Solar tomography

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The solar tomography (or time-distance helioseismology) is a new promising method for probing 3-D structures and flows beneath the solar surface, which is potentially important for studying the birth of active regions in the Sun's interior and for understanding the relation between the internal dynamics of the active regions, and the chromospheric and coronal activity. In this method, the time for waves to travel along sub-surface ray paths is determined from the temporal cross correlation of signals at two separated surface points. By measuring the times for many pairs of points from Dopplergrams, covering the visible hemisphere, a tremendous quantity of information about the state of the solar interior is derived. As an example, we present the results on the internal structures of supergranulation, meridional circulation, active regions and sunspots. An active region which emerged on the solar disk in January 1998, was studied from SOHO/MDI for nine days, both before and after its emergence at the surface. The results show a complicated structure of the emerging region in the interior, and suggest that the emerging flux ropes travel very quickly through the depth range of our observations.

Method of solar tomography

Solar acoustic waves are excited by turbulent convection near the solar surface and propagate through the interior with the speed of sound. Because the sound speed increases with depth, the waves are refracted and reappear on the surface at some distance away from the source. The wave propagation is illustrated in Figure 1. The waves excited at point A will reappear at the surface points B, C, D, E, F, and others after propagating along the ray paths indicated by curves.

The basic idea of solar tomography is to measure the acoustic travel time between different points on the solar surface, and then to use these measurements for inferring variations of the structure and flow velocities in the interior along the wave paths connecting the surface points. This idea is similar to the Earth's seismology. However, unlike in Earth, the solar waves are generated stochastically by numerous acoustic sources in the subsurface layer of turbulent convection. Therefore, the wave travel-time is determined from the cross-covariance function, \( \Psi(\tau, \Delta) \), of the oscillation signal, \( f(t, r) \), between different points on the solar surface

\[
\Psi(\tau, \Delta) = \int_0^T f(t, r_1) f^*(t+\tau, r_2) \, dt,
\]

where \( \Delta \) is the horizontal distance between the points with coordinates \( r_1 \) and \( r_2 \), \( \tau \) is the delay time, and \( T \) is the total time of the observations. Because of the stochastic nature of excitation of the oscillations, function \( \Psi \) must be averaged over some areas on the solar surface to achieve a signal-to-noise ratio sufficient for measuring travel times \( \tau \). The oscillation signal, \( f(t, r) \), is usually the Doppler velocity or intensity. A typical cross-covariance function shown in Figure 2 displays several sets of ridges which correspond to the first, second, and third bounces of packets of acoustic-wave packets from the surface.

The origin of the multiple bounces is illustrated in Figure 1. Waves originated at point A may reach point B directly (solid curve), or after one-bounce at point C (dashed curve), or after two-bounces (dotted curve), and so on. Because the sound speed is greater in the deeper layers, the direct waves arrive first, followed by the second-bounce and third-bounce waves.

The cross-covariance function represents a solar 'seis-

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[Figure 1. A cross-section diagram through the solar interior, illustrating the wave propagation inside the Sun.]
mogram'. Figure 3 shows the cross-covariance signal as a function of time for a distance of 30 degrees. It consists of three wave packets corresponding to the first, second, and third bounce times. Ideally, the seismogram should be inverted to infer the structure and flows using a wave theory. However, in practice, as in terrestrial seismology, different approximations are employed, the most simple and powerful of which is the geometrical acoustic (ray) approximation.

Generally, the observed solar oscillation signal corresponds to displacement or pressure perturbation, and can be represented in terms of normal modes, or standing waves. Therefore, the cross-covariance function can be expressed in terms of normal modes, and then represented as a superposition of traveling wave packets. An example of the theoretical cross-covariance function of $p$-modes of the standard solar model is shown in Figure 4. This model reproduces the observational cross-covariance function very well in the observed range of distances, from 0 to 90 degrees. The theoretical model was calculated for larger distances, including points on the far side of the Sun, which are not accessible for observation. A backward propagating ridge originating from the second bounce ridge at 180 degrees is a geometrical effect due to the choice of the range of the angular distance from 0 to 180 degrees.

This is illustrated by the green curve in Figure 1. The waves reaching point F after the reflection at point D propagate more than 180 degrees, but are considered as propagating the distance AF which is less than 180 degrees. In the theoretical diagram (Figure 4) one can notice a weak backward ridge between 30 and 70 degrees and at 120 min. This ridge is due to reflection from the solar core. However, it has not been detected in observations.

By grouping the modes in narrow ranges of the angular distance from 0 to 180 degrees, one can observe the presence of several ridges in the theoretical time-distance diagram, which correspond to different bounce times. The lowest set of ridges (first-bounce) corresponds to waves propagated to the distance, without additional reflections from the solar surface. The middle ridge (second-bounce) is produced by the waves arriving to the same distance after one reflection from the surface, and the upper ridge (third-bounce) results from the waves arriving after two bounces from the surface. The backward ridge associated with the second-bounce ridge is due to the choice of the angular distance range from 0 to 180 degrees.
lar phase velocity, $v = \omega_b/L$, where $L = (\bar{k}(l+1))^{1/2}$, and applying the method of stationary phase, the cross-
coherence function can be approximately represented in the form

$$
\Psi(\tau, \Delta) \approx \sum_k \sum_v \cos \left[ \omega_0 \left( \tau - \frac{\Delta}{v} \right) \right] \exp \left[ -\frac{\Delta^2}{4} \left( \tau - \frac{\Delta}{u} \right)^2 \right].
$$

(2)

where $\Delta v$ is a narrow interval of the phase speed, $u = (\partial \omega_0/\partial k)$ is the horizontal component of the group velocity, $k_0 = L/R$ is the angular component of the wave vector, $R$ is the solar radius, $\omega_0$ is the central frequency of a Gaussian frequency filter applied to the data, and $\Delta \omega$ is the characteristic width of this filter. Therefore, the phase- and group travel-times are measured by fitting individual terms of eq. (2) to the observed cross-coherence function using a least-squares technique. This technique measures both phase ($\Delta/v$) and group ($\Delta/u$) travel-time of the $p$-mode wave packets. The previous time-distance measurements provided either the group-time, or the phase times. It was found that the noise level in the phase-time measurements was substantially lower than in the group-time measurements. Therefore, we use the phase times in this paper. We also employ the geometrical acoustic (ray) approximation to relate the measured-phase times to the internal properties of the Sun. More precisely, the variations of the local travel-times at different points on the surface relative to the travel-times averaged over the observed area are used to infer variations of the internal structure and flow velocities using a perturbation theory.

In the ray approximation, the travel-times are sensitive only to the perturbations along the ray paths given by the Hamilton equations. The variations of the travel-time obey Fermat’s Principle

$$
\delta \tau = \frac{1}{\omega} \int \delta k \, dr,
$$

(3)

where $\delta k$ is the perturbation of the wave vector due to the structural inhomogeneities and flows along the unperturbed ray path, $\Gamma$. Using the dispersion relation for acoustic waves in the convection zone, the travel-time variations can be expressed in terms of the sound speed, magnetic field strength and flow velocity.

The effects of flows and structural perturbations are separated from each other by taking the difference and the mean of the reciprocal travel-times

$$
\delta \tau_{\text{diff}} = -2 \int_\Gamma \frac{(nU)}{c} \, ds;
$$

(4)

$$
\delta \tau_{\text{mean}} = -\int_\Gamma \frac{\delta c}{c} \, S \, ds,
$$

(5)

where $c$ is the adiabatic sound speed, $n$ is a unit vector tangent to the ray, $S = k\Delta$ is the phase slowness. Magnetic field causes anisotropy of the mean travel-times, which allows us to separate, in principle, the magnetic effects from the variations of the sound speed (or temperature). So far, only a combined effect of the magnetic fields and temperature variations has been measured reliably. One-dimensional tests by Kosovichev and Duvall and two-dimensional numerical simulations by Jensen et al. have shown that eqs (4) and (5) provide a reasonable approximation to the travel-time variations. The development of a more accurate theory for the travel-times, based on the Born approximation is currently under way.

Typically, we measure times for acoustic waves to travel between points on the solar surface and surrounding quadrants symmetrical relative to the North, South, East and West directions. In each quadrant, the travel-times are averaged over narrow ranges of travel distance $\Delta$. Then, the times for northward-directed waves are subtracted from the times for south-directed waves to yield the time, $\tau_{\text{NS}}$ which predominantly measures north–south motions. Similarly, the time differences, $\tau_{\text{EW}}$, between westward- and eastward-directed waves yields a measure of eastward motion. The time, $\tau_{\text{diff}}$, between outward- and inward-

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**Figure 5.** The regions of ray propagation (colour areas) as a function of depth, $z$, and the radial distance, $\Delta$, from a point on the surface. The rays are also averaged over circular regions on the surface, forming three-dimensional figures of revolution. The dashed lines show the inversion grid.
directed waves, averaged over the full annuli, is mainly sensitive to vertical motion and the horizontal divergence. The time, \( t_{\text{mean}} \), which measures sound speed and magnetic perturbations, is also averaged over the full annuli (see refs 3 and 8).

The next step is to determine the variations of the sound speed and flow velocity from the observed travel-times using eqs (4) and (5). It is assumed that the convective structures and flows do not change during the observations and can be represented by a discrete model. In this model, the 3-D region of wave propagation is divided into rectangular blocks. The perturbations of the sound speed, and the three components of the flow velocity are approximated by the linear functions of coordinates within each block, e.g. for the flow velocity

\[
U(x, y, z) = \sum U_{jk} \left[ \frac{1-x-x_i}{x_{i+1}-x_i} \right] \left[ \frac{1-y-y_j}{y_{j+1}-y_j} \right] \left[ \frac{1-z-z_k}{z_{k+1}-z_k} \right],
\]

where \( x_i, y_j, z_k \) are the coordinates of the rectangular grid, \( U_{jk} \) are the values of the velocity in the grid points, and \( x_i \leq x \leq x_{i+1}, y_j \leq y \leq y_{j+1}, \text{and } z_k \leq z \leq z_{k+1}. A \text{ part of the } x-z \text{ grid is shown in Figure 5 together with the ray system used in inversion.}

The travel-time measured at a point on the solar surface is the result of the cumulative effects of the perturbations in each of the traversed rays of the 3-D ray systems. Figure 5 shows, in the ray approximation, the sensitivity to subsurface location for a certain point on the surface. This pattern is then translated for different surface points in the observed area, so that overall the travel-times are sensitive to all subsurface points in the depth range 0–20 Mm, in this example.

We average the equations over the ray systems corresponding to the different radial distance intervals of the data, using approximately the same number of ray paths as in the observational procedure. As a result, we obtain two systems of linear equations that relate the data to the sound speed variation and to the flow velocity, e.g. for the velocity field,

\[
\delta t_{\text{diff}; \lambda \mu \nu} = \sum_{jk} A_{jk}^{\lambda \mu \nu} U_{jk},
\]

where vector-matrix \( A \) maps the structure properties into the observed travel-time variations, and indices \( \lambda \) and \( \mu \) label the location of the central point of a ray system on the surface, and index \( \nu \) labels the annuli. These equations are solved by a regularized least-squares technique using the LSQR algorithm\(^9\). Jensen et al.\(^10\) suggested to speed up the inversion by doing most of the calculation in the Fourier domain.

The results of test inversions of Kosovichev and Duvall\(^3\) demonstrate a very accurate reconstruction of sound speed variations and the horizontal components of subsurface flows. However, vertical flows in the deep layers are not resolved because of the predominantly horizontal propagation of the rays in these layers. The vertical velocities are also systematically underestimated by 10–20% in the upper layers. Similarly, the sound speed variations are underestimated in the bottom layers. These limitations of the solar tomography should be taken into account in interpretation of the inversion results.

**Inversion results**

Helioseismic tomography has been successfully used to infer local properties of large-scale zonal and meridional flows\(^11\), convective flows and structures (refs 3 and 8),
structure and dynamics of active regions\textsuperscript{12} and flows around sunspots\textsuperscript{3}. Here we present some results of tomographic inversion for large-scale convective cells (supergranulation), meridional flow, sunspot and an emerging active region.

**Quiet-Sun convection**

The data used were for 8.5 h on 27 January 1996 from the high-resolution mode of the MDI instrument. The results of inversion of these data are shown in Figure 6. We have found that, in the upper layers, 2–3 Mm deep, the horizontal flow is organized in supergranular cells, with outflows from the centre of the supergranules. The characteristic size of the cells is 20–30 Mm. Comparing with MDI magnetograms, it was found that the cell boundaries coincide with the areas of enhanced magnetic field. These results are consistent with the observations of supergranulation on the solar surface\textsuperscript{13}. However, in the layers deeper than 5 Mm, we do not see the supergranulation pattern. This suggests that supergranulation is only 5 Mm deep. An alternative interpretation suggesting a depth of 8 Mm was presented by Duvall\textsuperscript{14}.

The vertical flows (Figure 7) correlate with the supergranular pattern in the upper layers. Typically, there are upflows in the 'hotter' areas and downflows in the 'colder' areas. In the hotter areas however, the sound speed is higher than the average.

**Meridional circulation**

Meridional flows from the equator towards the north and south poles have been observed on the solar surface in direct Doppler-shift measurements\textsuperscript{15}. The MDI observations by Giles et al.\textsuperscript{11} have provided the first evidence that such flows persist to great depths, and, thus, possibly play an important role in the 11-year solar cycle. The poleward flow can transport the magnetic remnants of sunspots generated at low latitudes to higher latitudes and, therefore, contribute to the cyclic polar-field reversal.

The meridional flows in the solar interior were detected by the time-distance method. Figure 8 shows the differences between the travel-times of acoustic waves propagating poleward and equatorward at different latitudes $\lambda$. These travel-time differences correspond to the mean meridional flow averaged over the penetration depth of the acoustic waves, which was 4–24 Mm in the measurements. By using eq. (4) Giles et al.\textsuperscript{11} estimated that the maximum mean speed of the flow is $\approx 20 \text{ m s}^{-1}$. They have also found that the flow velocity is almost constant over the observed range of depth.

**Tomography of sunspots and active regions**

An important problem of astrophysics is understanding the mechanisms of solar activity. The solar tomography provides a tool for studying the birth and evolution of active regions and complexes of solar activity. In Figure 9, we show the results for the emerging active region observed in January 1998. This was a high-

![Figure 7](image-url)  
**Figure 7.** The vertical-flow field (arrows) and the sound speed perturbation at the North-South position indicated by arrows in Figure 6.

![Figure 8](image-url)  
**Figure 8.** The average travel-time difference (south minus north) as a function of latitude, $\lambda$, for surface separation of pairs of points in the range 12–73 Mm. The individual points are shown (squares) and the 1σ errors (vertical lines). The solid curve is the best-fit 2 parameter model. The velocity scale on the right axis, in which 12.1 m/s flow corresponds to a 1 s time difference, is obtained from eq. (3) (ref. 11).
latitude region of the new solar cycle which was started in 1997.

Figure 9 shows the distribution of the wave speed variations in a vertical cross-section in the region of the emerging flux and in a horizontal plane at a depth of 18 Mm, for three 2-h intervals, a, at 17:00 UT, 11 January 1998; b, 3:00 UT, 12 January 1998; and c, 5:00 UT, 12 January 1998. The perturbations of the magnetosonic speed shown in this figure are associated with the magnetic field and temperature variations in the emerging magnetic ropes. The positive variations are shown in red, and the negative variations are shown in blue.

Figure 9 a shows no significant variations in the region of the emergence, which is at the middle of the vertical plane. The MDI magnetogram shown at the top indicates only very weak magnetic field above this region. Figure 9 b shows a positive perturbation associated with the emerging region. The strongest perturbation in this panel is at the bottom of the observed region. During the next 2 h (Figure 9 c), the perturbation is propagated to the top of the box. From these data, we conclude that the emerging flux propagated through the characteristic depth of 10 Mm during 2 h. This gives an estimate of the speed of emergence $\approx 1.3 \text{ km/s}$. This speed is somewhat higher than the speed predicted by theories of emerging flux. The typical amplitude of the sound speed variation in the perturbation is about $0.5 \text{ km/s}$. This may correspond to a magnetic field strength of 500 G at the top of the box, or a temperature variation of 800 K.

After the emergence we observed the gradual increase of the perturbation in the subsurface layers with the formation of sunspots. The observed development of the active region suggests that the sunspots were formed as a result of the concentration of magnetic flux close to the surface.

Figure 10 is an example of the internal structure of a large sunspot observed on 17 January 1998. An image of the spot taken in the continuum is shown at the top. The sound speed perturbations in the spot are much stronger than in the emerging flux, and reach more than $3 \text{ km/s}$. It is interesting that beneath the spot the perturbation is negative in the subsurface layers and becomes positive in the deeper interior. This data also shows the connection to the spot of a small pore which is on the left side of the spot. The negative perturbations beneath the spot are, probably due to the lower temperature. However, the effects of temperature and magnetic field have not been

Figure 9. The sound speed perturbation in the emerging active region (a) on 11 January 1998, 17:00 UT; (b) 12 January 1998, 3:00; and (c) 12 January 1998, 5:00. The horizontal size of the box is 415 Mm, the vertical size is 18 Mm. The panels on the top are MDI magnetogram showing the surface magnetic field of positive (red) and negative (blue) polarities. The perturbations of the sound speed range from $-1.6$ to $1.3 \text{ km/s}$. The positive variations are shown in red, and the negative ones in blue.
Solar tomography, or time-distance helioseismology, provides unique information about three-dimensional structures and flows associated with magnetic field and turbulent convection in the solar interior. This method is at the very beginning of its development. In this paper, we have reviewed some basic principles of this technique, based on the geometrical ray approximation, and presented some initial inversion results. Developing wave-form solar tomography is one of the most challenging problems of helioseismology.

Using time-distance seismology, we have been able to measure the structure of supergranulation flows and detect an active region before it appeared on the surface. The inversion results also have shown interesting dynamics of supergranulation, meridional circulation, emerging active regions and the formation of sunspots in the upper convection zone. Further studies of the Sun’s interior by the time-distance seismology will shed light on the mechanisms of solar activity.

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