Seismic sun

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Even though the interior of the Sun is not directly accessible to observations, it is nonetheless possible to infer the physical conditions inside the Sun using the theory of stellar structure and the accurately measured frequencies of solar oscillations. The helioseismic data has provided a powerful tool to probe the Sun and also to test physical theories describing its internal constitution.

1. Introduction

THE Sun has been verily described as the Rosetta Stone of astronomy. This is very apt since our nearest cosmic laboratory is readily available for studying a variety of physical processes operating both inside and outside the object. Astrophysicists have an abiding hope that the study of the Sun can serve as a guide for theory of structure and evolution of stars in general, and pulsating stars in particular. Clearly, its internal layers are not directly accessible to observations. Nonetheless, it is possible to construct a reasonable picture of the interior with the help of structure equations governing its equilibrium, together with the boundary conditions provided by observations. The principal question concerning the structure of the Sun is about checking the correctness of the theoretically constructed solar models. Fortunately, it turns out that the Sun is transparent to neutrinos released in the nuclear reaction network operating in the energy-generating core and also to seismic wave motions generated through the solar body. These complementary probes enable us to see inside the Sun and to infer the physical conditions prevailing in the solar interior and relate them to larger issues in astronomy and physics. The internal layers, in fact, provide an ideal celestial laboratory for testing atomic and nuclear physics, and high-temperature plasma physics and neutrino physics.

2. Standard solar model

The standard solar model (SSM) is constructed using a variety of simplifying assumptions. The Sun is assumed to be spherically symmetric maintaining mechanical and thermal equilibrium, with negligible effects of rotation, magnetic field, mass loss or accretion of material and tidal forces on its overall structure. The energy generation takes place in the central regions by thermonuclear reactions converting hydrogen into helium mainly by the proton–proton chain. The energy is transported outwards by radiative processes except in the outer unstable zone, extending over approximately a third of the solar radius below the surface where the energy flux is carried largely by convection modelled in the framework of a local mixing length theory. There is supposed to be no mixing of material outside the convection zone, save the slow gravitational diffusion of helium and heavy elements beneath the convection zone into the radiative interior, and there is no wave transport of energy or material. The standard nuclear and neutrino physics is adopted for constructing theoretical models satisfying the observed constraints, namely,

$$\begin{aligned}&\text{Mass } (M_\odot) = (1.9889 \pm 0.0002) \times 10^{33}\ \text{g},\\ &\text{Radius } (R_\odot) = (6.9599 \pm 0.0007) \times 10^{10}\ \text{cm},\\ &\text{Luminosity } (L_\odot) = (3.846 \pm 0.006) \times 10^{33}\ \text{erg s}^{-1},\\ &\text{Age } (t_\odot) = (4.6 \pm 0.1) \times 10^{9}\ \text{yrs, and}\\ &\text{Chemical composition } (Z/X) = 0.0245 \pm 0.002. \qquad (1)\end{aligned}$$

Here X and Z respectively, refer to the fractional abundance by mass of hydrogen and elements heavier than helium.

The manner in which the pressure, density and temperature vary throughout the solar interior can be determined by solving the equations of mechanical and thermal equilibrium applicable to the spherically symmetric Sun[1], where these variables are all taken to be functions of the radial coordinate, r. The structure equations are integrated numerically, with appropriate boundary conditions, with the auxiliary physical input of the opacity, nuclear energy generation rate and equation of state to construct solar models. The conventional approach to the theory of solar structure is to adopt at zero-age, a homogeneous chemical composition and the mass, and then to evolve the Sun over the solar age to yield the present luminosity and radius by adjusting the initial helium abundance and the mixing-length parameter which determines the convective flux in the convection zone.

3. Seismic waves

It has been observed since the early 1960s that the solar surface undergoes a series of mechanical vibrations; these manifest as Doppler shifts oscillating with a period centered around 5 min[2]. The pulsations have now been identified as acoustic modes of oscillation of the entire Sun[3-5]. Just like any musical instrument, the Sun also oscillates in a number of characteristic modes whose frequencies are determined by the internal structure and

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dynamics. The solar surface is seen to oscillate simultaneously in millions of modes, with the amplitude of an individual mode of the order of a few centimeters per second. Remarkably, the frequencies of many of these modes have been determined to an accuracy of better than 0.01%. In much the same manner as the geophysicists are able to study the internal layers of Earth from seismic disturbances, the helioseismic tool furnished by the rich spectrum of velocity fields observed at the solar surface can probe the Sun's internal layers to an extraordinary degree of precision. The accurately measured frequencies of oscillations, in fact, provide very stringent constraints on the admissible solar models.

In order to determine the frequencies of these oscillations to high accuracy, one needs continuous observations extending over very long periods. From most observatories on the surface of Earth it is not possible to observe the Sun continuously for more than 15 h due to the day–night cycle. Thus, to get longer coverage of the Sun various strategies have been tried, which include observations from the geographic south pole, from a network of sites located around the Earth and observations from a suitably located satellite. There are several ground-based networks observing the Sun more or less continuously with a variety of instruments. The most prominent among these is the Global Oscillations Network Group (GONG) which includes six stations located in contiguous longitudes around the world[6]. GONG has been observing the Sun more or less continuously since 1995 and frequencies of approximately half a million modes have been calculated for different periods of observations[7]. Apart from earth-based networks, many instruments located on satellites have been observing the solar oscillations. The most important among these is the Michelson Doppler Imager (MDI) instrument[8] on board the Solar and Heliospheric Observatory (*SOHO*) satellite, which was launched on 2 December 1995. The higher spatial resolution provided from space has enabled MDI to study oscillations with small-length scales.

Solar oscillations may be regarded as a superposition of many standing waves, whose frequencies are controlled by the physical properties of the solar interior. There are two distinct types of wave-modes that the Sun can support: high-frequency acoustic modes (p-modes) for which pressure gradient provides the main restoring force; and low-frequency gravity modes (g-modes) for which buoyancy is the dominant restoring force, and separating these two classes of modes are the fundamental modes (f-modes) which are essentially the surface gravity modes. The eigenmodes of oscillations can be characterized by three quantum numbers: the angular degree, l; azimuthal order, m; and radial order, n. The oscillation amplitudes are small, and so they can be analysed using a linear perturbation theory. Further, since the Sun is spherically symmetric to a good approximation, the eigenmodes of oscillations can be expressed in terms of the spherical harmonics, $Y_l^m(\theta,\phi)$. Thus, for example, the radial component of velocity can be expressed as

$$v(r,\theta,\phi,t) = v_{nl}(r) Y_l^m(\theta,\phi) e^{i\omega_{nlm}t}. \qquad (2)$$

Here r is the radial distance from the center, θ the colatitude and ϕ the longitude and ω_{nlm} is the frequency of oscillations. It is often convenient to express frequencies in terms of mHz and define $\nu = \omega/2\pi$ as the cyclic frequency. In absence of rotation and magnetic field, the frequencies will be independent of the azimuthal order m; the rotation and other symmetry-breaking forces lift this degeneracy giving rise to splitting of the modes for a given value of n, l. The mean frequency of a given multiplet ν_{nl} is determined by the spherically symmetric structure of the Sun, while the frequency splittings are determined by the rotation rate, magnetic field and other asphericities in solar interior. Extensive observations from GONG and MDI instruments have provided the mean frequencies of modes with degree, l, from 0 to 4000 and frequencies from 1 to 10 mHz (refs 7, 9–11). These include the p- and f-modes, while the low-frequency g-modes have not yet been unambiguously detected.

The p-modes are believed to be excited by turbulent convection in the subsurface layers[12]. The propagation characteristics of these seismic waves are affected by sound speed in the solar material, which increases inwards due to rising temperature with depth. Thus, a wave excited near the surface and propagating inwards is refracted away from the radial direction until at some depth it suffers a total internal reflection and bounces back to the surface. Near the solar surface the waves tend to get reflected because of sharply declining density. In this way, the acoustic waves are trapped within a cavity and the wave may travel around the Sun several times, establishing a standing wave pattern, in the process providing a global diagnostic of the solar interior.

Acoustic waves propagate through the body of the Sun along ray paths, as shown in Figure 1. The penetration depth of a given wave depends on its horizontal wavelength, or the angle of inclination to the radial direction – shorter waves (oscillations with large l) are confined within relatively shallow cavities below the surface, while the longer waves (small l) propagate deeper penetrating practically to the central regions. The radial modes ($l = 0$) propagate radially and hence suffer no refraction, thus penetrating all the way to the center. As different modes are trapped in different regions of solar interior, they sample properties of the region where they are trapped. This improves the diagnostic potential of solar oscillations since by studying the properties of a large variety of modes it is possible to infer the conditions over a sizeable fraction of solar interior. The disturbances observed at the photosphere naturally encounter the 'murky' surface layers which influence the oscillation frequencies to a significant extent; these surface effects must be properly filtered out while analysing the seismic data. Clearly, the characteristics of p-modes are mainly determined by the sound speed inside the Sun, but other properties like density, rotation velocity, magnetic field also affect the waves to smaller extent. Consequently, the accurately determined frequencies of solar oscillations provide a powerful tool to probe the structure and dynamics inside the Sun.

4. Probes of the solar interior

The initial attempts to learn about the solar interior were concerned with the boundary conditions at the surface. The spectroscopic data was extensively collected for studying the solar atmosphere, and the theory of solar structure was widely used to surmise the physical conditions below the surface for obtaining the observed temperature and luminosity.

Since the 1960s, there have been valiant attempts to measure the flux of neutrinos generated by the nuclear reactions operating in the solar core (cf., Bahcall, this issue). The neutrino flux is sensitive to the temperature and composition profiles in the central regions of the Sun. It was, therefore, expected that the steep temperature dependence of some of the nuclear reaction rates will determine Sun's central temperature to better than a few per cent. The persistent discrepancy between the measured solar neutrino counting rates and the predictions of standard models raised doubts about the reliability of structure calculations, based on the assumption of standard physical properties for neutrinos. This had prompted solar physicists to look for some independent means to explore conditions inside the Sun and the techniques of geo-seismology were adopted by using the precisely measured eigenfrequencies of global oscillations to determine the sound speed and density variations through most of the solar body.

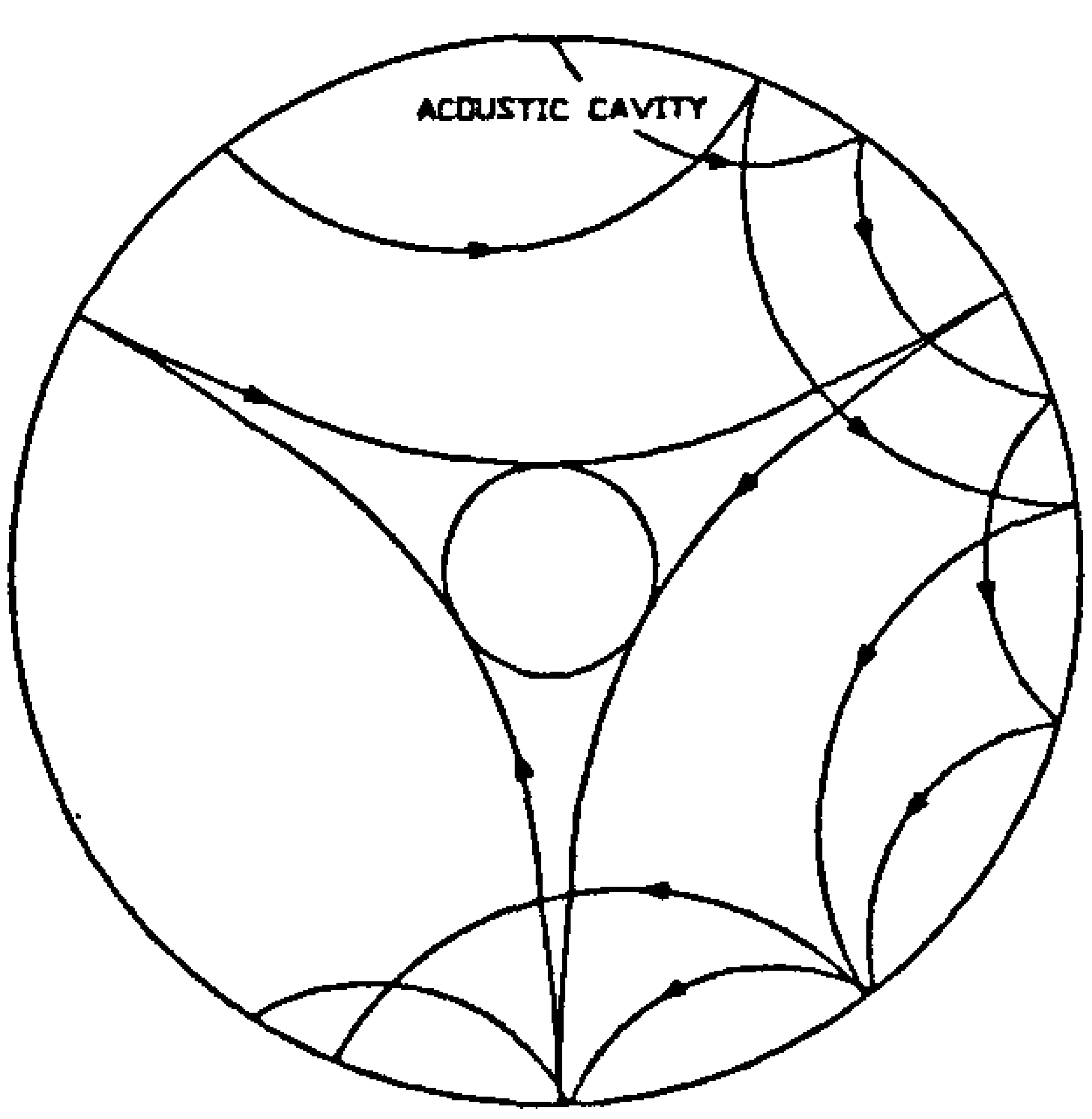

Figure 1. Propagation of acoustic waves corresponding to different values of l. The distance between two successive points at which a ray intersects the surface is a measure of its horizontal wavelength. Modes with low l or large horizontal wavelength penetrate into deeper layers.

The helioseismic database of oscillation frequencies may be analysed in two ways: (i) forward method, and (ii) inverse method. In the forward method, an equilibrium solar model, constructed using the structure equations, is perturbed to obtain the eigenfrequencies of solar oscillations in a linearized theory, and these are compared with the accurately measured oscillation frequencies. The fit is, of course, seldom perfect; but the comparison suggested that the thickness of the convection zone is close to 200,000 km, deeper than what was previously estimated, and helium-abundance by mass in solar envelope was indicated to be 0.25. The direct method has had only a limited success, though, since it is not possible to produce a perfect fit for the seismic data by merely fitting a set of adjustable parameters characterizing the specific models. As a result, the values for various parameters may be non-unique. A number of inversion techniques[13] have, therefore, been employed to extract more information about the solar interior.

One of the major accomplishments of the inversion techniques has been the effective use of the observed solar oscillation frequencies for a reliable inference of the internal structure of the Sun[14,15]. Thus, the profile of the sound speed, $c = \sqrt{\Gamma_1 P/\rho}$ (where $\Gamma_1 = (\partial \ln P/\partial \ln \rho)_s$ is the adiabatic index), has now been established through the bulk of the solar interior to an accuracy of better than 0.1%, and the profiles of density, adiabatic index and other thermodynamic quantities are known to somewhat lower accuracy. It also appeared from the variation of sound speed beneath the convection zone that the adopted opacities for solar modelling near the base of the convection zone, were low by about 15–20%. This was later confirmed by the use of Livermore opacity calculations[16]. In Figure 2 are shown the plots of the relative difference in sound speed, and density between the Sun, as inferred from helioseismic inversions and a standard solar model with gravitational settling of helium and heavy elements[17]. There is a reasonably close agreement except for a noticeable discrepancy near the base of the convection zone and a smaller discrepancy in the energy-generating core. The bump below $0.7\,R_\odot$ could be attributed to a sharp change in the gradient of helium-abundance profile arising from diffusion in the reference model. A moderate amount of turbulent mixing (induced by say, a rotationally induced instability) immediately underneath the convection zone can alleviate this discrepant feature. The dip in the relative sound speed difference around $0.2\,R_\odot$ is not yet well understood; it could be due to inaccurate composition profile in the solar model, possibly due to use of incorrect nuclear reaction rates.

The sound speed profile in ionization zones is affected by the variation in Γ_1 and it is possible to use this to determine the helium-abundance in solar convection zone. The inverted sound speed profile can be employed to compute the quantity,

$$W(r) = \frac{r^2}{GM_\odot} \frac{dc^2}{dr}, \qquad (3)$$

which is shown in Figure 3. The small peak around $r = 0.98\,R_\odot$ in this curve is due to the HeII ionization zone, which can be calibrated to measure the helium abundance. The sharp change in its gradient around $r = 0.713\,R_\odot$ indicates the base of the convection zone and this curve can thus be used to measure the depth of the convection zone. The helium abundance in solar envelope is found to be[18] 0.249 ± 0.003. This value is less than what was used in the earlier standard solar models and the discrepancy was attributed to the fact that some of the helium would diffuse into the interior through gravitational settling. An examination of the inverted sound speed profile, below the convection zone, also reinforces this conclusion[19]. The incorporation of gravitational settling in radiative interior, indeed, results in a significant improvement in solar models. This also reduces the life time of main sequence stars and the estimated age of globular clusters is also diminished. Clearly, this will have wider implications for the age problem in the context of standard big bang model of cosmology.

The dip in Γ_1 inside the ionization zone is also determined by the equation of state and the inverted sound speed in this region provides a test for the equation of state[20]. It is found that standard equations of state, which were widely used in stellar evolution calculations, were not good enough to model the solar interior. More sophisticated equations of state, like the MHD (Mihalas, Hummer and Dappen)[21], or OPAL[22] equation of state, are found to produce good accordance with helioseismic data. Further, the OPAL equation of state is found to be in better agreement with solar data compared to the MHD equation of state[18]. Even these equations of state show slight discrepancy in the core and this discrepency has recently been attributed to the neglect of relativistic correction for electrons[23].

In solar models the second derivative of temperature and hence that of the sound speed is discontinuous at the base of the convection zone. This discontinuity in the function $W(r)$, eq. (3) can be utilized to identify the position of the base of the convection zone[24]. The sound speed as well as the frequencies of p-modes are very sensitive to the depth of the convection zone and therefore seismic inversions enable a very accurate measurement of its thickness. Using recent data the depth of the convection zone is estimated to be[25] $(0.2865 \pm 0.0005)\,R_\odot$. Further, the position of the base of the convection zone is controlled by the opacity of solar material. We can then

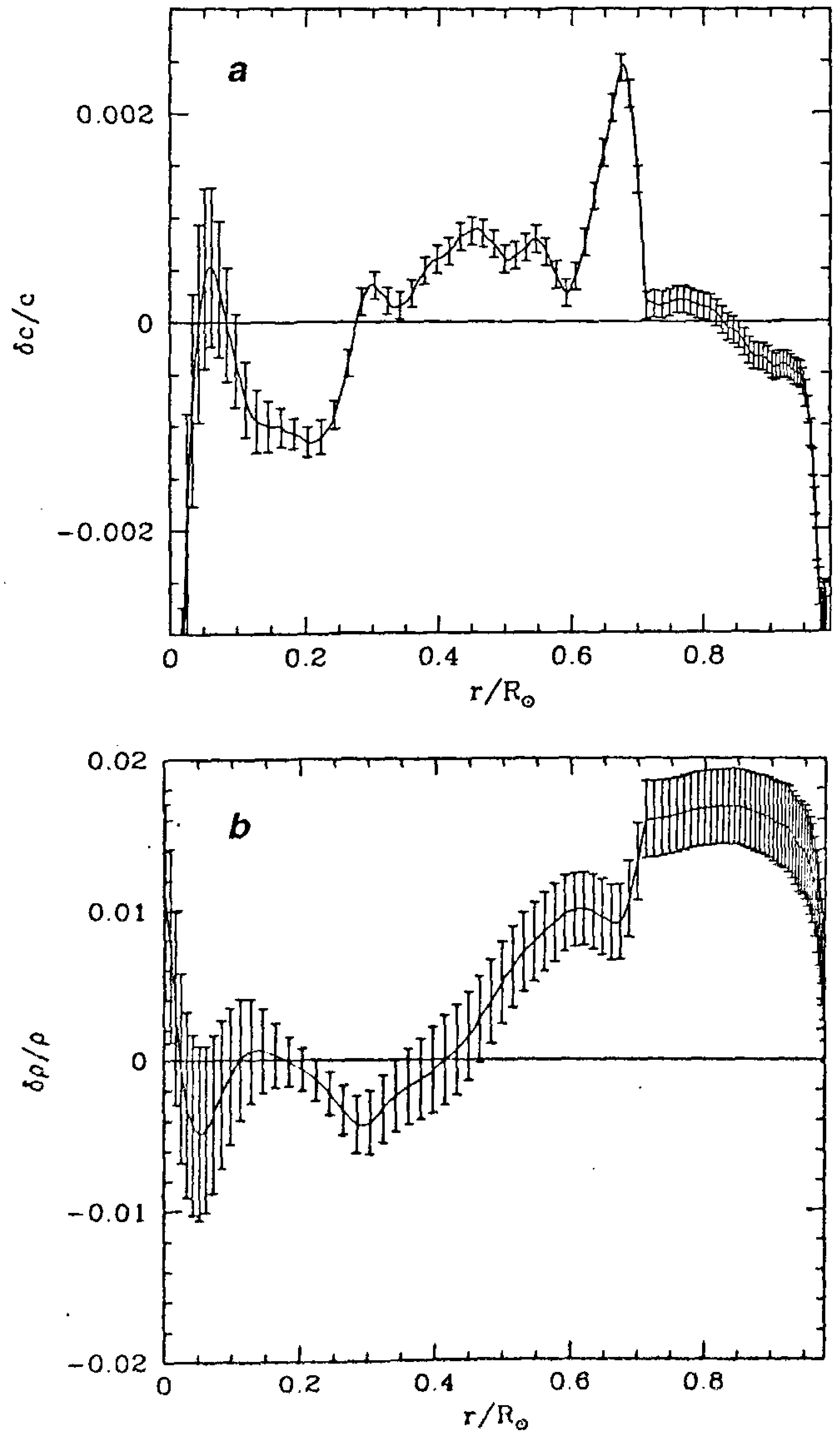

Figure 2 *a, b*. Relative difference in sound speed and density profiles between the Sun and a standard solar model[17].

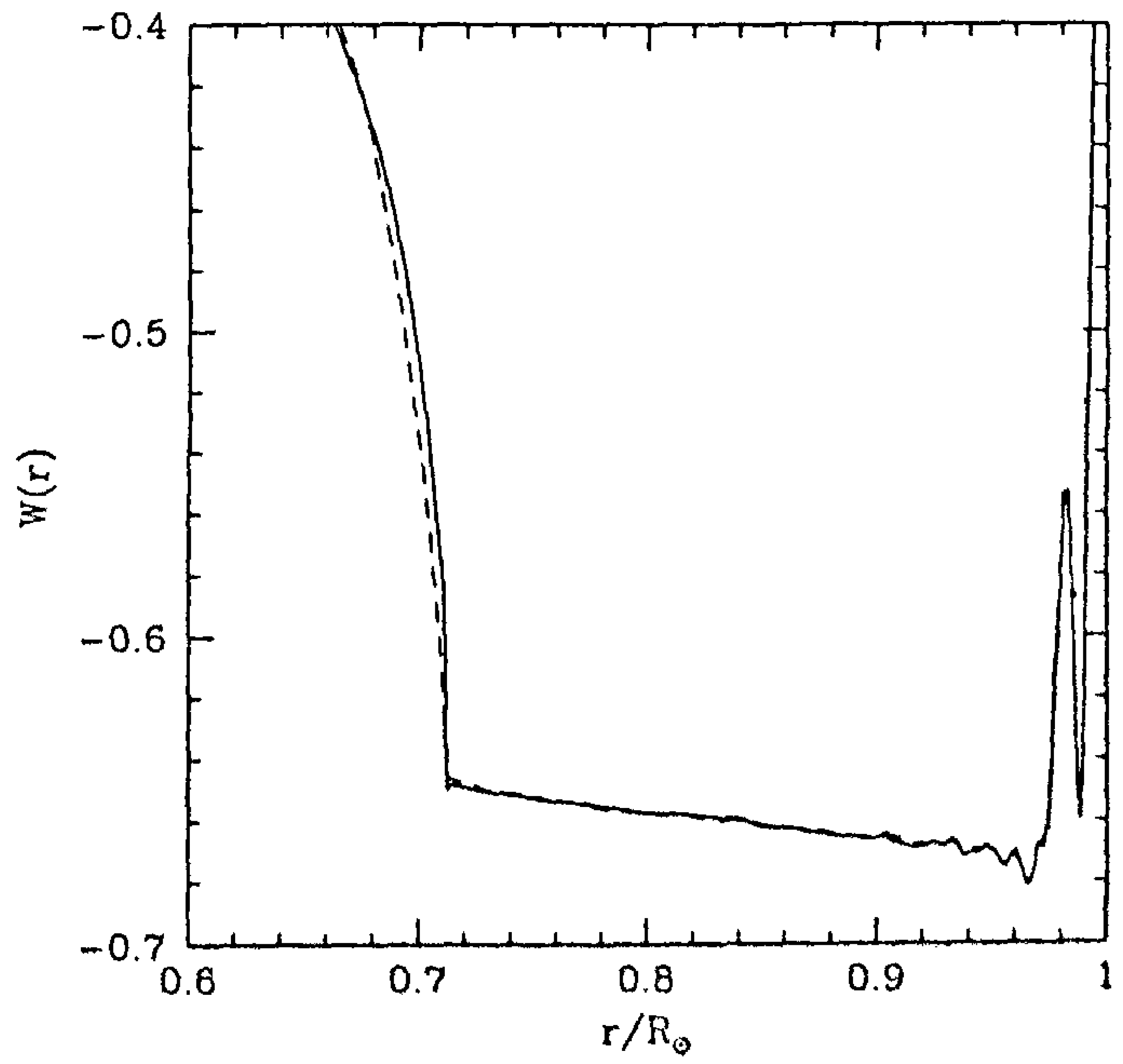

Figure 3. The function $W(r)$ for a solar model is shown by the continuous line, while the dashed line represents the same for the Sun using inverted sound speed profile.

estimate the opacity at the base of the convection zone[26] and it has been found that the current OPAL opacity tables[27] are consistent with helioseismic data to within an estimated error of 3%.

The convective eddies inside the convection zone are expected to penetrate beyond the theoretical local boundary, but there is no satisfactory theory to describe this overshoot. A significant overshoot can alter the stellar-evolution calculations and so far the extent of penetration is treated as a parameter in stellar evolution calculations. Now with the availability of helioseismic data, it has become possible to estimate this extent of overshoot below the base of the solar convection zone. The discontinuity in the derivatives of sound speed at the base of the convection zone introduces an oscillatory component[28] in the frequencies as a function of radial order n. The amplitude of this signal depends on the magnitude of discontinuity, which in turn depends on the extent of overshoot below the solar convection zone. Thus by measuring the amplitude of this oscillatory signal we can determine the extent of overshoot below the convection zone[29,30]. The measured oscillatory signal is found to be consistent with no overshoot and on the basis of this result an upper limit of 1/20 of the local pressure scale height has been obtained[31] for the overshoot distance. This is, of course, too small to affect the stellar evolution calculations significantly.

The frequencies of f-modes, which are surface gravity modes, are largely independent of stratification in the solar interior and are essentially determined by the surface gravity. These frequencies, which have now been measured reliably by GONG and MDI data, provide an important diagnostic of the near-surface regions, as well as an accurate measurement of the solar radius[32,33]. Furthermore, the changes in solar radius with time by a few km have also been recorded. Another application of the accurately measured f-mode frequencies is their potential use as a diagnostic of solar oblateness and of near-surface magnetic fields, in addition to the possibility of investigating solar cycle variation in these quantities.

The primary inversions which have provided information about the physical quantities like the sound speed, density and adiabatic index in the solar interior are based on the equations of mechanical equilibrium. The equations of thermal equilibrium are not used, because on time scales of several minutes, no significant energy exchange is expected to take place in moving elements. The frequencies of solar oscillations are, therefore, largely unaffected by the thermal processes in the interior. However, having obtained the sound speed and density profiles in solar interior through primary inversions, we can employ the equations of thermal equilibrium to determine the temperature and chemical composition profiles inside the Sun[34–36], provided input physics like the opacity, equation of state and nuclear energy generation rates are known. In general, the computed luminosity resulting from these inferred profiles would not necessarily match the observed solar luminosity. The discrepancy between the computed and measured solar luminosity can, in fact, provide a test of input physics, and using these constraints it has been demonstrated that the nuclear reaction cross-section for the proton–proton reaction, needs to be increased slightly to $(4.15 \pm 0.25) \times 10^{-25}$ MeV barns[36]. This cross-section has a controlling influence on the rate of nuclear energy generation and neutrino fluxes, but it has never been measured in the laboratory and all estimates are based on theoretical computations. More recently, this cross-section has been revised upwards[37] to a value close to what was estimated helioseismically.

The inferred helium-abundance profile agrees with that in the standard solar model, incorporating diffusion of helium and heavier elements, except in layers just below the solar convection zone. This is the region where the solar rotation rate has a sharp gradient in radial direction (Christensen-Dalsgaard and Thompson, this issue). The inferred helium-abundance profile, for example, shown in Figure 4 is essentially flat in this region. This indicates the presence of some sort of mixing process, possibly by rotationally induced instability which has not been properly accounted. The mixing in this region can also explain the anamolous low-lithium abundance in solar envelope. The destruction of lithium by nuclear reactions can take place at temperatures exceeding 2.5×10^6 K, but at the base of the solar convection zone the temperature is still not high enough to burn lithium. Thus, if the mixing extends a little beyond the solar convection zone to a radial

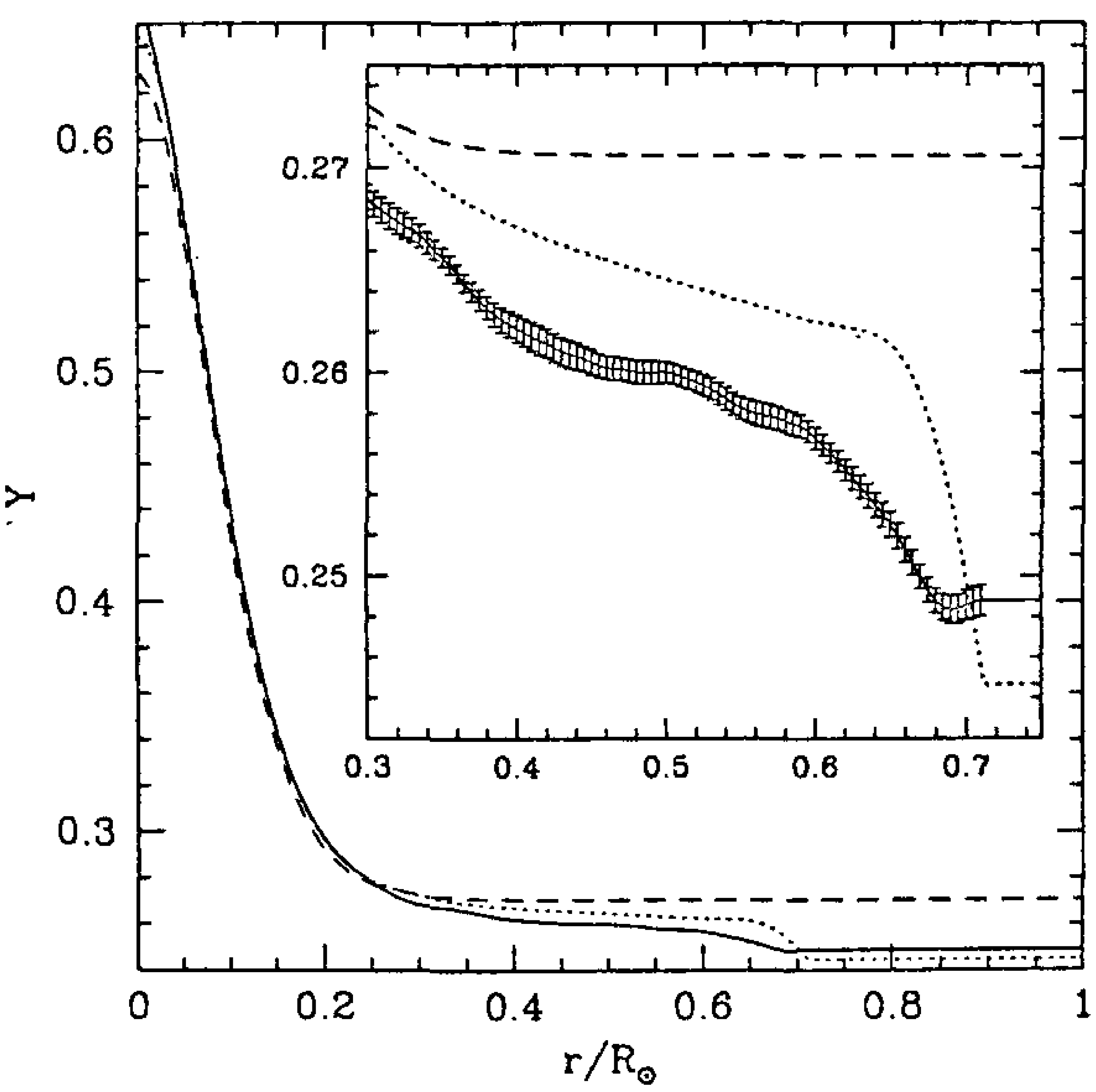

Figure 4. Fractional helium abundance by mass in the Sun as obtained from inversions is shown by the continuous line. The dashed line represents the abundance profile for a solar model without diffusion, the dotted line shows that for a model incorporating diffusion of helium and heavy elements.

distance of 0.68 $R_\odot$, the temperature can reach high enough value to explain the low abundance of lithium. This is exactly the region where the inferred composition profile is flat, indicating the operation of a mixing process.

With an allowance of up to 10% uncertainty in opacity values, the central temperature of the Sun is found to be $(15.6 \pm 0.4) \times 10^6$ K (ref. 38). The inferred temperature and composition profiles may be used to compute the neutrino fluxes in the seismic solar models and the predicted neutrino fluxes come close to what is obtained for the current standard solar models. This suggests that the known discrepancy between the observed and predicted neutrino fluxes is likely to be due to non-standard neutrino physics. Thus, helioseismology has turned the Sun into a laboratory to study properties of neutrinos.

Apart from spherically symmetric structure of solar interior, it is also possible to determine helioseismically the rotation rate inside the Sun from the accurately measured rotational splittings (cf., Christensen-Dalsgaard and Thompson, this issue), $D_{nlm} = (\nu_{nlm} - \nu_{nl-m})/2m$. It turns out that first-order effects of rotation yield splittings which depend on odd powers of m, and these odd splitting coefficients have been used to determine the rotation rate as a function of depth and latitude. On the other hand, magnetic field or other asphericities give only even order splittings and hence these can be separated from the rotational effects. These even splitting coefficients allow us to study the departures from spherical symmetry inside the Sun. Further, the local helioseismic techniques (cf., Kosovichev and Duvall, this issue) allow us to study other large-scale flows, including meridional flows in solar interior.

The Sun's oblateness has been measured to be about 10^{-5} at the solar surface[39], and there does not seem to be any evidence of temporal variation in oblateness. The oblateness is indeed, consistent with what is expected from the helioseismically inferred rotation rate in solar interior. The resulting quadrupole moment[40] turns out to be $(2.18 \pm 0.06) \times 10^{-7}$, which yields a precession of perihelion of planet Mercury's orbit by about 0.03 arc sec/century, validating the general theory of relativity.

The continuing efforts in helioseismology will hopefully reveal the nature and strength of the magnetic field present in the solar interior and will also help in ascertaining the causes that drive the cyclic magnetic activity, and also locate the seat of the solar dynamo (cf., Choudhuri, this issue). The global and local seismology of the Sun is clearly poised to reveal its interior to a remarkably accurate detail. The uninterrupted accruing of the seismic data, can enable us to study the temporal variation of mode frequencies and amplitudes, which will indicate what changes are taking place in solar structure and dynamics. We may also learn how the Sun's magnetic field changes with the solar activity cycle and what causes the Sun's irradiance to vary with the sunspot cycle.

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