Rotation of the solar interior

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Helioseismology has allowed us to infer the rotation in the greater part of the solar interior with high precision and resolution. The results show interesting conflicts with earlier theoretical expectations, indicating that the Sun is host to complex dynamical phenomena, so far hardly understood. This has important consequences for our ideas about the evolution of stellar rotation, as well as for models for the generation of the solar magnetic field. Here we provide an overview of our current knowledge about solar rotation, much of it obtained from observations from the *SOHO* spacecraft, and discuss the broader implications.

SOLAR rotation has been known at least since the early seventeenth century when, with the newly invented telescope, Fabricius, Galileo and Scheiner observed the motion of sunspots across the solar disk. Indeed, at the last solar maximum 10 years ago, one of us made naked-eye observations of sunspots from the sunset walk at the TIFR, Mumbai: over several evenings the day-to-day change in position of sunspots, visible to the naked eye during the haze just before sunset, clearly showed that the Sun was rotating.

It is hardly surprising that the Sun and other stars are observed to rotate. Stars are born of contracting interstellar gas clouds which share the rotation of the Galaxy. As the clouds contract, they rotate more rapidly, much as an ice skater makes herself spin around faster by pulling in her arms to her body, because she is reducing her moment of inertia while her angular momentum is conserved. Although the details of star formation within the contracting clouds are uncertain and involve mass loss and interaction with disks around the star which will transport angular momentum from one part of the cloud to another, it is plausible that newly formed stars should be spinning quite rapidly. This is indeed observed: the rotation of the stellar surface causes a broadening of the lines in the star's spectrum, owing to the Doppler effect, and from measurements of this effect it is inferred that many young stars rotate at near the break-up speed, where the centrifugal force at the equator almost equals gravity.

Stars tend to slow down when they get older. At least for stars of roughly solar type, the observations show that the rotation rate decreases with increasing age. The Sun's slowdown is thought to take place through angular-momentum loss in the solar wind, magnetically coupled to the outer parts of the Sun. The extent to which the slowdown affects the deep interior of the Sun then depends on the efficiency of the coupling between the inner and outer parts. In fact, simple models of the dynamics of the solar interior tend to predict that the core of the Sun is rotating up to fifty times as rapidly as the surface. Such a rapidly rotating solar core could have serious consequences for the tests of Einstein's theory of general relativity based on observations of planetary motion: a rapidly rotating core would flatten the Sun and hence perturb the gravitational field around it. Even a subtle effect of this nature, difficult to see directly on the Sun's turbulent surface, might be significant.

Very detailed observations have been carried out of the solar surface rotation by tracking the motion of surface features, such as sunspots and, more recently, by Doppler-velocity measurements. It was firmly established by the nineteenth century, by careful tracking of sunspots at different latitudes on the Sun's surface, that the Sun is not rotating as a solid body: at the equator the rotation period is around 25 days, but it increases gradually towards the poles where the period is estimated to be in excess of 36 days. This *differential rotation* is not as surprising as it might seem: since the Sun is a sphere of gas, it is not constrained to rotate at a uniform rate. Nevertheless, the origin of the differential rotation, and how it is continued in the solar interior, are evidently interesting questions.

The origin of the differential rotation is almost certainly linked to the otherwise dynamic nature of the outer parts of the Sun. The Sun's radius is 700 Mm (i.e. 700,000 km). In the outer 200 Mm, energy is transported by *convection*, in rising elements of warm gas and sinking elements of colder gas: this region is called the *convection zone*. The convection zone can be seen directly using high-resolution observations of the solar surface, in the *granulation* with brighter areas of warm gas just arrived at the surface, surrounded by colder lanes of sinking gas. The gas motions also transport angular momentum, and hence provide a link between rotation in different parts of the convection zone. Also, convection is affected by rotation, which may introduce anisotropy in the angular momentum transport. Indeed, it is likely that this transport is responsible for the differential rotation, although the details are far from understood.

Similarly complex dynamical interactions are also found in the giant gaseous planets (Jupiter, Saturn, Uranus and Neptune) which, like the Sun, are vigorously convecting as they rotate. Here the interaction probably gives rise to the banded structures immediately visible on Jupiter, and more faintly on Saturn. Even closer to home, the Earth's atmosphere and oceans are rotating fluid systems and exhibit, among other things, large-scale circulations and meandering jets, such as the jet stream. In all these systems, rotation plays a significant role in the observed dynamical behaviour.

Helioseismic probes of the solar interior

In recent years, the observation that the Sun is oscillating simultaneously in many small-amplitude global resonant modes has provided a new diagnostic of the solar interior. As discussed elsewhere in this issue (in the article by Chitre and Antia), the frequencies of these global modes depend on conditions inside the Sun, and so by measuring these frequencies we are able to make deductions about the state of the interior. This field is known as *helioseismology*. The observed oscillations are sometimes called five-minute oscillations, because they have periods in the vicinity of five minutes. The modes are distinguished not only by their different frequencies, but also by their different patterns on the surface of the Sun. These patterns are described by spherical harmonics (see Figure 1 for some examples) which are characterized by two integer numbers, their *degree l* and their *azimuthal order m*. As Chitre and Antia explain (see also below), different modes are sensitive to different regions of the Sun, depending on their frequency, degree and azimuthal order. By exploiting the different sensitivities of the modes, helioseismology is able to make inferences about localized conditions inside the Sun.

One of the factors that affect the mode frequencies is the Sun's rotation. The dominant effect of rotation on the oscillation frequencies is quite simple: the oscillation patterns illustrated in Figure 1 actually correspond to waves running around the equator; if the images were animated, they would essentially look like rotating beach balls. Patterns travelling in the same direction as the rotation of the Sun would appear to rotate a little faster, patterns rotating in the opposite direction a little more slowly. When observed at a given position on the Sun, the oscillations in the former case would have slightly higher
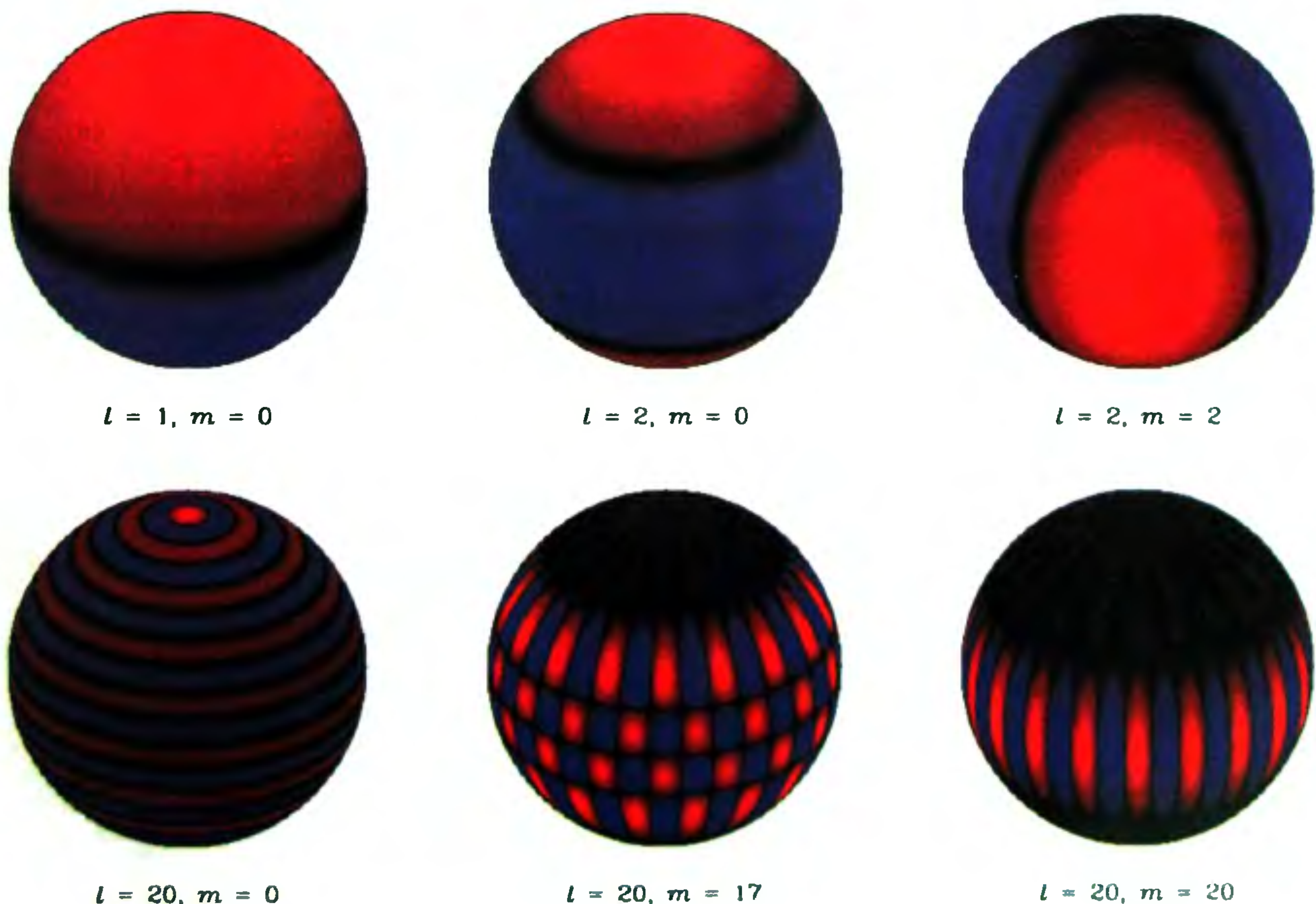

$l = 1, m = 0$ $l = 2, m = 0$ $l = 2, m = 2$

$l = 20, m = 0$ $l = 20, m = 17$ $l = 20, m = 20$

Figure 1. Examples of spherical harmonic patterns for different values of the degree l and order m. Red and blue represent positive and negative regions, black represents regions where the spherical harmonic is close to zero.

frequency, and in the latter case slightly lower frequency, than if the Sun had not been rotating. The frequency difference between these two cases therefore provides a measure of the rotation rate of the Sun.

In reality other effects must be taken into account to describe the frequency shifts caused by rotation, which are often referred to as the *rotational frequency splitting*. The *Coriolis force* affects the dynamics of the oscillations and hence their frequencies, although it turns out that for the modes observed in the five-minute region this effect is modest. However, the variation of rotation rate with position in the Sun must be taken into account. Each mode feels an average rotation rate, where the average is determined by the mode's frequency, degree and azimuthal order: the precise form of this spatial average is described by a weight function. These weight functions vary widely from mode to mode. As already indicated in Figure 1, modes with $m = l$ are concentrated near the equator, increasingly so with increasing l, whereas modes of lower azimuthal order extend to higher latitudes. Thus modes with $m = l$ feel only an average of the rotation near the equatorial plane, whereas modes of lower azimuthal order sense the average rotation over a wider range of latitudes. In a similar manner, as described by Chitre and Antia, the high-degree five-minute modes (i.e. with large values of l) sense only conditions near the surface of the Sun, whereas modes of low degree feel conditions averaged over much of the solar interior.

These properties can be illustrated by a few examples of weight functions (as shown in Figure 2). The observed modes include some that penetrate essentially to the solar centre, others that are trapped very near the surface, and the whole range of intermediate penetration depths, with a similar variation in latitudinal extent. Thus the observed frequency splittings provide a similarly wide range of averages of the internal rotation. It is this wealth of data which allows the determination of the detailed variation of rotation with position in the solar interior. Modes of high degree, trapped near the surface, provide measures of the rotation of the superficial layers of the Sun. Having determined that, its effect on the somewhat more deeply penetrating modes can be eliminated, leaving just a measure of rotation at slightly greater depths. In this way, information about rotation in the Sun can be 'peeled' layer by layer, much as one could an onion, in a way that allows us to obtain a complete image of solar internal rotation.

It is fairly evident that this process gets harder, the deeper one attempts to probe, since fewer and fewer modes penetrate to the required depth; furthermore, the effect of rotation decreases because of the smaller size of the region involved. Thus the rotation of the solar core is difficult to determine. Similarly, all modes are affected by the equatorial rotation while only modes of low m extend to the vicinity of the poles, and the polar regions have relatively little effect on the oscillations, complicating the determination of the high-latitude rotation. However, as we shall see, the quality of current data is such that the rotation rate can be determined quite near the poles, at least in the outer parts of the convection zone. We also
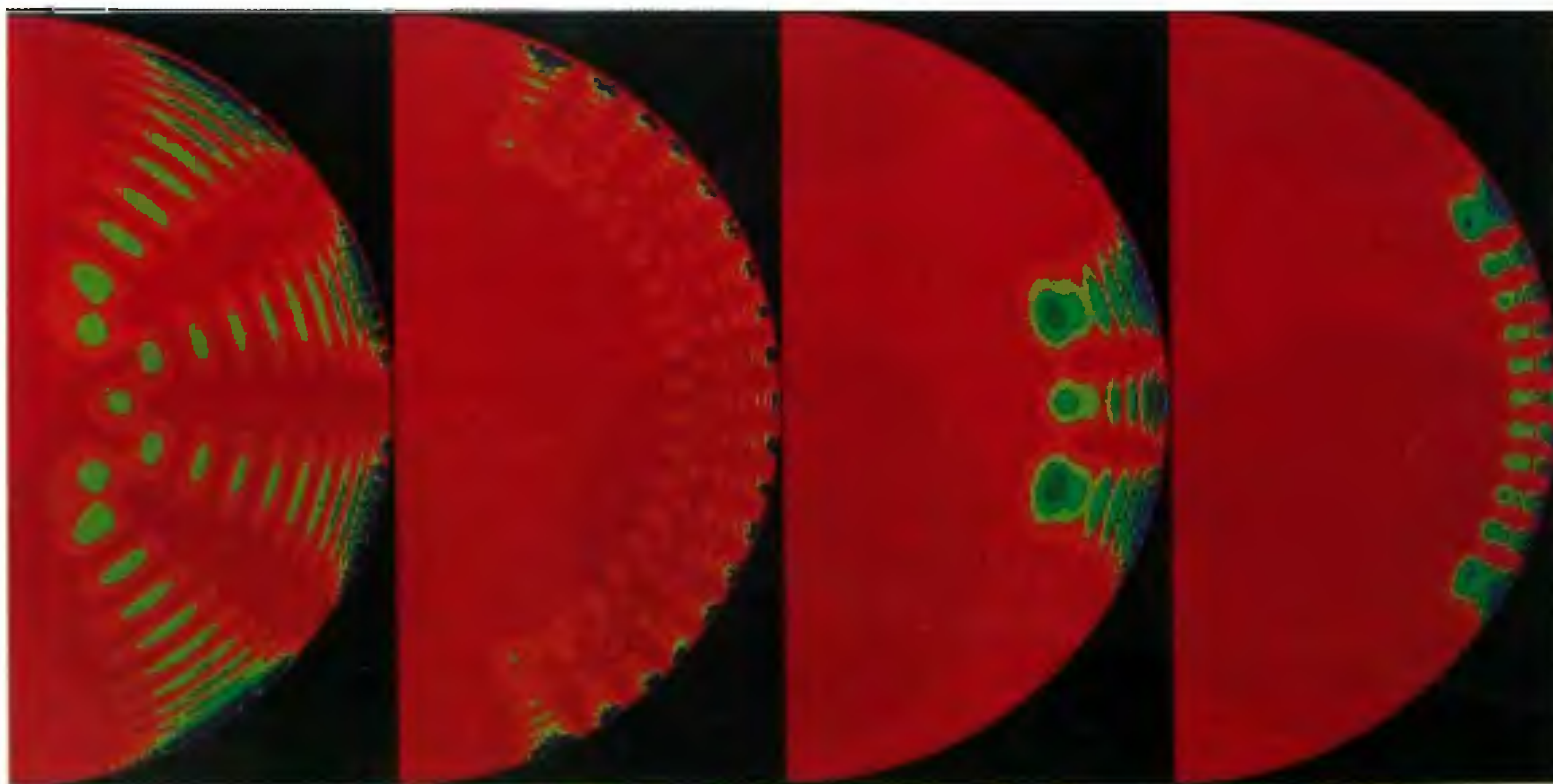

Figure 2. Weight functions determining the sensitivity of different modes to the solar internal rotation. Red indicates essentially no sensitivity, whereas green and blue show regions of successively higher sensitivity. All modes have frequencies near 2 mHz; their degree l and azimuthal order m are, from left to right: $(l, m) = (5, 2), (28, 10), (28, 26),$ and $(60, 50)$.

note an intrinsic limitation of the frequency splittings of global modes: the weight functions are symmetrical around the solar equator, as can be seen in Figure 2; thus we can infer only the similarly symmetric component of rotation. This must be kept in mind in the following, when interpreting the results. We note that this restriction can be avoided by applying *local helioseismology* techniques to the data: such techniques are described elsewhere in this issue by Kosovichev and Duvall.

The solar internal rotation

Early helioseismic data on rotational splittings provided information only about the modes with $m \simeq \pm l$; as a result, they were sensitive mainly to rotation near the equator. Observations from the Kitt Peak National Observatory, USA, showed around 1984 that there was relatively little variation of rotation with depth; in particular, there were no significant indications of a rapidly rotating core. A few years later, initial data on the dependence of the splitting on m were obtained at the Sacramento Peak and Big Bear Solar Observatories. Strikingly, they indicated that the surface latitudinal differential rotation persisted through the convection zone, whereas there was little indication of variation with latitude in the rotation beneath the convection zone.

In the last few years, the amount and quality of helioseismic data on solar rotation have increased dramatically, as a result of several ground-based and space-based experiments. The LOWL instrument of the High Altitude Observatory has provided high-quality data on modes of low and intermediate degree over the past more than five years. The BiSON and IRIS networks, observing low-degree modes in Doppler velocity integrated over the solar disk, have yielded increasingly tight constraints on the rotation of the solar core, while the GONG six-station network (including a station at the Udaipur Solar Observatory) is setting a new standard for ground-based helioseismology. Finally, the SOI-MDI experiment on the *SOHO* spacecraft has yielded a wealth of data on modes of degree up to 300, allowing for the first time a detailed analysis of the properties of rotation in the convection zone. The results we present below are the combined knowledge that has emerged from these observational efforts.

In discussing what we now know about the rotation inside the Sun, we shall start from the near-surface layers and work towards the centre. As we have already discussed, the outer 30 per cent of the Sun is convectively unstable. Before helioseismology, models predicted that the rotation inside the convection zone would organize itself on cylinders aligned with the rotation axis. Thus the rotation at depth at, say, equatorial latitudes would match the surface rotation at high latitudes, rather than the faster equatorial rotation at the surface, and so at a given

latitude the rotation in the convection zone would decrease with depth. Helioseismology has shown that this is not so: to a first approximation, it is more accurate to say that the rotation at a given latitude is nearly constant with depth, or to put it another way, the differential rotation seen at the surface imprints itself through the convection zone. This finding is clearly visible in Figure 3. In detail, the situation is more complicated. At low latitudes, immediately beneath the solar surface the rotation rate actually initially *increases* with depth. The equatorial rotation reaches a maximum at a depth of about 50 Mm (i.e. about 7 per cent of the way in from the surface to the centre of the Sun): at this point the rotation rate is about 5 per cent higher than it is at the surface. This is consistent with a variety of surface measurements of rotation. Tracking sunspots tends to give a slightly higher rotation rate than that obtained by making direct spectroscopic measurements of the velocity of the surface. Probably the reason is that the sunspots extend to some depth below the surface, and so are dragged along at a rate that is similar to the subsurface rotation a few per cent beneath the surface which helioseismology has revealed.

The rotational velocity at the surface of the Sun is about 2 km sec^{-1} (i.e. 7000 km h^{-1}), dropping off rather smoothly towards higher latitudes. However, it has now been found[2] that superimposed on this are bands of faster and slower rotation, a few m sec^{-1} higher or lower than the mean flow (Figure 4). The origin of this behaviour is not understood, but it is reminiscent of the more
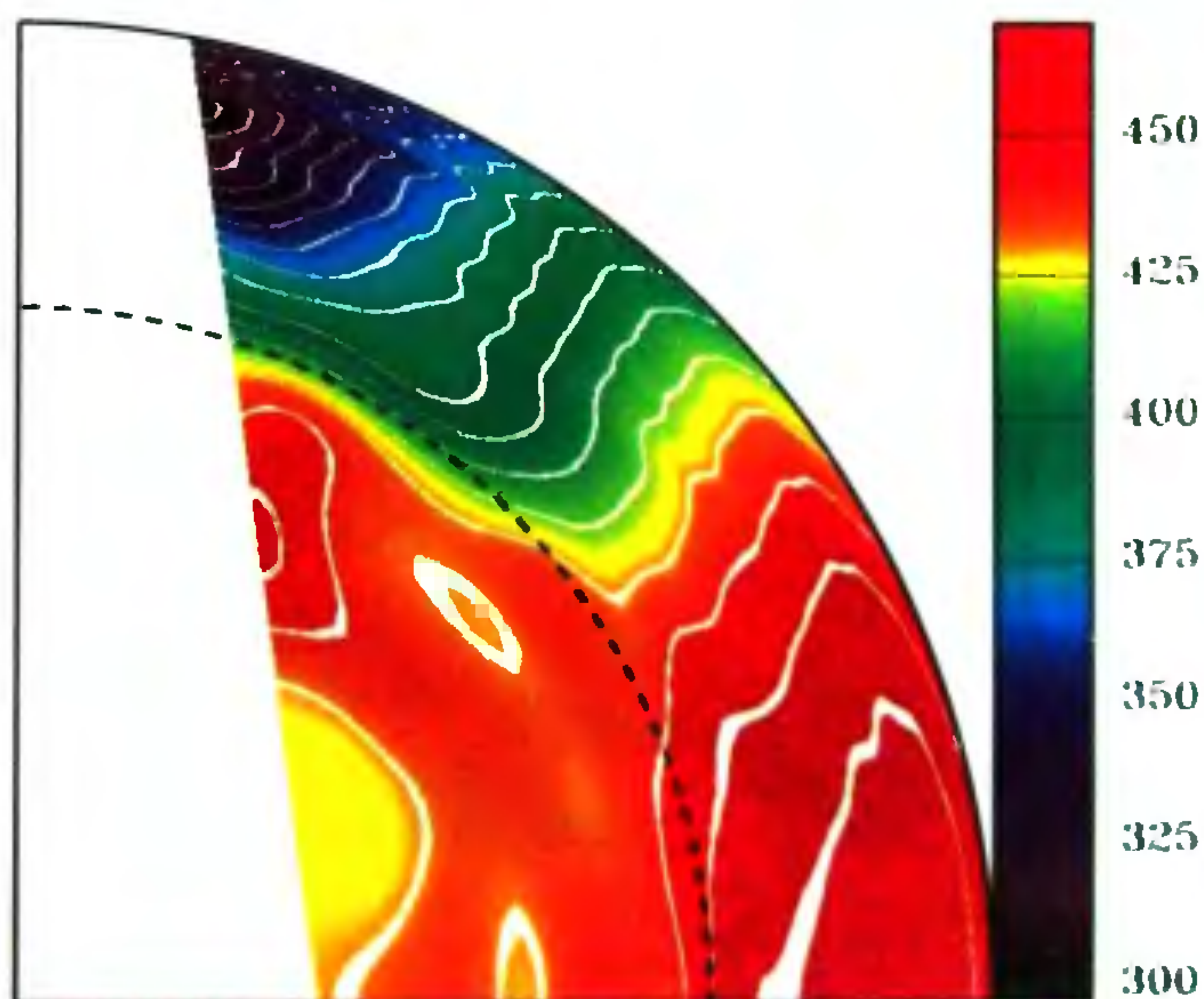

Figure 3. Rotation of the solar envelope inferred from observations by the SOI-MDI instrument on board the *SOHO* satellite[1]. Regions of faster rotation are red, regions of slower rotation are blue and black. The values quoted on the colour key are the frequency of the rotation (i.e. the reciprocal of the rotation period), in nano-Hertz (nHz): 300 nHz corresponds to a period of roughly 39 days, while 450 nHz corresponds to a period of about 26 days. The dashed line indicates the bottom of the convection zone.

pronounced banded flow patterns seen on Jupiter and Saturn. Evidence for such bands had been obtained previously from direct Doppler measurements on the solar surface. However, the seismic inferences have shown that they extend to a depth probably exceeding 40 Mm beneath the surface[3]. Moreover, these bands migrate from high latitudes towards the equator over the solar cycle.

It has been customary to represent the directly measured surface rotation rate in terms of a simple low-order expansion in sin ψ, where ψ is latitude on the Sun. This in fact quite successfully captured the observed behaviour; however, since the solar rotation axis is close to the plane of the sky, direct measurements of rotation near the poles are difficult and uncertain. Strikingly, the helioseismic results have shown a marked departure from this behaviour, at latitudes above about 60°: relative to the simple fit, the actual rotation rate decreases quite markedly there. The origin or significance of this behaviour is not yet understood. There is also evidence, hinted at in Figure 3, of a more complex behaviour of rotation at high latitudes. Some analyses have shown a 'jet', i.e. a localized region of more rapid rotation, at a latitude around 75° and a depth of about 35 Mm beneath the solar surface. Also, evidence has been found that the rotation rate shows substantial variations in time at high latitudes, over time scales of the order of months. It is probably fair to say that the significance of these results is still somewhat uncertain, however. Also, it should be kept in mind, as mentioned above, that the results provide an average of

rotation in the northern and southern hemispheres and, evidently, an average over the observing period of at least 2–3 months. Thus the interpretation of the inferred rotation rates in terms of the actual dynamics of the solar convection zone is not straightforward.

At the base of the convection zone, a remarkable transition occurs: the variation of rotation rate with latitude disappears, so that the region beneath the convection zone rotates essentially rigidly, at a rate corresponding to the surface rate at mid-latitudes (Figure 5). The region over which the transition occurs is very narrow, no more than a few per cent of the total radius of the Sun. This layer has been called the *tachocline*. Why the differential rotation does not persist beneath the convection zone is not yet known, but it is possible that a large-scale weak magnetic field permeates the inner region and enforces nearly rigid rotation by dragging the gas along at a common rate[5]. Such a field is quite possible, as a relic from the original collapsing gas cloud from which the Sun condensed.

The discovery of the tachocline, and of the form of the rotation in the convection zone, has led to an adjustment of our theories of the *solar dynamo* (see the article by Choudhuri in this issue). The Sun displays a roughly eleven-year cycle of sunspot activity, with the number of spots and their latitudinal distribution on the Sun varying over the cycle. Sunspots are formed where strong magnetic fields poke through the Sun's surface, and these magnetic fields are widely believed to be generated by some kind of dynamo action in the Sun. One idea is that the dynamo action consists of two components: a twisting
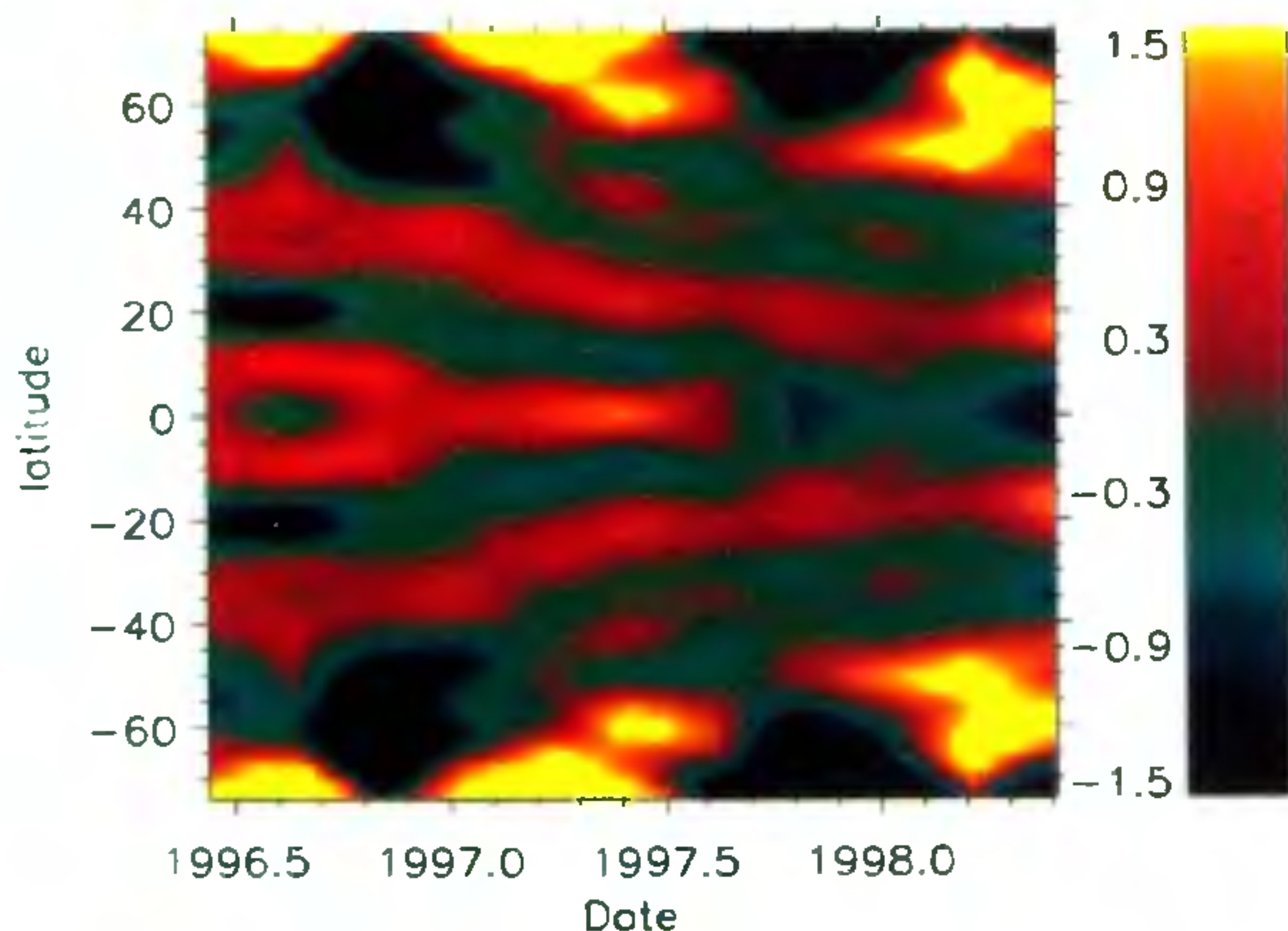

Figure 4. The evolution with time of the fine structure in the near-surface solar rotation. The time-averaged rotation rate has been subtracted from each of 11 independent inferences of rotation, for consecutive 72-day time intervals. The result is represented as a function of time (horizontal axis) and latitude (vertical axis), the colour-coding at the right gives the scale in nHz; 1.5 nHz corresponds to a speed of around 6/m sec^{-1} at the equator. The banded structure, apparently converging towards the equator as time goes by, should be noted. (From ref. 3.)
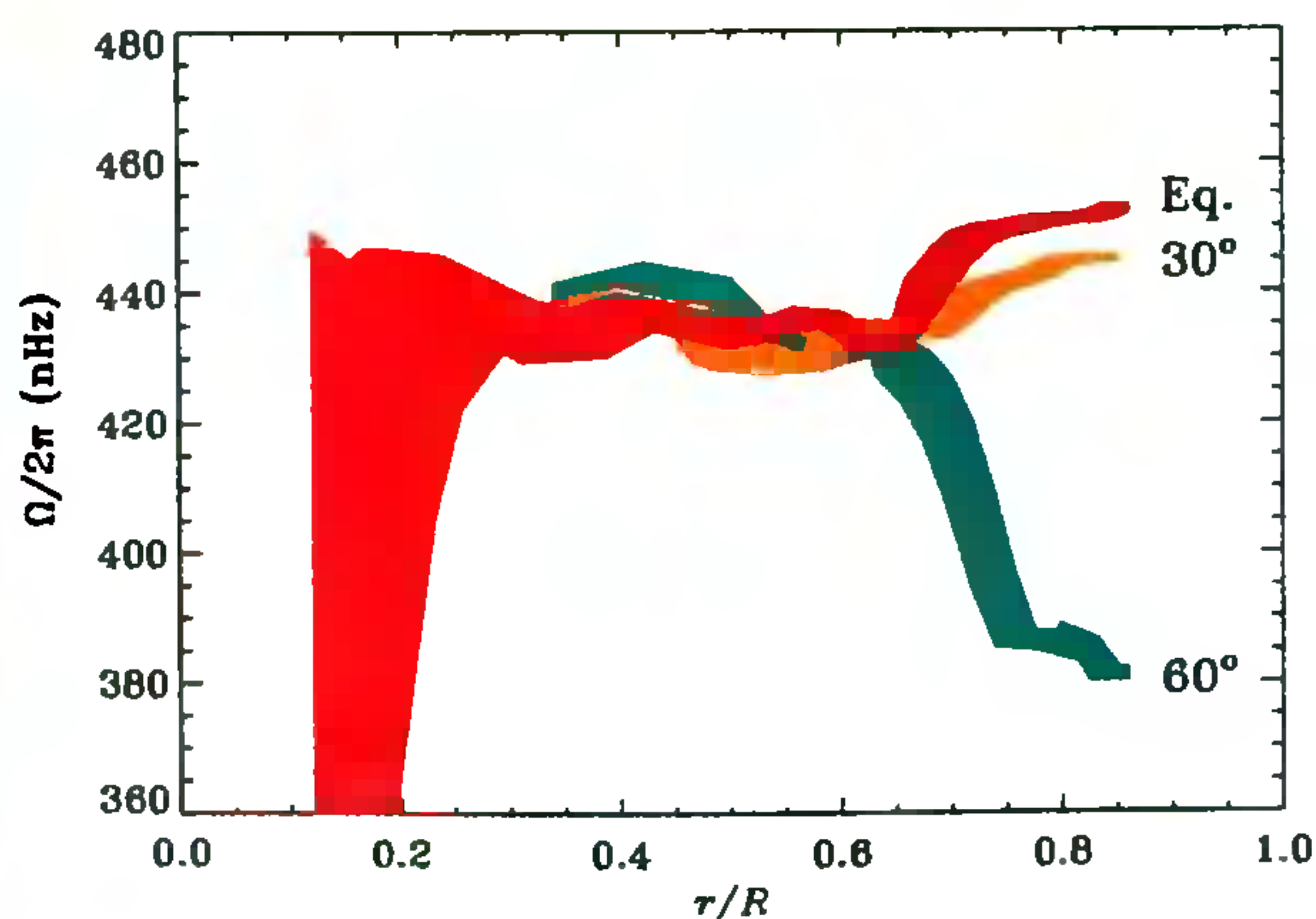

Figure 5. The inferred rotation as a function of depth inside the Sun at three solar latitudes: the equator (red), 30 degrees (orange) and 60 degrees (green). The vertical spread in the coloured bands shows the statistical uncertainty on the rotation rate ($\pm$ 1 standard deviation). Note that the result becomes much more uncertain in the deep interior. The values on the vertical axis are the rotation frequency in nHz (see caption to Figure 3). The values on the horizontal axis are the fractional radius inside the Sun, and run from the centre of the Sun ($r/R = 0.0$) to the visible surface ($r/R = 1.0$). The observations used to infer the rotation were from the LOWL instrument and the BiSON network[4].

of the magnetic field by the motion of convective elements, and a shearing out of the field by differential rotation. Prior to the helioseismic findings, the simulations of rotation implied that the radial gradient of differential rotation in the convection zone could provide the second ingredient, so it was thought that the dynamo action occurred in that region. Now, however, the tachocline with its very substantial radial gradient seems a more likely location for the dynamo.

Even deeper in the Sun, right down into the core where the energy-releasing nuclear reactions take place, the helioseismic results on the rotation are more uncertain due to the fact that so few of the observed five-minute modes (only the low-degree modes in fact) have any sensitivity to this region. Indeed, the results have been somewhat contradictory, some indicating rotation faster than the surface rate and others indicating rotation slower than or comparable to the rotation rate at the base of the convection zone; an example is illustrated in Figure 5. However, down to within 15 per cent of the solar radius from the centre, which is the deepest point at which present observations permit localized inferences to be made, all the modern results agree that the rotation rate is

not more than a factor two different from the surface rate: thus early models which predicted that the whole of the nuclear-burning core was rotating much faster are firmly ruled out. Again, this finding would be consistent with a magnetic field linking the core to the bulk of the radiative interior.

Modelling solar rotation

Although helioseismology has provided us with a remarkably detailed view of solar internal rotation, the theoretical understanding of the inferred behaviour is still incomplete. In the convection zone, the problem is to model the complex combined dynamics of rotation and convection, the latter occurring on scales from probably less than a few hundred kilometres to the scale of the entire convection zone and time scales from minutes to years. Viscous dissipation is estimated to occur on even smaller spatial scales, of the order 0.1 km or less. Capturing this range of scales is entirely outside the possibility of current numerical simulations; thus simplifications are required. Detailed simulations of near-surface
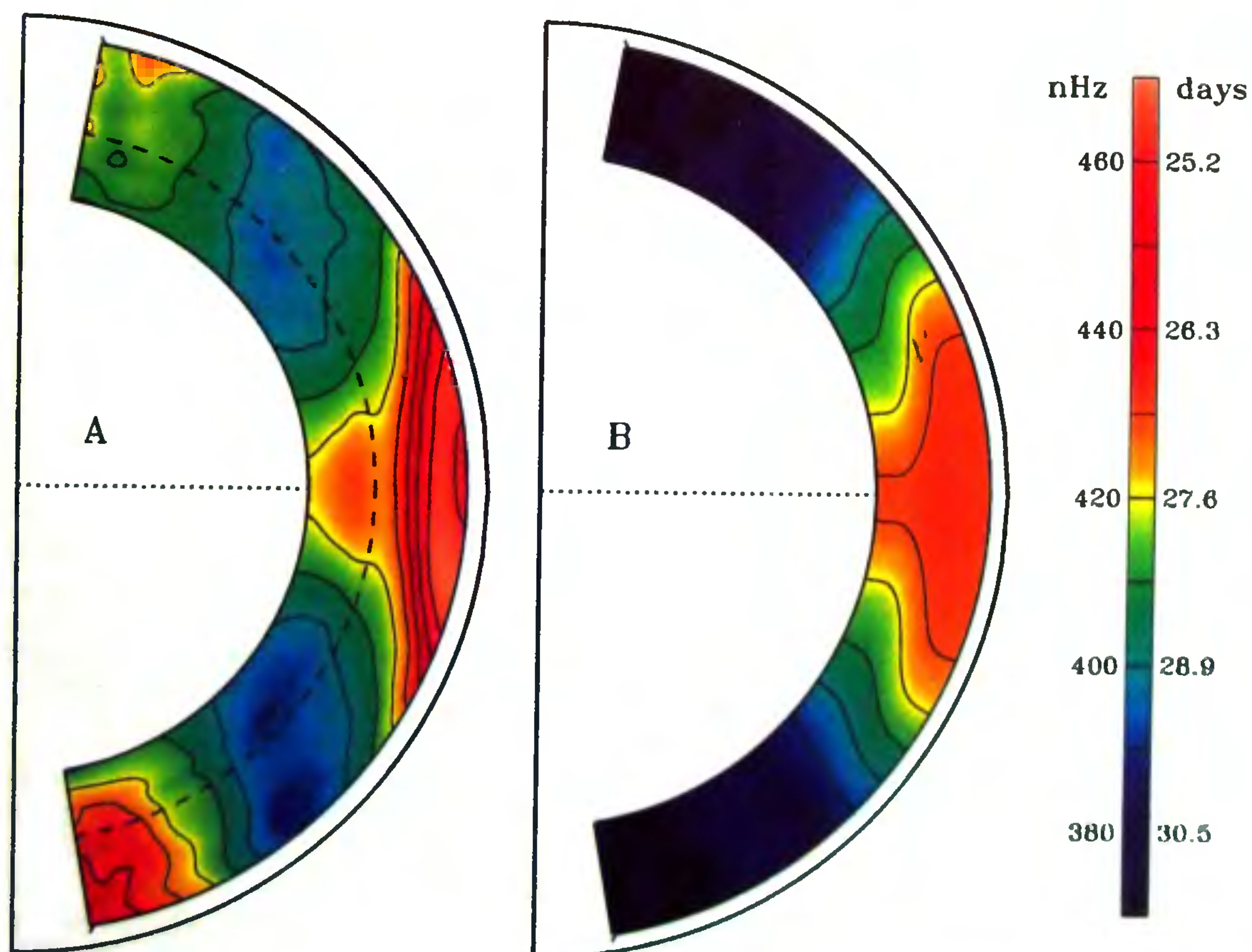

Figure 6. The results of two simulations of convection-zone rotation by Miesch, Elliott and Toomre, from the paper by Miesch[9]. The two simulations use different boundary conditions and parameter values, and illustrate some of the range of possible responses of the differential rotation to the form of the convection. Also note that Simulation A has a higher resolution and includes penetration into a stable region beneath the convection zone, whereas the convective motions in Simulation B are more laminar and there is no penetration beneath the convection zone.

convection, on a scale of a few Mm, have been remarkably successful in reproducing the observed properties of the granulation[6], but are evidently not directly relevant to the question of rotation. Simulations of the entire convection zone are necessarily restricted to rather large scales and hence cannot capture the near-surface details. Such simulations, therefore, typically exclude the outer 30 Mm of the convection zone. Early examples of such simulations by Gilman and Glatzmaier, of fairly limited resolution, showed a tendency for rotation to organize itself on cylinders[7]: the rotation rate depended primarily on the distance to the rotation axis; such a behaviour is predicted for simple systems by the *Taylor–Proudman theorem*. The rotation rate on a given cylinder would obviously be observable where the cylinder intersected the surface;·thus the observed decrease of rotation rate with increasing latitude would correspond to a similar decrease of rotation rate with depth at, say, the equator. The convection itself in these simulations was dominated by so-called *banana cells* – long, thin, large-scale convection cells oriented in the north–south direction.

The actual behaviour of rotation, shown in Figure 3, is obviously very different from these simulation results. The overall variation of rotation within the convection zone is evidently predominantly with latitude, with little variation in the radial direction except in the tachocline. Given the necessary simplifications of the calculations, their failure to model solar rotation is perhaps not surprising. In particular, the effects of smaller-scale turbulence (beneath the smallest scale resolved in the simulation) are typically represented as some form of viscosity; it was suggested by Gough, and later by others, that the effect of rotation on the small-scale motion might render this turbulent viscosity non-isotropic, with important effects on the transport of angular momentum within the convection zone. In fact, simple models of convection-zone dynamics, with parametrized anisotropic viscosity, have had some success in reproducing the helioseismically inferred rotation rate.

Recent advances in computing power have led to improved numerical simulations[8], which come closer to representing turbulent convective flow regimes such as exist in the Sun's convection zone. Figure 6 shows results from two such simulations by Miesch, Elliott and Toomre. The simulations can yield a range of differential rotation profiles, depending on the conditions imposed at the top and bottom boundaries of the simulation region, and on the parameter values adopted for the problem. Since it is not obvious what are the most appropriate boundary conditions and parameter values to choose, it is necessary to explore various possibilities and study the different responses. Simulation B has rotation contours at mid-latitudes which are nearly radial, as in the Sun (compare Figure 3), but the contrast in rotation rate between low and high latitudes is not as great as is observed in the Sun (about 70 nHz, rather than 130 nHz). In case A, the latitudinal variation of the Sun's rotation is better reproduced, but the mid-latitude contours do not look quite as similar to those in Figure 3. Nonetheless, these results are encouraging indications that we may be close to reproducing theoretically the gross features of the solar rotation inferred by helioseismology. There is still, though, much work ahead, both observational and theoretical, in getting a detailed understanding of the Sun's rotation and with that, we hope, a better understanding of the solar activity cycle and of large-scale rotating fluid systems on planets and stars.

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