String theory and Hawking radiation

Sumit R. Das

Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India

Recently string theory has succeeded in providing a microscopic basis for black hole thermodynamics and Hawking radiation. We review the main ideas which led to this development.

1. Introduction

In general relativity black holes are classical solutions with a region of space-time which is causally disconnected from the asymptotic region. The boundary of such a region is called the event horizon. No physical signal can travel from a point inside the event horizon to a point outside. Black holes are believed to be the endpoint of gravitational collapse of massive stars. Astronomers have by now identified several objects as black holes.

An observer who is sitting outside can never 'see' the interior of the horizon. According to her clocks, an infalling object takes an infinite amount of time to reach the horizon; as it approaches the horizon, it slows down and never quite makes it to the horizon. This is because of an effect called the gravitational red shift. Clocks stationed at different points in a gravitational field run at different rates: Generally a clock will appear to run slower as observed by someone who is at a location where the metric components are weaker than that at the location of the clock. As a result, if some physical process emits light at some frequency, it will appear to have a much lower frequency when detected at a position with much smaller metric components, so that there is a redshift. At the horizon, this redshift is infinitely large.

The infinite redshift might give the impression that the gravitational fields at the horizon must be infinitely large. This is not true. For a very massive black hole, local gravitational fields are very weak at the horizon. In fact for a neutral spherically symmetric black hole of mass \( M \) in four space-time dimensions, the magnitude of the space-time curvature, \( |R| \) (which is the measure of the strength of the gravitational field) at a distance \( r \) from the center scales as

\[
|R| = \frac{M}{r^4}, \tag{1.1}
\]

For such a black hole, the radius of the horizon is proportional to \( M \), so that the curvature at the horizon scales as \( 1/M^2 \) and can become arbitrarily small for large \( M \).

In fact, an infalling object will not feel anything special as it reaches the horizon. There is no particularly strong gravitational force and the object will happily proceed to cross the horizon and go 'inside'. The crossing will happen in a finite proper time (the time measured by a clock travelling with it). Thus even though the outside observer never sees the object crossing horizon, she knows that there is a region beyond the horizon where this object will go to.

In 1974 Hawking\(^1\) made a remarkable discovery: he showed that due to quantum effects a black hole is not really black. Rather, it emits a steady stream of particles of all kinds, with a spectrum which is approximately thermal at late times. There is a heuristic way of understanding this process. Due to quantum fluctuations, pairs of particles are always created in a vacuum. Normally they would annihilate quickly. Consider, however, such a process occurring near the horizon of a black hole: in this case, one member of the pair can go inside the black hole – never to come out again; the other member can fly off to infinity. Since the actual state of the particle which went in cannot be measured by an observer sitting far away, he/she would average over these states and this would result in a mixed state. The nontrivial fact being, essentially due to the large redshift at the horizon, that the resulting spectrum is thermal. This radiation is called Hawking radiation.

Because of its thermal nature, one can associate standard thermodynamic properties to a black hole. Remarkably, these properties are rather universal in nature and related to geometric properties of the black hole spacetime. The black hole entropy – called Beckenstein-Hawking entropy, \( S_{\text{BH}} \) – has a leading contribution given, in units \( \hbar = c = 1 \), by

\[
S_{\text{BH}} = \frac{A_H}{4G}, \tag{1.2}
\]

where \( A_H \) is the area of the horizon and \( G \) is the Newton's gravitational constant. This gave a rationale for an earlier conjecture by Beckenstein\(^2\) that one should assign an entropy to a black hole to avoid violations of the second law of thermodynamics, and that entropy should
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Figure 1. Redshift and Hawking radiation by pair creation at horizon.

be the horizon area. For all kinds of black holes in all possible number of space-time dimensions, this is the leading result for large black holes. In the similar way the temperature, called the Hawking temperature $T_H$ is given by

$$ T_H = \frac{k}{2\pi}, \quad (1.3) $$

where $k$ is the surface gravity at the horizon. This is the acceleration felt by a static object at the horizon measured by clocks at infinity. Finally the luminosity is related to the absorption cross-section by the principle of detailed balance

$$ \Gamma_i(k) = \frac{\sigma_i(k)}{e^{\frac{\hbar}{d} \pm 1} (2\pi)^d}. \quad (1.4) $$

Here $\Gamma_i(k)$ is the rate of emission of a particle of type $i$ in a momentum state $k$ with energy $\omega$ and $\sigma_i(k)$ is the corresponding classical absorption cross-section of that particular wave by the black hole. The thermal factor which appears is the standard Bose–Einstein factor for bosons (with $-$ sign) and Fermi-Dirac factor for fermions (with $+$ sign) and the last factor is the standard phase space factor. $d$ is the number of spatial dimensions. $\sigma_i(k)$ is calculated by solving the relevant classical wave equation in the black hole background.

Hawking radiation is a rather robust result. In its derivation, gravity is treated as classical while fields corresponding to particles which are radiated are treated as quantum: this is the semiclassical approximation. A priori this is expected to be a good approximation near the horizon of a very massive black hole. For such a black hole, curvatures are small so that gravity is weakly coupled and its quantum effects are expected to be small. Furthermore, the standard methods of quantum field theory are expected to be valid.

Ever since its discovery, Hawking radiation appeared as a puzzling phenomenon. In other areas of physics, a ‘hot’ body actually means a system which has a large number of degenerate microstates for a given macrostate. Normally we do not care to measure the microstates of the system; so we average over them. This is statistical mechanics, i.e. because of this averaging process we have an incomplete knowledge of the system which is now described in terms of thermodynamic variables. Entropy $S$ is related to the number of microstates, $\Omega$, by the well known Boltzmann

$$ S = \log \Omega, \quad (1.5) $$

which forms the cornerstone of the microscopic derivation of thermodynamics. In a similar way the temperature is given by

$$ \frac{1}{T} = \frac{\partial S}{\partial E}. \quad (1.6) $$

It is conceptually important to know that such a microscopic basis exists. It ensures that in principle there is a complete description of the system in terms of a wave function which has a unitary evolution via Schrödinger equation. The thermodynamic description in terms of a mixed state (i.e. a density matrix $\rho$ for which $\rho^2 \neq \rho$) is only a coarse-grained description, which we use since we do not care to keep track of the details of the microstates.

In the framework of General Relativity it is rather unclear whether the thermal properties of black holes has such a microscopic origin. There exists a set of results in classical general relativity, called no hair theorems, which state that the field of a black hole is uniquely specified by its mass $M$, angular momentum $J$, and the values of the various gauge charges $Q_i$. These parameters specify the macrostates. However, given finite values of these parameters, the horizon area is generically non-zero: in fact, very large for large values of these parameters. The no hair theorems then suggest, however, there are no further microstates for each such macrostate! This makes it rather difficult to believe – as long as we are in the domain of classical general relativity – that the black hole entropy has a statistical origin.

If black hole thermodynamics does not have a statistical origin, there could be a serious problem with the way we understand physical laws. This would indicate that there is a fundamental source of uncertainty in the description of physical phenomena involving black holes. The problem may be highlighted by considering the following thought experiment devised by Hawking$^1$. Consider an initial pure state of some kind of matter, which then collapses to form a black hole. The black hole now radiates in a nearly thermal fashion and as it radiates its mass decreases – it evaporates. Suppose this continues till the black hole evaporates completely. Then at the end, we have thermal radiation in space and
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there is no black hole any more. A pure state has evolved into a mixed state. In other words, the detailed information about the initial state has been lost and all we have is thermal radiation. This would, of course, violate quantum mechanics, which evolves pure states into pure states – as is consistent with unitarity.

Hawking in fact claimed that this is what would indeed happen. It is remarkable that one reached this conclusion putting together two well verified laws of nature – quantum mechanics and general relativity – in a domain where we expect both to be valid.

While such a ‘information loss’ could be possible, most physicists would like to believe that the conclusions are based on rather incomplete knowledge about the quantum theory of gravity. Recall that if there was no microscopic description of an ordinary thermal gas, its thermal nature would have implied a similar loss of information. The statistical basis for thermodynamic laws for gases was put on a firm footing only after the advent of quantum mechanics and its application to atomic phenomena. In a similar way we might expect that if we address the problem of black hole evolution in a consistent theory of gravity which obeys the usual laws of quantum mechanics, there cannot be such an information loss.

Over the past twenty years string theory has established itself as a consistent quantum theory of gravity. Till a few years back, string theory provided only a perturbative theory of gravity. Recent developments related to the discovery of a large number of symmetries of the theory, called duality symmetries, it is becoming increasingly clear that the theory also describes gravity nonperturbatively. If this is so, we should be able to describe black holes and their evolution in a way consistent with quantum mechanics.

In fact it is well known that highly excited states in string theory have large degeneracies. Such massive states would form black holes at sufficiently strong coupling. In 1993 Susskind suggested that this degeneracy accounts for the Beckenstein–Hawking entropy.

In the past three years, this expectation has been borne out to a large extent. We now know that string theory provides a microscopic basis of black hole thermodynamics, at least for a class of black holes. Black hole entropy arises in the usual way due to a large degeneracy of states and black hole radiation is usual quantum mechanical unitary decay whose thermal nature appears only when we choose to ignore the details of the microstates and average over them. String theory provides quantitative predictions for thermodynamic quantities and emission rates which are in exact agreement with the semiclassical answers, in the regime where the latter is expected to be valid. While we still do not have a complete picture of black hole evolution and we do not have a reliable microscopic model for all kinds of black holes, these results go a long way towards a resolution of the information loss problem – in favor of unitary evolution.

In the rest of the article we will discuss the main conceptual elements of this development. Our discussion will be simplified and we will not explain the details of various derivations.

2. Black holes as quantum states

Since string theory is a quantum theory of gravity, black holes must appear as quantum states. How does this relate to the fact that in general relativity black holes appear as classical solutions?

The situation is analogous to electrodynamics. In classical electrodynamics, a point charge is described as a solution of the equations of motion – Maxwell’s equations. The solution describes an electric field satisfying standard Coulomb’s law. This is not the way we describe point charges in quantum electrodynamics. In QED point charges are described as states in a quantum field theory and its evolution is described by the Schrödinger equation for its wave functional. However we know that when the effective charges are large enough, the results of the quantum theory agree with the classical prediction.

Of course quantum electrodynamics is a complicated interacting theory which we don't know how to solve exactly. In fact we do not quite know how to write down the state of a point charge in an exact fashion. There is however one limit in which it is easy to write down this state. This is the limit in which the fine structure constant is very small. In this limit the state of a point charge is described as a state in the theory of a free Dirac field – and we know how to quantize this exactly! But in this limit the state is described merely by a point source and there is no electric field since the latter is proportional to the coupling! We do know, however, how we can start with such a description and proceed to describe electric fields. This happens due to interactions, which we can put in in a perturbative fashion. The effect of these interactions in fact is to produce the correct electric field, as shown for example by the fact that this procedure reproduces the correct Rutherford scattering.

2.1 String theory

In string theory, the quantized normal modes of a closed string appear as an infinite tower of particles. Unlike theories of point particles, consistent quantization imposes severe restriction on what string theories we can have. At the perturbative level, this requires that the theory must live in ten space-time dimensions and has to be supersymmetric. There are five such known perturbative theories which are consistent – and all of

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them have the feature that their spectrum contains a massless spin-2 field which has properties identical to that of a graviton. In fact while we don’t know a complete formulation of the theory we know its low energy behavior: this is given by a usual field theory—supergravity. And we also know that the spin-2 field which appears in the perturbative spectrum is indeed the graviton in this supergravity theory. In other words, the low energy limit of string theory contains standard general relativity. In addition, it contains other massless fields which can be interpreted as the fields required to describe other forces and matter of nature—electroweak and strong interactions.

We will be interested in ‘Type II’-string theories, which are of two kinds, Type IIA and Type IIB. For these theories, the bosonic part of the low energy supergravity action may be written as

\[
S = \frac{1}{2\kappa^2} \int d^{10}x \left[ e^{-2\phi} \left( R + 4(\nabla \phi)^2 - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) + \sum_{(p)} \frac{1}{2p!} F_{\mu_{1}...\mu_{p}} F^{\mu_{1}...\mu_{p}} \right],
\]

(2.1)

where in the sum in (2.1), \( p = 1, 3, 5 \) for Type-IIB and \( p = 2, 4 \) for Type-IIA. \( H_{\mu\nu\lambda} \) is the field strength of a rank-2 antisymmetric tensor gauge field \( B_{\mu\nu} \) which appears in both the theories and \( F_{\mu_{1}...\mu_{p}} \) is the field strength of a rank-(\( p - 1 \)) antisymmetric tensor gauge field. The scalar \( \phi \) is called a dilaton. The coefficient is related to the gravitational constant in ten dimensions and to the basic coupling constant in string theory—the string coupling \( g \) and the string tension \( T_{s} \) by the expression

\[
k^2 = \frac{4\pi^2 g^2}{T_{s}},
\]

(2.2)

where \( R \) is the scalar curvature of a metric \( g_{\mu\nu} \). Note that the gravitational part of the action in (2.1) is not of the standard Einstein–Hilbert form in general relativity, because of the presence of the \( e^{-2\phi} \) factor. For this reason, the metric which appears in (2.1) is called the ‘string frame metric’. One can perform a definition of the metric to recast the action in the Einstein–Hilbert form. In ten dimensions this involves the definition of an ‘Einstein frame’ metric

\[
g_{\mu\nu}^E = e^{-\frac{\phi}{2}} g_{\mu\nu},
\]

(2.3)

and in terms of the Einstein frame metric the factor \( e^{-2\phi} \) disappears from the coefficient of the Ricci scalar.

To describe a four-dimensional world, six of the dimensions have to be compact, i.e. the coordinates which describe them are more like angles. For energies much smaller than the inverse size of the compact directions, these compact directions are invisible and one has a lower dimensional description. This procedure, called dimensional reduction, is the well known Kaluza–Klein procedure. At these low energies, any field which carries a momentum in the compact direction is massive and decouples from the low energy Lagrangian. The effective lower dimensional action for fields which do not depend on the compact coordinates may be written down following a standard procedure starting from (2.1). We will not do this, but mention several important points about the Kaluza–Klein procedure which will be essential for us.

First, if \( i \) denotes a compact direction, the components \( g_{\mu i} \) of the ten dimensional metric behave as electromagnetic gauge fields. In fact higher dimensional theories of gravity were invented by Kaluza and Klein in an attempt to unify gravitation and electromagnetism where electromagnetism of the four dimensional world is actually gravity in higher dimensions. The objects which carry charges under this electromagnetic gauge field (which will be referred as a Kaluza–Klein gauge field) are those which have momentum along the compact direction. Secondly, components of higher rank antisymmetric tensor gauge fields which are along the compact direction would appear as lower rank gauge fields in the non-compact dimensional description. For example, a rank-\( p \) gauge field which has one index along a compact direction \( A_{\mu\nu...i} \equiv A^{(i)}_{\mu\nu...} \) now becomes a \((p-1)\)-rank gauge field. If there are two components along compact directions it becomes a rank-(\( p - 2 \)) gauge field and so on. Thirdly, the gravitational constant \( \kappa_{d}^2 \) of the lower dimensional theory is related to the ten dimensional gravitational constant \( \kappa^2 \) by

\[
\kappa_{d}^2 = \frac{\kappa^2}{V_{10-d}},
\]

(2.4)

where \( V_{10-d} \) is the volume of the \((10-d)\)-dimensional compact internal space. Finally as we dimensionally reduce, the relationship between the string frame metric and the Einstein frame metric changes. If we have \( d \) noncompact space-time dimensions, the relationship is

\[
g_{\mu\nu}^E = \exp \left[ \frac{4\phi}{d - 2} \right] g_{\mu\nu},
\]

(2.5)

Several consistent choices of such compact manifolds are known, but we don’t know any criterion which chooses one such compactification over another. It is nevertheless interesting that compactifications exist which lead to models which are supersymmetric extensions of the standard model with three generations of quarks and leptons.
2.2 Black holes in string theory

In any case, string theory contains gravity—and therefore it contains black holes. In fact any solution of general relativity is necessarily a solution of string theory. String theory of course has more solutions and it has more fields. Clearly the conceptual problem about information loss can be posed in string theory and should have a definitive resolution. This is regardless of whether we know enough about string theory to make contact with the observed world. Hawking radiation is a feature of any theory which contains general relativity in any number of dimensions—the fact that other fields are present is not important. If string theory is a consistent quantum theory of gravity, one should be able to describe Hawking radiation in a unitary fashion.

To describe black holes, however, we need to know nonperturbative features of the theory. Dramatic developments since 1994 have given us a large body of knowledge about nonperturbative properties of string theory. Admittedly, we do not have a complete formulation of string theory at the moment. However we do know that these nonperturbative properties are correct and should be present in a complete formulation. Knowledge about these non-perturbative properties came with the realization that string theory probably admit a class of symmetries called duality symmetries. Some of these symmetries (called S duality) relate a particular perturbative string theory at weak coupling to the strong coupling behavior of another string theory. Yet other symmetries (called T dualities) relate a string theory on one background to (generally) another string theory on a different background. Of course we cannot prove duality: this would require a complete formulation of the theory. The evidence for T-duality comes from perturbation theory, which we do know. S-duality is nonperturbative and evidence for this comes from various sources—low energy effective actions, classical solutions, and above all, the extremely impressive self-consistency of consequences derived from the assumption that this is a symmetry. A crowning achievement of this development has been the realization that the different perturbative string theories are simply different vacua of a single theory. We don't know this grand theory—but it is probably not a string theory in the conventional sense. In fact we know that in a certain regime the low energy behavior of this theory is governed by eleven dimensional supergravity.

One of the things which we have learnt in the ‘duality experience’ is the ability to predict accurately the properties of a certain class of states at strong coupling. As we will see soon, this ability will turn out to be crucial in understanding black holes. The second important outcome has been the discovery of a class of solitonic objects, called D-branes which turned out to be very useful examples of black holes where nonperturbative results could be derived.

Let us first discuss the regime of parameters we need to work in order to understand the black hole problem from a microscopic viewpoint. Consider for example a neutral black hole of mass $M$ in four dimensions. The space-time metric is given by

$$ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)} + r^2d\Omega_2^2.$$  (2.6)

Here $d\Omega_2^2$ is the metric on a unit 2-sphere and $G$ is Newton’s constant. The horizon is at $r_H = 2GM$ and the curvature at the horizon is of the order of $1/(GM)^2$.

In string theory, the coupling is given by a dimensionless coupling constant $\alpha'$ and there is only one length scale in the theory, the string length $l_s$ which is related to the string tension $T_s$ by $T_s = 1/(2\pi\alpha')$. We can express every dimensional parameter in the four dimensional reduction of the theory in terms of this string length (e.g. the size of the compact directions can be expressed in units of the string length). Thus on one-dimensional grounds we must have

$$G - g^2 l_s^2, \quad M = \frac{m}{l_s}.$$  (2.7)

The dimensionless quantity which controls the gravitational field of the black hole is therefore given by $g^2 m$.

Since $g$ is the coupling constant of the theory, the classical limit is given by $g \to 0$. However to describe a nontrivial classical solution $g^2 m$ must be finite, i.e. one must have

$$g \to 0 \quad m \to \infty \quad g^2 m = \text{finite}.$$  (2.8)

In fact, when $g^2 m \gg 1$ the horizon is large and the curvature at the horizon is weak—this is the regime where we can trust the semiclassical calculations of Hawking. Thus the effective coupling we are interested in is $g^2 m$ – the semiclassical limit is the regime where this effective coupling is strong.

On the other hand we might expect that the microscopic description of black holes as states in a quantum theory would be tractable when the effective coupling is weak. In this regime the geometric description of a black hole is not very good and Hawking’s semiclassical results do not apply. If our microscopic results about black hole states were valid only in this limit, there would be no way to make contact with the semiclassical picture and we would not be able to decide whether large black holes have a microscopic description.

Fortunately string theory provides examples of black hole states for which special symmetry properties ensure
that we can reliably extrapolate weak coupling answers to the strong coupling domain. These happen to be charged black holes. At least for this class of black holes, string theory does provide a microscopic description which explains the thermodynamic behavior exactly.

3. D-branes

The perturbative spectrum of string theories consists of the various normal modes of a string. However, string theory is not a theory of strings alone. Once one goes beyond perturbation theory one finds that there are various kinds of extended objects in the spectrum: these are generically branes. The existence of these objects is in fact intimately related to and required by duality symmetries of the theory. p-branes are p dimensional extended objects whose worldvolume is therefore p + 1 dimensional. They have a finite energy density determined in terms of the string tension and the string coupling. Out of the p-branes, only one of them, the fundamental 1-brane, also called the F-string, is present in the perturbative spectrum of the theory. All the others are solitonic states.

If a p-brane is wrapped around a p-dimensional compact manifold, the object has a finite total mass. At energies much smaller than the scale of compactification, this will appear as a point like object in the non-compact world. Since this has a mass, it will produce a gravitational field. Furthermore a p-dimensional extended object naturally couples to a p + 1 form gauge field via a coupling

\[ \int d\sigma^{\mu_1\mu_2...\mu_{p+1}} A_{\mu_1\mu_2...\mu_{p+1}} \]

(3.1)

where the integration is over the (p + 1)-dimensional worldvolume of the p-brane. This is similar to the way a point particle couples to a Maxwell gauge field. In other words, such p-dimensional objects can also produce a p + 1 form gauge field. In fact the type of p-branes present in the spectrum of a particular string theory is largely dictated by the presence of the corresponding gauge field in the perturbative spectrum.

There is a special class of such solitonic states which has played an important role in black hole physics. These are called D-branes and appear in what are called the Type-II string theories. There are two Type-II theories, named Type-IIA and Type-IIB, respectively. The D-branes which appear in Type IIA are of dimensionality p with p = 0, 2, 4, 6 with p + 1 dimensional worldvolumes and produce p + 1 form gauge fields. They will be called D-p-branes. The D-branes which appear in Type-IIB theory are of dimensionality 1, 3, 5 and produce rank-2, rank-4 and rank-6 gauge fields. The possible values of p are thus determined by the presence/absence of the corresponding gauge field in the spectrum. Note that some of these gauge fields could be 'magnetic' in nature. Thus the D-1 brane produces an electric type rank-2 gauge field, while a D-5 brane produces a magnetic type rank-2 gauge field. However the gauge field strength of the latter has rank-3 and this is dual, in ten dimensions, to a rank-7 field strength which in turn comes from a rank-6 gauge field – the one appropriate for coupling to a five-dimensional object. In this article we will deal mostly with Type-IIB theory.

3.1 Classical solutions

In the semi-classical domain discussed above, D-p-branes are represented by classical solutions of the low energy supergravity equations of motion. The string metric is given by

\[ ds^2 = [A(r)]^{-\frac{1}{3}} \left[ \left( 1 - \frac{\rho_0}{r^2} \right) dt^2 + \sum_{i=10-p}^9 dx_i dx_i \right] + [A(r)]^{-\frac{1}{3}} \left[ \frac{dr^2}{1 - \frac{\rho_0}{r^2}} + r^2 d\Omega_{p+1}^2 \right]. \]

(3.2)

Here the brane is along the directions \( x^{10-p}...x^9, n = 7-p \), \( r \) is the transverse radial coordinate, \( r^2 = \sum_{i=1}^{9-p} (x^i)^2 \), and \( A(r) \) is the function, i.e.

\[ A(r) = 1 + \frac{\rho_0}{r^2} \sinh^2 \alpha. \]

(3.3)

The branes also produce a dilaton, i.e.

\[ e^{2\Phi} = [A(r)]^{\frac{2}{3}}, \]

(3.4)

and a p + 1-rank gauge field with the components

\[ A_{0,0,0=\ldots,n} = \frac{1}{2} \frac{\rho_0}{r^2} \sinh (2\alpha). \]

(3.5)

This is Coulomb's law in the transverse directions. Since there are both electrically and magnetically charged D-branes the charge is quantized, in terms of integers \( Q_p \) given by

\[ Q_p = \frac{1}{8\pi g} \frac{\sinh 2\alpha \cdot \rho_0}{\Gamma^2(t) r^n}. \]

(3.6)
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It is sometimes useful to visualize a solution with integer $Q_p$ as a collection of $Q_p$ objects of unit charge each – in common parlance $Q_p$ D-p-branes.

The solution (3.2) in fact represents a black p-brane extended along the directions $x^{10-p} \ldots x^9$, which becomes a black hole when these longitudinal directions are compactified on say a p-torus. In terms of the noncompact $(10 - p)$ dimensional world, this object carries an electric Maxwell like charge, since the rank-$(p + 1)$ gauge field (3.5) becomes an electric potential under dimensional reduction and $Q_p$ is the quantized charge.

The horizon is at $r = r_0$; has the shape of $S^{n+1} \times T^p$ and has an area in Einstein metric given by

$$A_H = V_p \Omega_{n+1} r_0^n \cosh \alpha,$$  

(3.7)

where $V_p$ is the volume of the p-compact directions. The standard semi-classical methods leading to Hawking radiation apply to this particular black brane, and the Hawking temperature is

$$T_H = \frac{n}{4\pi r_0 \cosh \alpha}.$$  

(3.8)

The Beckenstein–Hawking entropy is

$$S_{BH} = \frac{A_H}{4G} = \frac{2\pi A_H}{k^2}.$$  

(3.9)

Viewed as a black hole in the noncompact directions, the horizon area is $A_H/V_p$, but the lower dimensional Newton constant is also decreased by the same factor (see eq. (2.4)) so that the entropy of this black hole is the same as that of the black brane.

There is a special limit of the classical solution which is of great importance to what follows. This is the extremal limit defined by

$$r_0 \to 0 \quad \alpha \to \infty \quad (r_0^n \sinh^2 \alpha = r_p^n \text{ fixed}).$$  

(3.10)

In this limit the mass and charge of the object remains finite, while the area and the temperature goes to zero. It is easily seen that in this limit the mass of the object is proportional to the charge.

The main significance of the extremal limit is that in this limit the solution retains some of the supersymmetries of the original theory. Superstring theory is supersymmetric. However a generic classical solution would break the supersymmetry. The extremal solution retains half of the supersymmetries and are examples of what are called BPS states. BPS states are the lowest energy states of the system with a given value of the charge – these are the ground states in this sector. Because of this, these are stable states – a fact that is consistent with the fact that the Hawking temperature is zero – these extremal black branes do not radiate.

3.2 Microscopic theory

In string theory D-branes of course appear as quantum states. These are in fact quantum states of a class of solitonic objects. The description of the low energy quantum excitations of such objects is remarkably simple. Clearly the collective dynamics of these low energy modes would be governed by a $(p + 1)$-dimensional field theory (one of them being time), just as the dynamics of point like solitons are described by a one dimensional field theory. It turns out that for a collection of D-branes of charge $Q_p$ this field theory, which lives on the brane is in fact a gauge theory with a gauge group $SU(Q_p)$ with 16 supersymmetries. The coupling constant of this Yang–Mills theory is given by

$$g^2_{YM} = g.$$  

(3.11)

Therefore the limit when this object has a reliable semi-classical description in terms of the classical solution described above may be read off from the solution of eq. (3.2) and the expression for the charge given in eq. (3.6),

$$g \to 0 \quad Q_p \to \infty \quad g^2 Q_p = \text{ fixed and large}.$$  

(3.12)

From twenty we see that this is a well-known limit in gauge theories, the ’t Hooft large-$N$ limit

$$g_{YM} \to 0 \quad Q_p \to \infty \quad g^2 Q_p = \text{ fixed},$$  

(3.13)

and the ’t Hooft coupling $g^2 Q_p$ is strong. To understand the microscopic behavior of these D-p-branes we have to understand the strong coupling limit of this supersymmetric gauge theory.

In fact, this gauge theory is in turn a low energy description of a theory of open strings whose ends are stuck on the brane. This is how D-branes are defined in string theory. Just as the low energy spectrum of closed strings is described by supergravity, the low energy spectrum of a theory of open strings is given by a supersymmetric Yang–Mills theory. Since the ends of the open strings are constrained to move along the brane, the super Yang–Mills theory lives on the $(p + 1)$ dimensional worldvolume of the brane.

Ground states of the D-p brane system are then described by the ground states of the Yang–Mills theory, while the excited states are described by the excited states of the Yang–Mills theory. Thus if we can obtain the spectrum of this gauge theory we will know the degeneracy of states and see whether the Beckenstein–Hawking entropy is indeed given by the Boltzmann formula.

However the system is described by the semiclassical solution only when the ’t Hooft coupling is large. It is
easy to determine the spectrum and hence the degeneracies of states at weak coupling — but we don’t know how to solve the theory at strong coupling. How do we then compare the microscopic results with the classical answers?

This is where the supersymmetry of the extremal solutions comes to the rescue. Because of supersymmetry, the BPS states of any theory have well-known renormalization properties. In particular the relationship between the mass and the charge follows solely from the supersymmetry algebra and is therefore an exact relation in the full quantum theory. If we know that a BPS state with a given charge is present in the perturbative spectrum, we know that such a state exists at strong coupling as well, and it would continue to have the same mass — even though it would be difficult to write down the state in terms of the free field operators. Thus the number of BPS states for a given value of the charge is independent of the value of the coupling and therefore can be evaluated at weak coupling. For such states, we can perform a weak coupling calculation and compare them with the semiclassical answer.

A collection of $Q_p$ D-branes in their BPS ground state as described by the classical solution eq. (3.2) has a rather small degeneracy — in fact, it has precisely 256 states. However one can construct other BPS states by introducing a momentum along one of the brane directions. Since the brane directions have been taken to be compact, this momentum is necessarily quantized

$$P = \frac{N}{R}, \tag{3.14}$$

where $R$ is the radius of this compact direction and $N$ is an integer. Under dimensional reduction, this momentum becomes a Kaluza–Klein charge, and we now have a black hole with two kinds of charges: a charge $Q_p$ under the Maxwell gauge field arising from the dimensional reduction of the rank $(p + 1)$ gauge field in ten dimensions and a Kaluza–Klein charge $N$. The corresponding classical solution may be obtained from (3.2) by performing a Lorentz boost in the direction of momentum and then taking the extremal limit. It turns out that even after this the horizon area is zero in the extremal limit and therefore there is no Beckenstein–Hawking entropy.

On the other hand, the BPS state of such a two charge state has a macroscopically large degeneracy. This is what was shown in $^8$ for states similar to D-brane states in heterotic string theory. It appears that this degeneracy does not have anything to do with black hole entropy!

This conclusion turns out to be wrong. The point is that in the extremal limit the horizon has shrunk to a point. The curvatures are large at this horizon and there is no reason why one should trust the supergravity solution here. In fact since the curvatures are large compared to the string scale, stringy effects will be important. In fact it was argued in $^9$ that due to stringy corrections we should define the Beckenstein–Hawking entropy not as the area of the event horizon, but the area of a 'stretched horizon'. The stretched horizon is the place where the curvature is of the order of the string scale and has a nonzero area. Sen showed that the area of this stretched horizon is in fact proportional to the logarithm of the degeneracy of states. However there was no way to determine the proportionality constant and therefore no definitive way to understand whether black hole entropy has a microscopic origin in terms of the degeneracy of string states.

It is clear now that to have a reliable microscopic calculation of the degeneracy of states BPS states are useful. On the other hand for the corresponding classical solutions to remain reliable at the horizon we need to have BPS solutions which have large horizon areas. If we can find examples of such black holes in string theory we will be able to test the conjecture that black holes are string states and the black hole entropy originates from the degeneracy of such states.

4. The five dimensional black hole: Classical solution

The first example of such a black hole was a five dimensional charged black hole $^9$. This is constructed out of $Q_1$ D-5-branes along $x^4 ... x^9$, $Q_1$ D-1-branes along $x^5$ and $N$ units of momentum along $x^5$. When dimensionally reduced this becomes a five dimensional black hole with three kinds of charges. The coordinates $x^4 ... x^9$ are coordinates on a 5-torus. The classical solution for the general non-extremal version is given by the following string metric

\[ \text{Figure 2. Open strings on D-branes.} \]
\[ ds^2 = A_t(r)^{-1} A_s(r)^{-1} \left[ -dr^2 + dx_1^2 + \frac{r_0^2}{r^2} (\cosh \alpha r + \sinh \alpha x^5) \right] \]

\[ A_t(r)^{-1} A_s(r)^{-1} [(dx^6)^2 + \ldots (dx^7)^2] \]

\[ A_t(r)^{-1} A_s(r)^{-1} \left[ \frac{dr^2}{1 - \frac{r_0^2}{r^2}} + r^2 d\Omega_5^2 \right] \]

(4.1)

where

\[ A_t(r) = 1 + \frac{r_0^2}{r^2} \sinh^2 \alpha_1, \]

\[ A_s(r) = 1 + \frac{r_0^2}{r^2} \sinh^2 \alpha_5. \]

(4.2)

There is also a nontrivial dilaton \( \phi \) and a two-form gauge field with gauge field strength \( H_3 \) having both magnetic and electric components, i.e.

\[ e^{2\phi} = \frac{A_t(r)^{-1}}{A_s(r)^{-1}}, \]

\[ H = (r_0^2 \sinh 2\alpha_5) e_3 + (r_0^2 \sinh 2\alpha_1) \star e_3, \]

(4.3)

where \( e_3 \) is the volume form on the three-dimensional sphere in the transverse space and \( \star \) denotes the six-dimensional dual in the six-dimensional space-time formed by the transverse space and \( x^5 \). The charges are related to the parameters in the classical solution by

\[ Q_1 = \frac{V_4 r_0^2 \sinh 2\alpha_1}{32 \pi^2 g^2 \alpha^4}, \]

\[ Q_5 = \frac{r_0^2 \sinh 2\alpha_5}{2g^2 \alpha^4}, \]

\[ N = \frac{V_4 r_0^2 \sinh 2\alpha}{32 \pi^2 g^2 \alpha^4}. \]

(4.4)

Here \( V_4 \) is the volume of the four torus along \( x^1 \ldots x^3 \), and \( R \) is the radius of the \( x^3 \) direction. The mass \( M \), Bekenstein–Hawking entropy \( S_{BH} \), and the Hawking temperatures, respectively are:

\[ M = \frac{RV_4 r_0^2}{32 \pi^2 g^2 \alpha^4} (\cosh 2\alpha_1 + \cosh 2\alpha_5 + \cosh 2\alpha), \]

\[ S_{BH} = 2\pi \sqrt{Q_1 Q_5 N} \coth \alpha_1 \coth \alpha_5 \coth \sigma, \]

\[ T_{BH} = \frac{1}{2\pi r_0 \cosh \alpha_1 \cosh \alpha_5 \cosh \sigma}. \]

(4.5)

In the extremal limit

\[ \alpha_1, \alpha_5, \sigma \to \infty \quad r_0 \to 0 \]

(4.6)

while the Hawking temperature vanishes. In this extremal limit the mass becomes a sum of the contribution of the masses from the 5-brane, 1-brane and the momentum. It is also clear from the metric that in this limit, the momentum along \( x^5 \) is made out of only left moving waves, since only the combination \( (dt + dx^5)^2 \) appears in eq. (4.1).

In the following we will be interested in a special departure from extremality, where we keep \( \alpha_1, \alpha_5 = \infty \), but \( r_0, \sigma \) finite. This means that we have both left and right moving waves. Indeed the thermodynamic quantities split up into sums of left and right moving parts, i.e.

\[ S_{BH} = S_L + S_R \quad E = E_L + E_R, \]

\[ \frac{1}{T_H} = \frac{1}{2} \left( \frac{1}{T_L} + \frac{1}{T_R} \right), \]

\[ S_{LR} = \frac{RV_4 r_0^2 e^{-2\alpha}}{16 \pi^3 g^2 \alpha^4}, \]

\[ T_{LR} = \frac{g^2 e^{2\alpha}}{2\pi r_0}, \]

\[ E_R = \frac{4\pi^2 V_4 R^2 r_0^2 e^{-2\alpha}}{g^2 \alpha^3}, \]

\[ E_L = \frac{N}{R} + E_R. \]

(4.8)

In the expression (4.8) \( E_L, E_R \) stands for the energies of the extremal 1-branes and 5-branes respectively. Note that

\[ T_{LR} = \frac{S_{LR}}{2\pi^3 Q_1 Q_5 R}, \]

(4.9)

a relation which will be useful later.

The semi-classical picture of these black holes is reliable when the charges are all large, \( g \to 0 \) \( Q_1, Q_5 \), \( N \to \infty \) with \( (gQ_1), (gQ_2) \) and \( g^2 N \) held fixed and large The extremal state is a ground state in the given charge sector and has zero temperature. When we excite this, we have a nonzero temperature and the black hole starts radiating. The radiation will continue till the black hole loses enough mass so that it becomes extremal again – this will take an asymptotically long time. The important point is that for large values of the charges, the curvatures at the horizon are weak at all stages of the evolu-
tion and one would expect Hawking’s results to be reliable. The information loss problem can be addressed in this example and one may hope that one can have a definitive resolution of the problem since high curvatures are never encountered.

5. Microscopic theory of the five-dimensional black hole

The microscopic theory of this black hole may be written down from what we know about D-branes. We have a system of 1-branes, 5-branes and momentum which forms a bound state. The low energy dynamics is again described by a gauge theory, which has ‘matter’ fields in addition to gauge fields. Determining the dynamics of this theory is rather complicated, but the final result is rather simple. The low energy modes are just massless quanta of waves moving along the 1-brane. However the 1-brane is now multiply wound $Q_1 Q_5$-times around the $x^5$ direction. This means is the following: these modes can carry momenta which are quantized not in units of $1/R$ but in units of $1/Q_1 Q_5 R$, though the total momentum is quantized in units of $1/R$. These quanta have four ‘flavors’. This is because the 1-brane can oscillate freely in the four directions lying on the four torus (in the $x^6, \ldots, x^9$) which is still transverse to the 1-brane but lying on the 5-brane. These account for four polarizations. The 1-brane cannot oscillate freely in the direction transverse to the five brane since the system is bound.

The extremal limit corresponds to the situation when all the quanta are moving in the same direction. They are all moving at the speed of light and cannot catch up with one another – which is an intuitive explanation why this state is stable. Nonextremal states of the type discussed above (with $\alpha_1, \alpha_5$ still infinite) correspond to waves in both directions: this seems to tie up with the clean left-right separation of the semiclassical thermodynamic quantities.

5.1 Thermodynamics

What we have is statistical mechanics of massless particles in one spatial dimension and we have to figure out the degeneracy of states with a given total energy $\varepsilon$ and total momentum $P$. The total energy of this gas is identified with the quantity $E_L + E_R$ of the classical solution while the total momentum is the difference $E_L - E_R$. Likewise one can introduce two ‘temperatures’, $T_L$ and $T_R$. Note that the temperature

$$\frac{1}{T} = \frac{1}{2} \left( \frac{1}{T_L} + \frac{1}{T_R} \right)$$  \hspace{1cm} (5.1)

which is conjugate to the total energy. The reason why we have different left and right moving temperatures is that there is a net momentum in the system, so that there is another intensive quantity $\alpha$ which is conjugate to the total momentum

$$\alpha = \frac{1}{2} \left( \frac{1}{T_L} - \frac{1}{T_R} \right).$$  \hspace{1cm} (5.2)

The effective coupling constant of this gas is $g_{\text{eff}} = g Q_1$.

When the effective coupling is weak we have a free gas and the statistical mechanics may be easily worked out. We have four flavors of bosons and four flavors of fermions. The thermodynamic relations which follow are

$$T_i = \left( \frac{2E_i}{\pi L} \right)^\frac{1}{\hat{t}},$$ \hspace{1cm} (5.3)

$$S_i = \left( 2\pi E_i L \right)^\frac{1}{\hat{t}}.$$

In the eq. (5.3) $i$ stands for L or R and $L$ is the size of the one-dimensional line on which the gas lives. In our case, $L = 2\pi Q_1 Q_5 R_1$, since we have a 1-brane wound $Q_1 Q_5$ times around $x^5$ which has a radius $R_1$.

In the extremal limit, we only have (say) right movers and the energy is equal to the momentum

$$E_R = 0, \quad E_L = P = \frac{N}{R}.$$  \hspace{1cm} (5.4)

Note the momentum is integer quantized, since this is the total momentum as well. Then eq. (5.3) implies that

$$T_R = S_R = 0, \quad T = 0,$$  \hspace{1cm} (5.5)

while the total entropy is just the left moving piece
S = S_L = 2\pi \sqrt{Q_0 Q_N} N. \quad (5.6)

This is exactly equal to the extremal entropy of the corresponding black hole, given by eq. (4.7).

The near-extremal system corresponds to a large number of left movers and a few right movers, i.e. we have

\[ E_L = \frac{N}{R} + \frac{n}{Q_0 Q N}, \quad (5.7) \]
\[ E_R = -\frac{n}{Q_0 Q N}. \]

Note that the total momentum \( P = E_L - E_R \) is still an integer. This leads to an entropy

\[ S = 2\pi \sqrt{Q_0 Q_N} N + n + \sqrt{n}, \quad (5.8) \]

which is again in agreement with the semiclassical answers for \( n \ll N \). Since the entropy as a function of the excess energy over extremality agrees with the semiclassical answer, the temperature of the gas of bosons and fermions considered above also agrees with the Hawking temperature. The agreement of the extremal entropy was first shown in \(^9\) using a related but slightly different argument. The low energy states of non-extremal D-branes and the necessity of multiple winding were found \(^10\). The agreement of non-extremal entropy and temperature was shown \(^11,13\), and the argument for multiply wound long string as the effective model of the five-dimensional black hole was suggested \(^13\).

### 5.2 Hawking radiation

In the near-extremal situation we have both right and left moving waves and they can now collide to form a mode of supergravity which can now leave the brane system and propagate to the asymptotic region. This would be the mechanism of decay of a non-extremal state back into the extremal state.

In the semiclassical theory, a slightly non-extremal black hole decays into the extremal black hole by Hawking radiation. The question is: does the quantum mechanical decay of the microscopic state described above accurately describe Hawking radiation?

To obtain the semiclassical decay rate we need to calculate the classical absorption cross-section of the particular wave. This is in principle a straightforward problem. One has to solve the relevant wave equation in the black hole background and calculate what fraction of an incident wave is absorbed by the black hole. In practice, however, it is rather difficult to perform this calculation analytically. A great simplification happens when we consider low energy waves. In this case, it was found in the 1970's that the absorption cross-section for massless scalars by four-dimensional black holes of various kinds approaches the area of the horizon at low energies \(^14\).

\[ \lim_{\omega \to 0} \sigma(\omega) = A_H. \quad (5.9) \]

This absorption is in fact dominated by the S-wave: higher angular momenta are suppressed. Likewise for higher spin fields, the lowest allowed angular momentum dominates the radiation.

The classical absorption problem for the five-dimensional black hole was first solved by Dhar \emph{et al.} \(^15\), and it was found once again that the absorption cross-section is exactly equal to the horizon area. It has already been argued \(^16\) that the emission rate of low energy scalars from the microscopic model was proportional to the horizon area. It therefore appeared that the microscopic model has a chance of describing Hawking radiation. However, if this is indeed the correct quantum description of the five dimensional black hole, the rate of decay must agree exactly, just like the entropy. Otherwise one would be left with some Hawking radiation which is not explained by standard quantum mechanics. In fact much later it was shown \(^16\) that the result of eq. (5.9) is quite general and is valid for (minimally coupled) massless scalar absorption by any spherically symmetric black hole in any number of dimensions.

However the microscopic description is a rather complicated bound state of D1 and D5 branes with momentum. Even though the low energy modes are given by the excitations of a long string as described above, even the tension of this long string is not known and its interaction with modes in the bulk would in general depend on the details of the bound state.

It was realized in ref. 17 that the decay rate of certain scalars is in fact independent of the details of the bound state of D1 and D5 branes and is in fact independent of the effective tension of the long string which describes the low energy excitations. Examples of such modes are the components of the ten dimensional graviton along the 5 brane, but perpendicular to the direction of the 1-brane, the antisymmetric tensors with these same polari-
This has a drastic simplification for low energies \( \omega \ll T_L \text{ and } T_L \gg T_R \) but \( \omega \sim T_R \). Then one has the temperature \( T \sim 2T_R \). Recalling the feature of one-dimensional thermodynamics that the temperatures are proportional to the entropy – for left and right movers separately, eq. (5.3), and using the fact that we have already shown that in the near extremal limit the entropy agrees with the Beckenstein–Hawking formula, we get in this limit

\[
\Gamma(k) = \frac{A_H}{e^{\omega/T} - 1} \frac{d^4k}{(2\pi)^4}.
\]

(5.14)

Comparison with the detailed balance relation given in eq. (1.4) shows that the microscopic absorption cross-section is \( \sigma = A_H \) – in exact agreement with the semiclassical answer, eq. (5.9).

What is even more remarkable is that the microscopic answer of eq. (5.12) agrees with the semi-classical answer for slightly larger energies. Now the absorption cross-sections are dependent on the energy, which is why they are called grey body factors. In a certain regime the classical grey body factors can be computed for the five-dimensional black hole and the result is in perfect agreement with eq. (5.12).

These results for the entropy, temperature and emission rates have been extended in several directions. They have been shown to be valid for four dimensional charged black holes of a certain class and the microscopic picture of Hawking radiation has been shown to hold for a variety of other particles, like charged particles. All such cases where exact agreement exists have the feature that the cross-section approaches a constant at low energies. For some other kinds of particles, the results are not so accurate. In several cases one can determine the energy dependence correctly from the microscopic model, but not the coefficients. It is believed that the details of the model start becoming important for such emissions.

In fact the above results have been derived from the effective ‘long string’ model. A complete and satisfactory derivation of this model starting from the gauge theory living on the intersecting brane system is still lacking, though some progress has been made in this direction.

6. Why does it work?

What is remarkable about the above calculations is that they are all performed in the limit where the effective theory of the branes which form the black hole is weakly coupled. On the other hand, we have argued that it is in the regime of strong coupling that we expect the microscopic model to agree with the semiclassical answers. The physical picture for Hawking radiation in these two
regimes are completely different. We understand why the derivation of the extremal entropy worked as explained above this is because of supersymmetry. Why do the results for slightly non-extremal states, especially the emission rates agree so well with the classical answers? In the nonextremal case the state is not supersymmetric and one might expect that the weak coupling results have nothing to do with the strong coupling results.

This is not very well understood at this moment. However there are some results about higher order corrections which indicate that while we are dealing with states which are not supersymmetric, the theory is still supersymmetric and if the departure from supersymmetry is not large, then at low energies some quantities do not get corrected as the coupling gets strong. By the same token, very little is known about neutral black holes. In string theory these appear as highly excited states of (unwound) strings. These are states which are very far from supersymmetry and weak coupling calculations would have really nothing to do with the semiclassical answers. Nevertheless, some progress has been made by viewing these objects in M-theory. The results are certainly not as accurate as for near-extremal black holes.

7. Concluding remarks

We now know that at least a class of large black holes seem to behave as perfectly ordinary objects. It is most likely true that for this class of black holes there is no information loss problem. While we do not know much about neutral black holes, it would be rather shocking if they too are not described as states in a unitary quantum theory. To say the least, it has been shown that information loss is avoidable in a consistent quantum theory which includes gravity: string theory. Turning the argument around, one might say that if string theory failed to provide a statistical basis for black hole radiation in a regime where it should have (i.e. for near-extremal black holes), one would have to discard the theory as a quantum theory of gravity. It is remarkable that almost everything we know about string theory went into our understanding of black holes: extra dimensions, infinite tower of states, dualities. Features of string theory which appeared to be unesthetic played a crucial role in this! Black Hole Thermodynamics and Hawking Radiation posed a 'disprovable' statement in string theory, and string theory has indeed passed an important test.

What is lacking is an understanding of local properties in a gravitational field in terms of string theory. We do not understand what happens to an object as it falls through the horizon of a black hole. We do not understand in any detail how is information actually retrieved and why are the standard arguments for information loss false. The calculations discussed above are of course correct but these are weak coupling calculations which happen to give the right answer in the strong coupling limit as well. The physical picture is, however, still at weak coupling and this physical picture has nothing to do with the physics in a black hole background. In a sense we have been too lucky: we got the answers without having a picture.

However, like in all other areas of physics, a physical picture is necessary in the right domain. Recently there have been some developments which are leading to such a picture. In this article we have seen that the low energy dynamics of black holes are described by gauge theories. In weak coupling we may think of the gauge theory as one living on the brane system. These degrees of freedom were then coupled to separate degrees of freedom in the bulk: the latter formed the particles which the black hole can emit. Recently, it has been argued that in a certain low energy limit this is not what is happening. Rather the gauge theory itself contains, in its Hilbert space, the bulk modes. This means that in this limit, gauge theory contains gravity. In fact the gravity description is good when the coupling is large. Questions in gravity may be then translated into questions in gauge theory and one may have a chance to answer detailed local questions about space-time physics in this manner. This proposal is vigorously pursued at this moment and it remains to be seen whether this will lead to a definitive resolution of the information loss problem.