle also host propagating strings. Strings propagating on an arbitrary geometry are governed by a two-dimensional theory called a conformal field theory. The conformal field theory describing string propagation is known for some, but not all backgrounds. Knowledge of the conformal field theory permits the computation of all physical quantities order by order in the string coupling constant. In backgrounds for which the conformal field theory is not known, only the weak coupling low energy (note 5) dynamics is understood; it is governed by IIA supergravity on the appropriate background.

M theory possesses at least four other classes of stringy vacua about which fluctuations are governed by theories of weakly interacting strings propagating in 10 flat spacetime dimensions. These string theories are called IIB, type 1, E_8 heterotic and SO(32) heterotic string theories, each of which reduces to a classical supergravity at low energies. In particular at weak

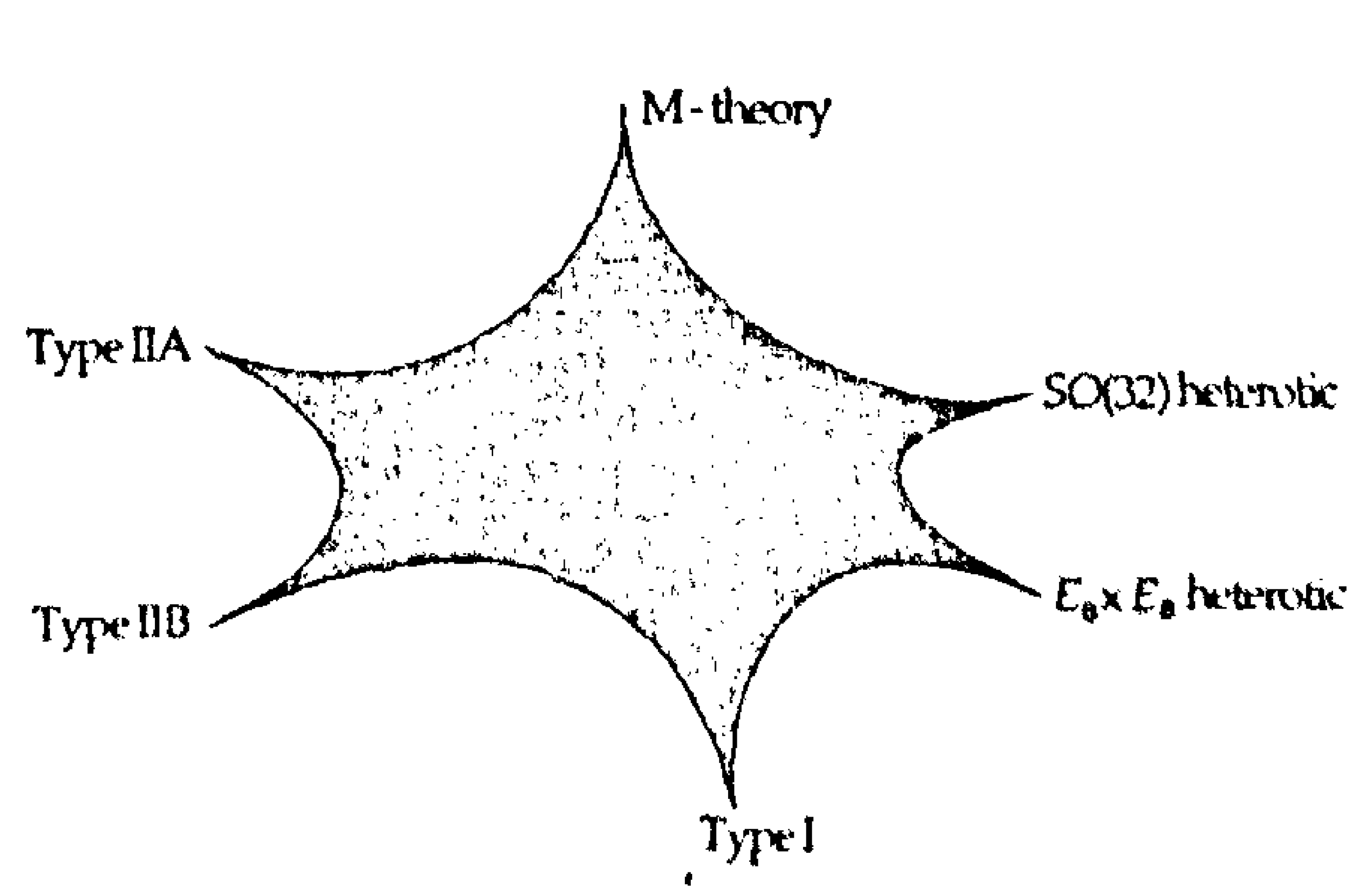

Figure 1. A schematic representation of the space of vacua of M theory. Vacua near the cusps of the diagram are partially understood. Almost nothing is known about all other vacua.

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coupling IIB string theory on weakly curved geometries and energies reduces to IIB supergravity.

Almost nothing is known about the dynamics about most of the very large number of M theory vacua that do not fall into one of the classes described above. As emphasized even dynamics about stringy vacua is completely understood only in the trivial case of zero string coupling. Indeed our ignorance about M theory is great. Until recently there have been no candidates for a complete (note 6) description of the dynamics of fluctuations about even one nontrivial M theory vacuum.

However, work inspired by a conjecture made by Maldacena in 1997 seems to have established that dynamics about certain families of M theory vacua is identical to that of certain well understood quantum field theories called gauge theories.

The gravity–gauge theory correspondence, the topic of this article, seems to have led to the first complete description of a quantum theory containing gravity.

In the next section I introduce background concepts, and describe the Maldacena conjecture. In section 3 I describe checks on the validity of the conjecture, and explain its implications for gauge theories at strong coupling. In section 4 I describe resulting insights into quantum theories of gravity. In section 5 I describe some extensions of the simplest version of the conjecture. Section 6 is a brief conclusion.

The article contains a large number of notes, all of which may be ignored by the reader uninterested in technical details. Further, certain technical portions of this review are identified in the text as being of interest only to the expert. The reader who skips these paragraphs should find his understanding of the rest of the article minimally impaired. I have included a number of references to original papers for the reader interested in specific details. The reader interested in learning more about the entire subject is refered to an exhaustive recent review[1].

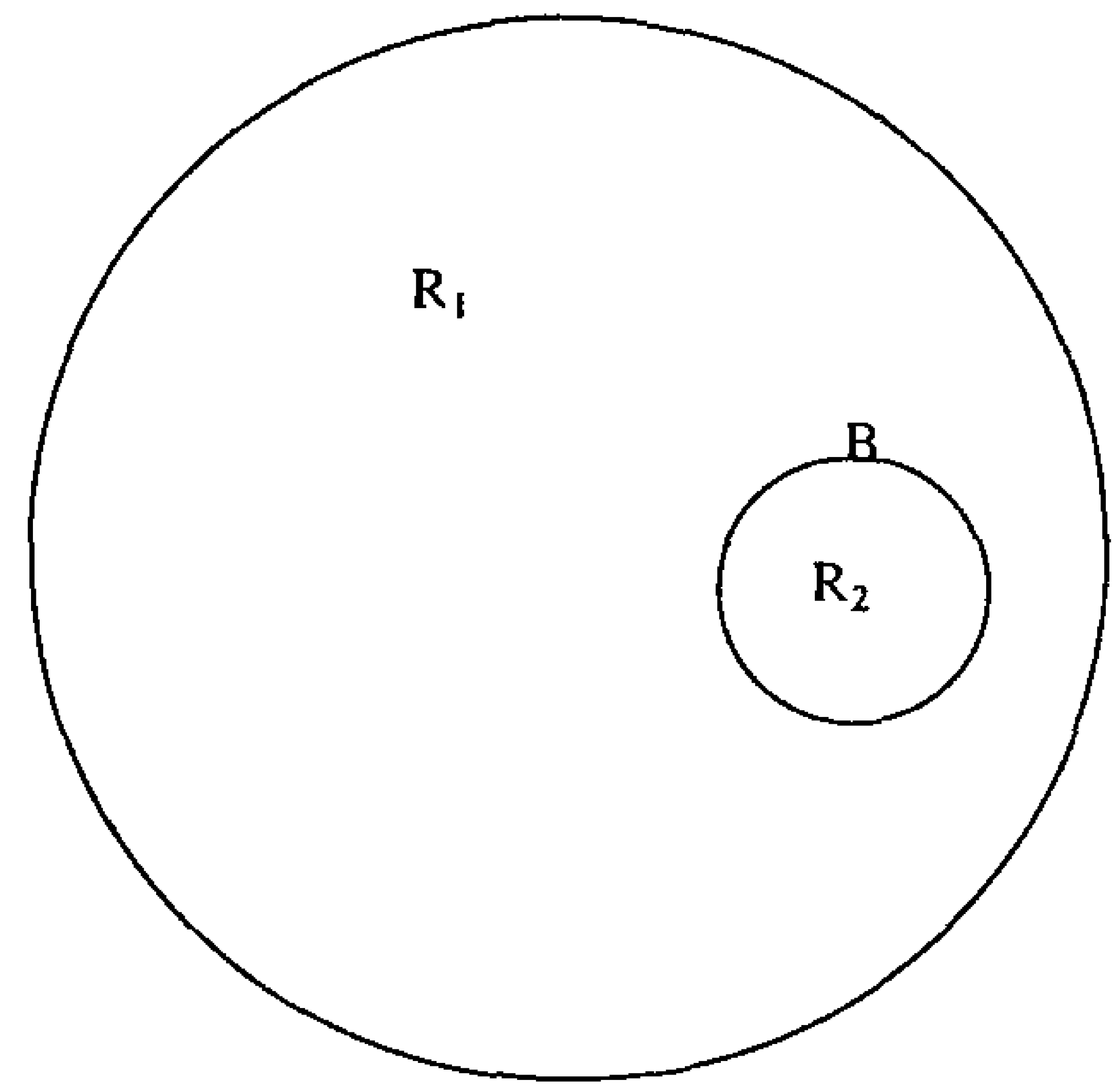

Figure 2. A spacetime consisting of 2 disjoint regions R_1 and R_2 separated by a boundary B (not to be confused with the outer circle, the boundary of the entire spacetime.

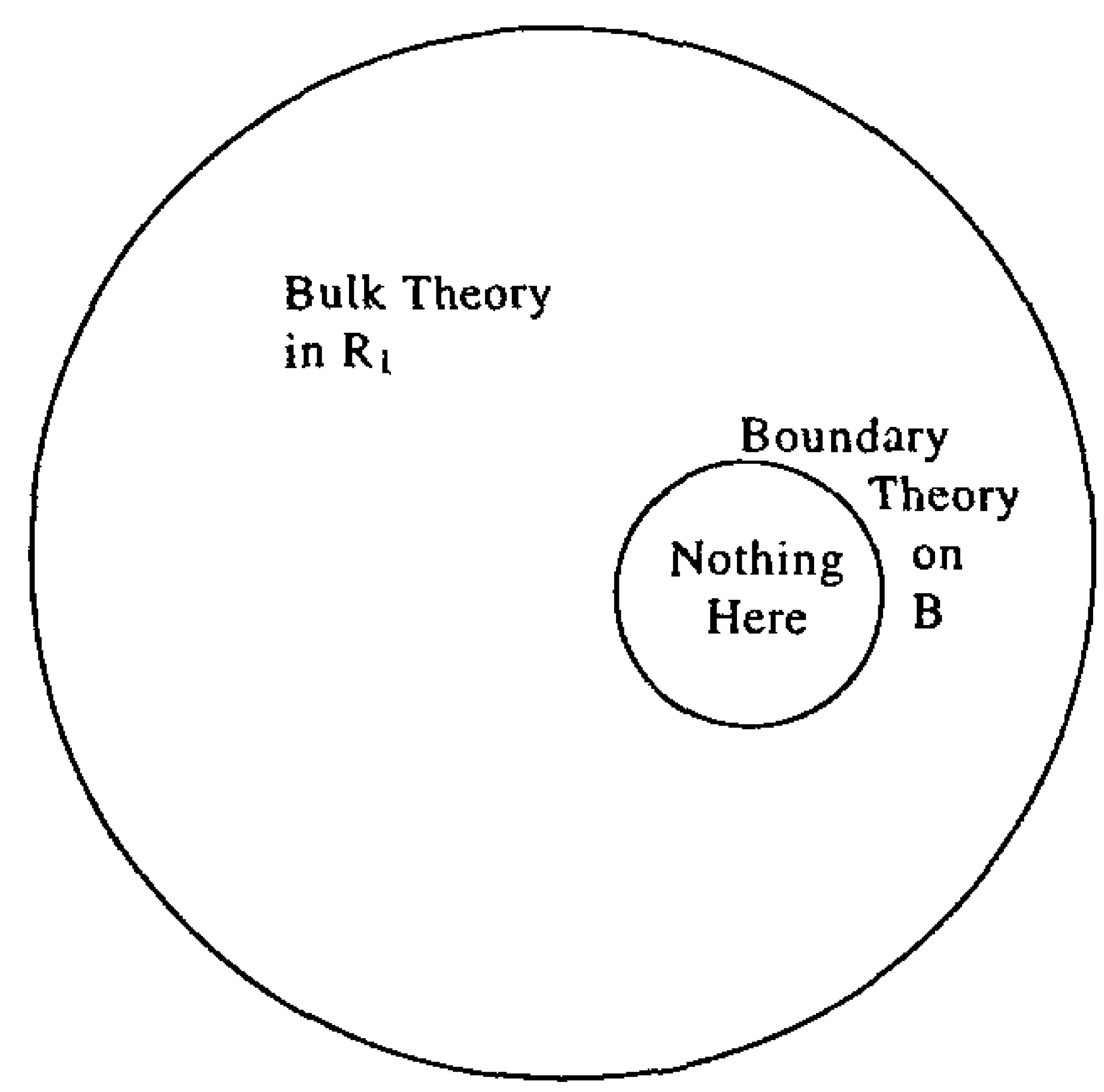

Figure 3. The bulk theory on the full spacetime, is, to an R_1 observer, equivalent to the bulk theory on R_1 interacting with a boundary theory on B, with nothing in R_2.

2. Background to the conjecture

In this section I describe some of the ideas that have come together in the Maldacena conjecture.

2.1 Bulk versus surface and holography

Consider a local quantum field theory defined on the union of two disjoint spaces R_1 and R_2 separated by a boundary B (Figure 2).

Since fields in R_1 couple to those in R_2, the dynamics of R_2 fields cannot be ignored even if one is interested only in the behavior of fields in R_1. However not all details of the theory in R_2 affect the behavior of fields in R_1. It is formally easy to show (note 7) that dynamics of R_1 fields in the theory formulated on the entire space time $R_1 + R_2 + B$ is identical to the dynamics of the same fields in the theory formulated on $R_1 + B$ interacting with a new surface theory, located on B (Figure 3). The surface theory and its interaction with the $R_1 + B$ modes is completely determined by the original theory on R_2, in a manner I describe in technical detail in note 8.

The bulk theory on R_2 defines a surface theory on B. When the theory in the bulk is a local quantum field theory, the converse is not true; computations in the surface theory do not yield answers to all bulk questions (note 9).

However quantum gravitational theories are not local (note 10). It is therefore possible that a bulk gravitational theory on R_2 is fully equivalent to the surface theory it defines on B. Such an equivalence of a bulk gravitational theory and a lower dimensional field theory is called holography[2,3].

The study of the thermodynamics of black holes provides some evidence for holography. The entropy of a large black hole is given by $S = (A/4G)$ where A is its surface area, and G is Newton's constant. Consider a large black hole in a gravitational theory on R_2. If gravity were governed by a local field theory, the entropy of any macroscopic object like a big black hole would be proportional to its volume in R_2. Of course gravitational theories are not local, indeed holography and approximate locality of the boundary theory on B predict that the entropy of a black hole is proportional to its 'volume' in the boundary theory on B, that is to its surface area in R_2 (Figure 4), providing an explanation for the form of the entropy formula.

The Maldacena conjecture provides a concrete realization of holography, as we shall see ahead.

2.2 Yang Mills theories, confinement and strings

The theory governing forces between quarks (the elementary constituents of the proton and the neutron) is called Quantum Chromodynamics (QCD) and is a generalization of the quantum version of ordinary electromagnetism, Quantum Electrodynamics (QED). While charges in electromagnetism are specified by a single number, QCD charges are characterized by vectors in an $N^2_c - 1$ dimensional space. N_c, the 'number of colors', is a parameter of the theory. $N_c = 1$ for QED, and $N_c = 3$ in real world QCD. We will have occasion to consider theories with arbitrary values of N_c in the sequel.

In most quantum field theories, the interaction strength is energy dependent. QCD interactions are weak at high energies or short distances but strong at low energies or long distances. The attraction between a quark and an antiquark at short distances is very similar to that between an electron and a positron. At longer distances interactions increase in strength and the analogy is no longer true; the attractive force between a quark and an antiquark tends to a constant independent of distance implying a long distance potential energy proportional to separation. This results in the phenomenon of *confinement*: the separation of bound states into free infinitely separated quarks is impossible.

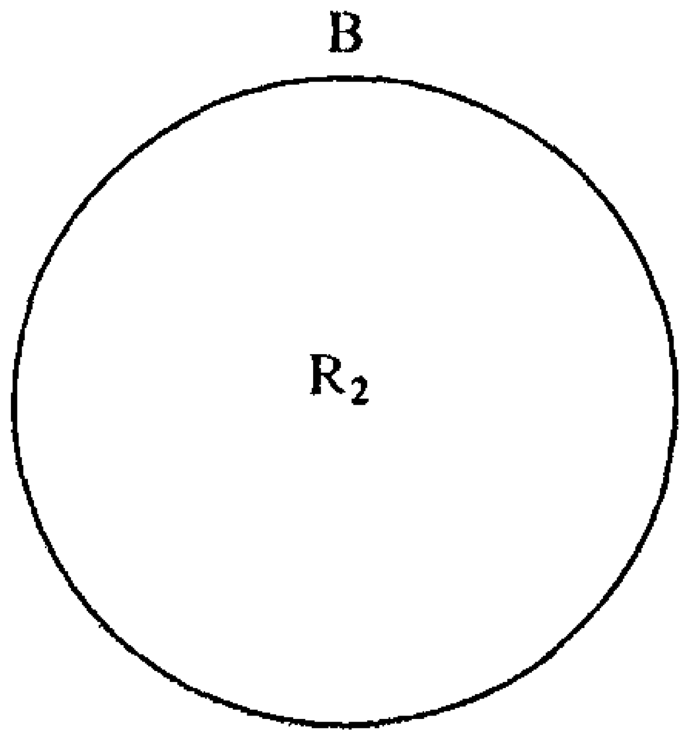

Figure 4. The boundary theory on B is the holographic projection of the bulk theory on R_2.

In the late 70s 't Hooft proposed a qualitative explanation for this behavior[4]. He managed to rewrite QCD as the sum of two copies of electromagnetism each of which contains both electrically and magnetically charged particles. He suggested that the vacuum of QCD, viewed in the new formulation, is a condensate or soup of magnetic particles just as a super conductor is a condensate of electrically charged particles. Electric flux lines in such a vacuum arrange themselves into narrow tubes, in analogy with magnetic flux tubes in a super conductor. A quark and an antiquark at large separation exchange unit electric flux in a long tube whose length, and hence energy, is proportional to the quark–antiquark separation.

Polyakov (note 11) conjectured[5] that it was possible to rewrite QCD exactly as a string theory describing fluctuations of electric flux tubes. Since long distance properties of QCD are dominated by strings of flux, long distance QCD phenomena are expected to be easy to analyze in the string theoretic framework. A technical analysis[4] of the QCD perturbation series demonstrates that QCD strings interact with strength $g = (1/N_c)$. As computations in string theories are easily performed only at weak string coupling, the stringy description of QCD is expected to be useful only at large N_c.

Based on the properties of string theories in less than 10 dimensions, Polyakov proposed that the QCD string should be thought of as propagating in 5 rather than 4 dimensions, as might naively have been supposed as QCD is of course a 4-dimensional theory. As string theories are generically gravitational, the identity of the string theoretic and field theoretic description of QCD may be expected to be holographic (note 12).

There exists a gauge theory closely related to QCD (note 13) that I will refer to as MSQCD through this article. MSQCD has all the fields that QCD possesses and more. In contrast with pure QCD, however, the strength of interaction in MSQCD is independent of energy and the theory looks similar at all length scales (note 14). In particular the quark antiquark attractive force in this theory is proportional to $1/r^2$ at all distances in contrast with pure QCD.

The Maldacena conjecture identifies a string theory holographically dual to MSQCD. This is rather remarkable given that the intuitive motivation for a stringy description of QCD (distance independent force between quarks) is absent in MSQCD. Maldacena's conjecture has convinced most string theorists that there also exists a stringy description of ordinary QCD. Finding it remains one of the most tantalizing open problems in high energy physics.

2.3 D-Branes and Maldacena's conjecture

As reviewed in previous articles, D-branes are solitons in string theory. A Dp-brane is a soliton which is infinite and flat in p spatial dimensions. Consider a bunch of Dp-branes that sit on top of each other. If you hit these branes with a hammer at some point, it sets them vibrating. The vibrations, initially localized to the spot at which the branes are hit, propagate outwards on the p spatial dimensional 'world volume' of the brane sheets. It turns out that two high-energy vibration quanta can collide with each other, metamorphose into a closed string, and leave the brane. However at low energies this process is suppressed, and if two vibrations collide, they can only produce another set of vibrations. Therefore the dynamics of vibrations at low energies is described by a p-dimensional quantum field theory called the worldvolume theory of the Dp-brane.

In this section we will focus on a specific example, the D3-brane which is a 3-spatial dimensional soliton in IIB string theory. At low energies string theory in the presence of D3-branes has two equivalent descriptions. Most directly one studies IIB theory on the spacetime background of Figure 5.

Distances from the brane are measured by a radial coordinate r. This background reduces to ordinary flat 10-dimensional space far (note 15) ($r > L$, where L is a function of the parameters of the theory) from the branes. Near the branes (the tube in Figure 4) it becomes a product of two spaces, a 5-dimensional noncompact infinite space called AdS_5, and a 5-dimensional sphere S^5.

I pause to explain my notation for various geometrical surfaces. A flat d-dimensional space infinite in all directions will be refered to as $\mathcal{R}^d$. A d dimensional sphere is denoted S^d; for instance a circle is S^1. A solid or filled in d dimensional sphere is called B_{d+1}; for instance B_2 is a circle along with all the points in its interior. Just as the circle has a generalization to higher dimensions in the sphere, a hyperbola may be generalized to higher dimensions. A generalized d dimensional hyperbola is called AdS_d, where AdS stands for Anti de Sitter space. AdS_3 is a special example reasonably familiar to many physicists, as it appears in the study of cosmological solutions to Einstein's General Theory of Relativity.

To obtain a second description of physics in the presence of D3-branes, apply the procedure (note 16) of section 2.1 to the spacetime of Figure 5. Let R_1 be the region $r > L$ (the horizontal line in Figure 5) and R_2 the region $r < L$ (the tube in Figure 5). From section 2.1, dynamics in R_1 may as well be described by IIB theory on R_1 interacting with a surface theory on its 9-dimensional boundary B (Figure 6).

R_1 is almost flat spacetime with the chunk $B_6 \times \mathcal{R}^4$ cut out from its center, where B_6 is a six-dimensional ball of radius L. Processes of energy $\omega << (1/R)$ in R_1 involve wavelengths much larger than L, and cannot see the difference between R_1 and flat 10-dimensional space. At such energies bulk physics is captured by IIB theory on flat space interacting with a 4-dimensional (3 space and time) 'boundary' theory at the origin of transverse $\mathcal{R}^6$ (Figure 7). The boundary theory is the holographic projection of IIB theory on R_2 (the tube in Figure 4) which is $AdS_5 \times S^5$. In order to evade the restriction on energies, we take a certain limit of the parameters of the IIB string theory in which the D3-brane is embedded; we take the string mass to be very large (note 17). It turns out that $1/L$ is proportional to the string mass which we take to be so large that all energies of interest obey the condition $\omega << (1/L)$.

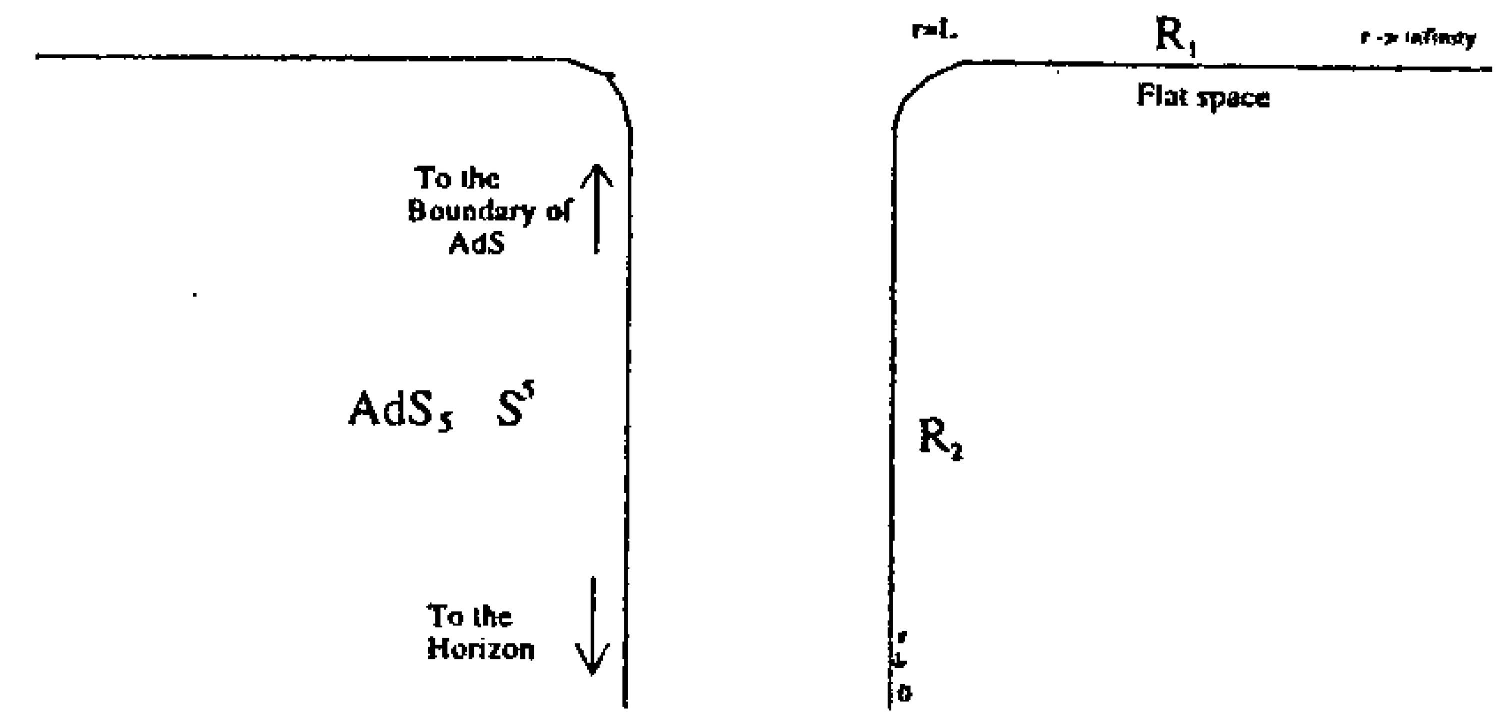

Figure 5. A crude depiction of the 3-brane geometry. Each point on either of the two lines represents a 9 dimensional subspace at constant r, the product of an S^5 (surrounding the 3-branes in 6 dimensional transverse space), and an R^4 (or $T^3 \times R$ for finite branes) the spacetime along the 3-branes. Arc length along the curves in the diagram measures proper distance along the radial direction. r decreases from ∞ to zero as one runs down the curve and $r = 0$ is infinitely far down the tube. The distance of any point from the axis of the diagram represents the radius of the S^5 at that point. For finite branes wrapped on a torus T^3 the size of the torus stays constant along the horizontal line, but shrinks on descending the tube, tending to zero at the bottom.

There happens to exist a completely independent description of IIB theory in the presence of N D3-branes. Four years ago Polchinski identified the sigma model[6] for strings propagating about this background, from which one may deduce that dynamics in the presence of D3-branes at low energies is governed by IIB theory in flat space interacting with the worldvolume theory of D3-branes, which turns out to be MSQCD with $N_c = N$ and an interaction strength related to the string coupling constant (note 18) located at the origin of transverse space.

We are therefore led to conjecture, following Maldacena[7], (see also[8,9]) that MSQCD (note 19) may be thought of as residing at the boundary of $AdS_5 \times S^5$ (note 20) and is the holographic projection of IIB string theory (note 21) on that space.

3. Checking and elaborating the conjecture

In this section I enumerate the direct calculational checks of Maldacena's conjecture. It turns out that checks are rather difficult to perform as the two sides of

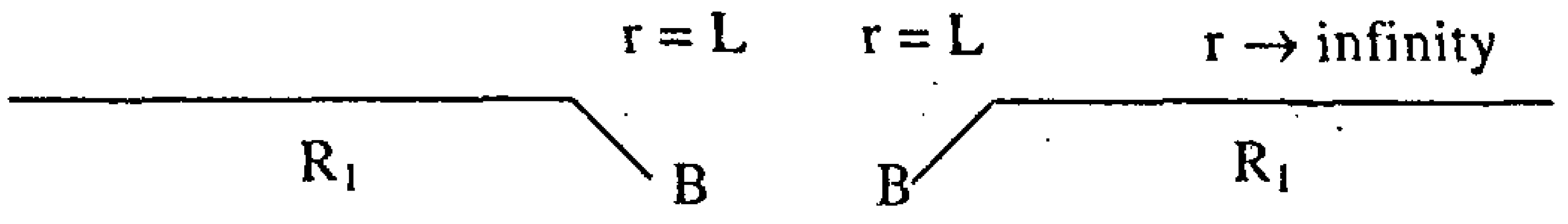

Figure 6. The D3-brane may also be described by IIB theory on the spacetime R_1 interacting with a theory on its boundary B.

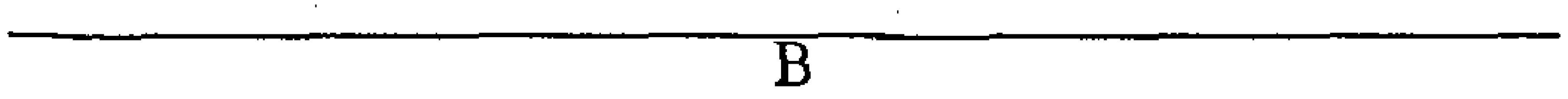

Figure 7. For low energy processes the system of Figure 6 may be replaced by IIB theory on flat 10 dimensional spacetime interacting with a 4 dimensional theory at the origin of transverse space.

Maldacena's proposed equivalence are simultaneously weakly interacting for no values of the parameters specifying MSQCD. However the checks that have been performed all work, verifying Maldacena's conjecture in a nontrivial fashion. Further, the conjecture leads to an intuitively compelling picture of the dynamics of MSQCD at large coupling. The reader willing to take these assertions on faith is advised to skip to section 4, as most of the rest of this section is of necessity rather technical.

IIB theory on $AdS_5 \times S^5$ is understood at weak coupling $g << 1$, when the deviation from flatness of the background over the distance $1/m_s$ is small and at energies $\omega << m_s$. In the limit $m_s \to \infty$ the energy condition (note 22) is always obeyed. The other two conditions are obeyed if $N >> g^2_{YM}N >> 1$ in which case IIB string theory is well approximated by classical IIB supergravity, and calculations are relatively easy to perform (note 23).

Computations directly in MSQCD with $N_c = N$ may in practice be affected only when the effective coupling $g^2_{YM}N = 4\pi gN << 1$. Thus parameter ranges for computability in IIB theory on $AdS_5 \times S^5$ and gauge theory do not overlap. Therefore Maldacena's conjecture is difficult to verify by direct computation.

Some tests of the conjecture are however possible. Firstly, the global symmetry group of MSQCD, the $\mathcal{N} = 4$, $d = 4$ superconformal group, matches the global symmetry group of a gravitational theory on $AdS_5 \times S^5$ (the killing super group of that space).

The conformal anomaly, a measure of the failure of conformal invariance in MSQCD on curved backgrounds, has been computed at strong coupling using supergravity. The conformal anomaly is known to obey a nonrenormalization theorem (its value does not depend on the interaction strength), and so this answer may be compared with a computation in perturbative gauge theory valid at weak coupling; the match is perfect[10].

MSQCD has a special class of operators called chiral operators. Up to a normalization, the two point function of such operators is easy to find; it is determined by the symmetry group of the theory. It turns out that all chiral operators that obey a technical restriction (are primary under the conformal group) correspond, under the Maldacena duality, to BPS (note 24) particles on AdS_5.

IIB theory in 10 flat dimensions possesses 256 BPS particles each of which is quantum mechanically described by a wave function varying on both AdS_5 and S^5. As the space of wave functions on S^5 is infinite dimensional (spanned by the infinite spherical harmonics on the sphere), each of these 256 modes give rise to an infinite tower of AdS_5 particles. Following this procedure, the full list of AdS_5 particles including their masses, spins and other quantum numbers, was worked out[11] in the middle of the 1980s. According to the Maldacena conjecture these particles must be identified with chiral primary operators in MSQCD at large N. Chiral operators in MSQCD have independently been enumerated[12], and at $N = \infty$ (where supergravity is exact), match perfectly with particles on AdS_5.

At finite N, supergravity is expected to receive quantum corrections that qualitatively modify dynamics, and indeed the reduction of wave functions on the sphere yield more particles than exist chiral operators. Agreement is recovered on modifying the list of AdS_5 particles allowing only those spherical harmonics on the sphere whose angular momentum is smaller than N. We have discovered the so-called stringy exclusion principle[13]. Perhaps quantum corrections to supergravity (noncommutatively?[14]) fuzz out spacetime at scale R/N, so that it is impermissible to deal with wave functions on the sphere that vary significantly on that length scale.

Having matched particles in AdS_5 with (conformal primary) operators in the gauge theory, the Maldacena conjecture may be used to compute correlation functions of these operators. Three point correlation functions of all chiral operators in MSQCD at infinite N and large $g^2_{YM}N$ have been computed[15] using supergravity. Remarkably, the result matches perfectly with a perturbative gauge theory computation (valid at small $g^2_{YM}N$). Since the two calculations have non-overlapping domains of validity, their matching was not predicted by Maldacena's duality but is instead a surprising statement about the behavior of MSQCD. This matching has inspired the conjecture[15] that 3, point functions in MSQCD are 'not renormalized' (given by the same formula at all interaction strengths). This conjecture, which may be regarded as a prediction of the gravity-gauge theory duality, has received some computational confirmation (note 25).

Higher point functions of chiral operators in MSQCD certainly depend on interaction strength. The part of the four point function of the stress energy super multiplet that depends exponentially on interaction strength has been computed using gravity at strong coupling[17] and at weak coupling using instanton perturbation theory[18]. The results of these two computations match perfectly, inspiring further confidence in the Maldacena conjecture as well as suggesting yet another (unproven) nonrenormalization theorem. Certain intermediate results in this computation also seem to support the Maldacena conjecture.

I am not aware of further quantitative checks of Maldacena's conjecture. It has however led to several predictions which seem to form a reasonable and self-consistent picture of the dynamics of MSQCD at strong coupling.

In the infinite N limit the free energy per unit volume of the Yang Mills theory at high temperatures takes the form $(2\pi^2/3)N^2T^3f(g^2_{\rm YM}N)$. From SYM perturbation theory $f(x) \approx 1 - \text{const.}\ x + \ldots$ at small x (ref. 19) A supergravity computation yields

$$f(x) \approx \frac{3}{4} + \frac{\text{const.}}{x^{3/2}} + \cdots$$

at large x (ref. 20). From supergravity, the force between a quark and an antiquark separated by distance r is found[21,22] to be proportional to $(g^2_{\rm YM}N)^{1/2}/r^2$ at large $g^2_{\rm YM}N$: The force at small $g^2_{\rm YM}N$ is proportional to $(g^2_{\rm YM}N)/r^2$.

Using supergravity, the $N = g^2_{\rm YM}N = \infty$ behavior of the four point function of a particular set of chiral operators has also been computed[23].

So far we have considered MSQCD formulated on spatial $\mathcal{R}^3$ and time. Turning on a temperature for the theory on such a space cannot have any dramatic effects. This is because MSQCD is conformally invariant and therefore has a symmetry relating high and low energy phenomena. By a scale or conformal transformation any finite nonzero temperature can be converted into any other. However when MSQCD is formulated on a spatial S^3 and time the radius ρ of the S^3 is a new scale in the problem. On this space the gauge theory at finite temperature has a scale-invariant parameter ρT and so the theory can behave very differently at high and low temperatures. Indeed at strong coupling using supergravity one discovers[24] a phase transition (note 26) at a temperature of order the inverse radius of the sphere (note 27). In the low temperature phase, quarks and anti quarks attract each other at long distances with a distance independent confining force. At high temperatures qq attraction falls of as $1/r^2$. Quarks therefore De confine above the critical temperature.

4. Lessons for quantum gravity

Assuming the validity of the Maldacena conjecture, we have a complete description of a two parameter family of quantum theories of gravity, at different coupling strengths, and on different background geometries.

Gravitational theories at strong coupling and on highly curved backgrounds will certainly be intensively studied in the future. We are however most interested in these theories at small curvatures and weak coupling, the limits in which classical gravity is some guide to dynamics. The Maldacena conjecture relates such gravitational theories to very strongly coupled gauge theories, which are well defined in principle, but completely intractable in practice. Therefore the lessons learned about quantum gravity from Maldacena's conjecture have all been qualitative in nature.

The conjecture has produced a proposal for an exact formula for the probabilities of the outcome of a collision (note 28) between particles in flat space IIB theory[25,26], at all couplings and energies, in terms of the correlation functions of a very strongly coupled gauge theory. Though these expressions are useless for calculational purposes, their formal structure may yield useful information under formal manipulations.

The holographic nature of at least some quantum theories of gravity has been established beyond reasonable doubt. Holography may well be a generic feature of all quantum gravitational theories in a yet to be understood precise sense of the term.

Holography has been understood in detail for gravitational theories on AdS spaces. AdS_5 is 5-dimensional. Four of its dimensions (those 'parallel' to its boundary) are identified with the space on which MSQCD propagates. The location of a supergravity process in the fifth (radial) AdS_5 dimension is associated with the spatial or energy scale of the equivalent gauge theory process. This fact[27,28] has been dubbed the UV-IR correspondence, and we elaborate below.

Events that occur near the boundary (top of the tube in Figure 4) of AdS_5 are associated with spatially localized events in the gauge theory. Conversely events that take place near the horizon of AdS_5 (bottom of the tube Figure 4) are spatially very spread out in the gauge theory. For instance, the process of a particle falling down the AdS tube translates into the process of a localized blob of gauge theory energy expanding. Particles cannot fall faster than light in AdS and blobs cannot expand faster than the speed of light in SYM. The Yang Mills theory defined with an upper bound on energies (note 29) is dual to the gravitational theory on truncated AdS, whose boundary is at a finite value (note 30) (not all the way to the top of the tube) of the radial coordinate.

Finally the gravity gauge theory correspondence seems to have answered the long standing question (see Das's article), 'Is information lost in the process of black hole formation and decay?'. All processes in a gravity theory on AdS, including the formation of black holes and Hawking radiation, may be described in the MSQCD, and therefore must have a unitary (information conserving) description, even though this seems difficult to reconcile with Hawking's semi-classical computation.

5. Extensions of the conjecture

The duality between gravity and gauge theories motivated in previous sections may be generalized in several ways.

5.1 D3-branes at singularities

D3-branes in flat space have two equivalent descriptions; this leads to the holographic identification of a gauge theory with a gravitational theory. D3-branes in curved space also allow two equivalent descriptions. However in the (decoupling) low energy limit (equivalently finite energy with $m_s \to \infty$) relevant to the derivation of Maldacena's conjecture, the amplitude of brane fluctuations goes to zero, so that the D3-branes only explore space in their immediate neighborhood and so behave as if they were in flat space.

However IIB backgrounds sometimes contain geometric singularities or surfaces of non-smoothness. Space does not look flat in any neighborhood no matter how small of a singular surface. D3-branes placed on a singular 3-plane behave differently from D3-branes in flat space even in the decoupling limit. Running through Maldacena's argument for these cases leads to new identities[29,30] between gravitational theories on backgrounds related to $AdS_5 \times S^5$ and rather exotic gauge theories. These theories have less supersymmetry than MSQCD, but are nonetheless conformal.

Some new features (that I will not attempt to describe; see[31,32] for instance) of these exotic gauge theories have been uncovered using their gravitational description.

5.2 Other branes

Various M theoretic vacua admit several extended solitons, many of which have been reviewed in Sen's article. The D3-brane about IIB vacua is an example of a particular soliton about a particular background. A Maldacena conjecture formulated for each of these solitons reads, 'The intrinsic worldvolume theory of a particular soliton (the equivalent of MSQCD) is holographically dual to M theory in the near horizon geometry (the equivalent of IIB theory in $AdS_5 \times S^5$) of that soliton'.

In this subsection we comment on this duality for the various maximally super symmetric solitons about various M theory vacua. The remainder of the subsection is technical, and may safely be skipped by a reader uninterested in the finer technical details of M theory and maximally super Yang Mills theories in various dimensions.

The intrinsic theory of M2-branes, obtained in the decoupling limit $m_{11} \to \infty$, is the conformal field theory to which $\mathcal{N} = 8$ $d = 3$ SYM flows at low energies. The intrinsic theory of M5-branes, obtained taking the same decoupling limit, is an intriguing and as yet undefined six-dimensional local quantum field theory which upon compactification on a two torus reduces at low energies to MSQCD. These theories are labeled by a single parameter N, the number of branes as opposed to the two parameters g_{YM} and N of MSQCD, and so are not weakly coupled at any parameter. It is therefore practically impossible to perform computations directly in these theories. The near-horizon geometry of M5 and M2-branes is $AdS_7 \times S^4$ and $AdS_4 \times S^7$ respectively. Using supergravity, the chiral operators in the theory of the M2 and M5-brane have been identified[33–35]. 3 point functions of these operators have also been computed[36] at infinite N for the theory of the M5-brane. Perhaps these new results will stimulate the discovery of a microscopic definition for these theories.

Consider now the various Dp-branes with $p < 3$ in type II theory. These are D0 and D2-branes in IIA theory, and D1-branes in IIB theory. The intrinsic theory obtained in the decoupling limit $m_s \to \infty$, $g^2_{YM} = g m_s^{3-p} =$ const. of each of these solitons is a $U(N)$ maximally super symmetric Yang Mills theory in $d = p + 1$ spacetime dimensions. Yang Mills theories in $d \leq 4$ are well defined (renormalizable). The naive coupling constant in such theories, g^2_{YM}, has mass dimension $4 - d$ and the strength of interactions is measured by $(g^2_{YM}N)/(\omega^{4-d})$ where ω is a typical energy involved in the process. Therefore when $d < 4$ these theories are weakly coupled at high energies, and strongly coupled at low energies. The near horizon geometry[37] of these branes turns out to be highly curved in that part of the geometry near the boundary that corresponds, according to the UV-IR correspondence reviewed in the previous section, to energies so high that the effective coupling $(g^2_{YM}N)/(\omega^{4-d}) << 1$. Provided $N >> 1$, spacetime is weakly curved in that region of the near-horizon geometry (far from the boundary) that corresponds to energies low enough to ensure that $(g^2_{YM}N)/(\omega^{4-d}) >> 1$.

In conclusion, analysis reveals that at large N computations in the D2-brane theory are best performed using perturbative Yang Mills at high energies and 11 dimensional supergravity, at lower energies, on a strange background which reduces to $AdS^4 \times S^7$ (the theory of the M2-brane) 'far down the tube'.

The D0-brane theory is approximated by perturbative Yang Mills at high energies, and by 11 dimensional supergravity in a particular background at lower energies. At still lower energies the spacetime description breaks down.

As an aside, note that in the limit $N \to \infty$ curvatures in the D0-brane near horizon geometry tend to zero over an infinite 10-dimensional spatial volume. Since bulk physics in the entire near horizon region is governed by the theory of D0-branes, $N \to \infty U(N)$ quantum mechanics (the world volume theory of D0-branes) must contain all information about M theory on an infinite flat vacuum[37–39]. A version of this statement, called the M(atrix) conjecture[40], predated the general understanding of Maldacena dualities by a year and a half.

The D1-brane theory is governed by perturbative Yang Mills at high energies, by IIB supergravity on one background at lower energies, on another background at still lower energies, and by a free two-dimensional orbifold conformal field theory[41] at even lower energies.

A similar analysis may be performed for D4-branes, D5-branes and NS5-branes. The intrinsic theory of the NS5-brane is of particular interest. It is expected to be a non-gravitational string theory propagating in 6 dimensions. The holographic description of the theory has recently been used[42,43] to understand some of its features.

I comment that a brane world volume/bulk duality has been conjectured and tested in great detail[44] in a toy model of string theory called topological string theory. Analysis of this or similar toy models may yield insights that lead to a clearer understanding, and perhaps even a proof of the more physical examples of the Maldacena conjecture.

5.3 A string theory of pure QCD?

String theory in flat space is understood beyond the supergravity approximation because the quantum description of the dynamics of a single string in flat space exists, i.e. the conformal field theory describing the propagation of strings in flat space is known. An equivalent description of string dynamics in $AdS_5 \times S^5$ is not available. In this sense, the string theory holographically dual to maximally super symmetric QCD has not really been identified. However we know something about the theory through the supergravity approximation. Further the limit $N \to \infty$, $g^2{}_{\mathrm{YM}}N \to \infty$, the supergravity approximation is exact.

In the absence of the much sought after conformal field theory for the string theory that is equivalent to pure QCD, attempts have been made to understand features of this string theory in the supergravity approximation. Below I describe the technical details of two of these attempts and why they do not succeed. The reader uninterested in technical niceties should skip to the final paragraph of this subsection.

One attempt[24] uses the fact that the intrinsic theory of the M5-brane wrapped on a torus T^2 (with radii R_1 and R_2, supersymmetry breaking periodicity conditions on fermions on R_2) and infinite in the other 4 dimensions is effectively governed by nonsupersymmetric $d = 4$ QCD at energies smaller than the the cut off scale, the smaller of $1/R_1$ and $1/R_2$. The bare coupling of the Yang Mills theory at the cut-off scale is $g^2{}_{\mathrm{YM}} = (R_1/R_2)$. In the limit $N \to \infty$, $\lambda = g^2{}_{\mathrm{YM}}N$ fixed, $g^2{}_{\mathrm{YM}}$ and hence R_1 tends to zero. M5-branes wrapping the 2-torus reduce to D4-branes wrapping the circle of radius R_2 (with supersymmetry breaking fermion boundary conditions on fermions) in this limit. The M theory decoupling limit $m_{11} \to \infty$ reduces to the IIA decoupling limit (note 31) $m_s \to \infty$, $(gN/m_s) = \lambda R_2 = \text{fixed}$. The large N near-horizon geometry deviates from flatness at length scale $(gN)^{1/2}/m_s$, and supergravity is a good approximation only if $gN = m_s R_2 \lambda >> 1$.

The theory thus obtained reduces to the $N = \infty$ limit of QCD at energies smaller than $1/R_2$. To obtain a theory that behaves like QCD at all energies, we take the limit $R_2 \to 0$. Since the QCD interaction strength constant decreases with energy λ is simultaneously taken to be zero in the manner dictated by QCD perturbation theory, $\lambda = b/\ln(R_2\Lambda_{\mathrm{QCD}})$. Unfortunately in this limit $gN = m_s R_2 \lambda \to 0$, so curvatures are large and supergravity is no longer a valid approximation. In summary it is possible to produce a stringy description of a rather beautiful regularization (discretization) of QCD. However as we take the discretization size to zero, in order to recover continuum QCD, the corresponding geometry becomes highly curved. In the absence of a sigma model describing string propagation in the background, we are unable to say anything about this string theory. We are not even able to assert the absence of a phase transition in the passage from large to small discretizations.

A second attempt at understanding pure QCD using gravity involves the near-horizon geometry of D3-branes in a new theory called the type 0 string theory[45] (and references therein), a nonsuper symmetric theory which may or may not be well defined. The intrinsic theory of type 0 D3-branes is quite similar to pure QCD. It is postulated that dynamics in the near-horizon geometry of D3-branes in such theories is holographically related to a QCD-like theory-the boundary of this space. However this near-horizon geometry is also highly curved near its boundary and supergravity breaks down there. Baring surprises, it seems that this approach to QCD suffers from all the complications of the previous approach.

On general grounds it seems unlikely that any supergravity description can capture the behavior of QCD at high energies. QCD is known to possess a high energy spectrum of states characteristic of a string theory, a feature impossible to reproduce in supergravity (note 32). Further at high energies QCD is perturbative, and it is difficult to believe that a theory can simultaneously be governed by perturbative gravity and perturbative gauge theory. Understanding the gravitational dual of nonsupersymmetric QCD at all energies probably calls for creative new ideas.

6. Conclusions

The central current question in theoretical string theory is 'What is M theory'? Although the answer to this question still seems rather distant, the gravity–gauge theory duality seems to have opened a tiny crack to the answer. Perhaps pushing hard enough will widen this particular crack into a window through which the luxurious fields of M theory are visible in all their splendor.

Notes

1. Compared to m_{11}, the 11-dimensional Planck mass, a fundamental constant in M theory.
2. Over the distance $1/m_{11}$.
3. Of radius $R_{11} << 1/m_{11}$.
4. The precise relation is: $g = (R_{11}m_{11})^{3/2}$, where g is the string-coupling constant.
5. Compared to $m_s = (m^3{}_{11}R_{11})^{1/2}$ the so called string mass.
6. I mean a formulation with which one can (given a large enough computer) answer any physical question to arbitrary accuracy.
7. By doing the path integral over R_2 fields.
8. The boundary theory and the interaction action are chosen such that $\langle e^{-S_{\rm int}}\rangle$ ($\langle\ldots\rangle$ refers to the expectation value evaluated in the vacuum of the boundary theory) is identical to the partition function of the bulk theory in R_2 evaluated as a function of boundary values of fields. If the boundary theory is a local quantum field theory, $S_{\rm int}$ may be written in the form $\int_B \phi_i(x)O_i(x)$. $\phi_i(x)$ are bulk fields in R_2 restricted to the boundary B and $O_i(x)$s form a basis in the space of local operators of the boundary theory. According to our prescription $\langle O_{i_1}(x_1)\ldots O_{i_n}(x_n)\rangle$ in the boundary theory is equal to

$$\frac{\delta}{\delta\phi_{i_1}(x_1)}\cdots\frac{\delta}{\delta\phi_{i_n}(x_n)}S_{\rm eff},$$

where $S_{\rm eff}$ is the effective action of the bulk theory on R_2 evaluated as a functional of boundary values of bulk fields on R_2. The bulk theory may thus be used to compute all boundary correlation functions, and so completely specifies the boundary theory.
9. Correlation functions of the boundary theory are identified roughly with S matrix elements of the theory in the bulk. S matrices, however, are not the only observables in a local field theory; these include local Greens functions. Distinct local quantum field theories can possess identical S matrices. Therefore a boundary theory does not fully specify local bulk dynamics.
10. The relabeling of all interior spacetime coordinates in an arbitrary fashion, (small diffeomorphisms) constitutes a redundancy of description (gauge symmetry) in such a theory. True observables are invariant under small diffeomorphisms. S matrices pass this test, but most local correlation functions do not. S matrix elements do not exhaust the list of observables of a gravitational theory, but form a larger subset of this list than in the case of a local quantum field theory.
11. Motivated by the analysis of QCD perturbation theory at large N, and QCD loop equations.
12. String theories are generically gravitational, and their observables include S matrices but not arbitrary Greens functions. QCD is nongravitational and possesses local correlation functions. All this suggests holography.
13. It is technically called $\mathcal{N} = 4$, $d = 4$, SYM where $\mathcal{N}$ refers to the amount of supersymmetry the theory possesses, d to the dimensionality of the spacetime in which it lives, and SYM is short for super symmetric Yang Mills.
14. Technically the theory is conformally invariant.
15. Deviations from flatness occur for r of order $L = ((4\pi gN)^{1/4})/m_s$ or smaller, where N is the number of branes, and g is the IIB string coupling constant and $m_s = \sqrt{R_{11}m_{11}^3}$ is the string mass.
16. We apply the results of that section even though IIB string theory is not local.
17. In this limit R_2 and R_1 modes decouple from each other. This implies in the second picture that the boundary theory decouples from R_1. Therefore the boundary theory on B and the bulk theory on R_1 must be independently well defined.
18. By $g^2{}_{\rm YM} = 4\pi g$.
19. With $N_c = N$ and $g^2{}_{\rm YM} = 4\pi g$.
20. With radius $L = ((4\pi gN)^{1/4})/m_s$.
21. With string coupling g and $m_s \to \infty$. The limit on m_s turns out to be unimportant, as m_s turns out not to be a true parameter in IIB theory on $AdS^5 \times S^5$.
22. When studying the gauge theory on S^3 it is natural to work on the unit sphere, and non-dimensionalize all gauge theory quantities with R. The condition on non-dimensionalized energy is $\tilde{\omega} << g^2{}_{\rm YM}N$. This condition is obeyed at all energies when $g^2{}_{\rm YM}N \to \infty$.
23. The exact sigma model for IIB on $AdS^5 \times S^5$ would enable us to compute at all $g^2{}_{\rm YM}N$ in the limit $N \to \infty$.
24. A particle is said to be BPS when it saturates a certain technical bound having to do with extended supersymmetry.
25. The first correction to the lowest order perturbative result was found to vanish[16].
26. Phase transitions can occur even on compact manifolds at infinite N.
27. The theory on $\mathcal{R}^3$ is the infinite radius (ρ) limit of the theory on S^3. The transition temperature, $\propto (1/\rho)$, therefore goes to zero, and the theory is in the 'high temperature' phase at all temperatures.
28. The S matrix.
29. Analogous to the theory defined on a discrete lattice in spacetime rather than on the spacetime continuum. The lattice version can be engineered to agree well with the full theory only for distances larger than the lattice spacing, or energies smaller than the inverse lattice spacing. One should not ask very short distance (high energy) questions in the lattice theory.
30. That is a function of the energy cut off in the Yang Mills theory.
31. The asymptotic string coupling is $g = (R_1m_{11})^{3/2}$ and $m^2{}_s = m^3{}_{11}R_1$. The D4-brane Yang Mills coupling is $g/m_s = R_1$.
32. More precisely, the Hagedorn spectrum of QCD is composed of glueball states, which should be identified with the excited oscillator states of the QCD string and therefore are out of the domain of the supergravity approximation. Hence all interesting QCD bound states are necessarily out of reach of a supergravity truncation of the QCD string.

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