Linear Algebra. Jin Ho Kwak and Sungpyo Hong. Birkhäuser Verlag AG, P. B. No. 133, CH 4051, Basel, Switzerland. 1998. 369 pp. Price: SFr 48.

Linear algebra is a branch of algebra which deals with linear vector spaces, linear operators as well as linear, bilinear and quadratic functions. The results of linear algebra have found application in such diverse fields as optics, quantum mechanics, display addressing, electric circuits, cryptography, computer graphics, economics, linear programming, solution of systems of differential equations, etc. The manipulation of matrices and determinants plays a central role in all applications of linear algebra.

The development of linear algebra started some centuries ago with the need to understand the nature of the solution of algebraic equations. Rapid development took place from the seventeenth century onwards with the invention of the concept of a determinant. With Cramer's rule and Gauss' method for the solution of a system of linear algebraic equations, the first stage of development was over. The importance of using matrices in the solution of equations was realized only in the nineteenth century. The work of Frobenius and others on the matrix rank made it possible to decide on the uniqueness of the solution of a system of linear equations. Matrices can be rectangular or square. While the theory of matrices deals with the general case of rectangular matrices, the square matrices assume an important role in most applications. The following summary concentrates mainly on square matrices.

The concept of the linear vector space took shape towards the end of the nineteenth century and has dominated developments in the field of linear algebra during the twentieth century. Elements of a vector space are vectors. Quantitative attributes of vectors can be easily fixed using a basis of linearly independent vectors (*coordinate system*) to span the vector space. Every vector in the space can be resolved uniquely into components in terms of the vectors of a given basis. More than one basis can exist in a space. Different vectors have different components in a given basis while the same vector has different sets of components in different bases.

Vectors can be subjected to the action of abstract entities called linear operators or linear transformations. The action of a linear operator on a vector results in another vector belonging to the same space. The successive applications of an operator and its inverse leave a vector unchanged. A new basis with orthogonal basis vectors can be produced (Gram–Schmidt process) by subjecting the basis vectors of a non-orthogonal basis to the actions of a series of operators. The action of a linear operator on the basis vectors helps to define the matrix representation for that operator. Clearly, the matrix inverse represents the inverse operator. The matrix representation of an operator depends upon the basis chosen. All elements of a matrix may be non-zero and different from one another. Such a matrix is highly asymmetric. A matrix which has a more symmetric and simpler form (e.g. a matrix having non-zero elements only on the diagonal) is easier to use. One of the problems of linear algebra is to find a basis in which a given operator has the simplest possible matrix representation. The Jordan canonical form can be shown to be the simplest one to which the most general matrix can be reduced by the appropriate choice of basis.

Just as every human being has individuality, every matrix also has its own *character*. Unlike human nature, the character of a matrix can be quantitatively defined in terms of its eigenvalues and eigenvectors. The eigenvalues are calculated by solving the characteristic polynomial equation associated with the given matrix. Interestingly, every matrix satisfies its own characteristic equation (Hamilton–Cayley theorem)! A basis can be formed out of the linearly independent eigenvectors of a matrix. Referring a matrix to this basis reduces the matrix to its simplest, diagonal form with the eigenvalues forming the diagonal elements. A general quadratic form can be written in terms of a symmetric matrix and the vector of coefficients. The diagonalization of the symmetric matrix reduces the quadratic form to a sum of squares, making it possible to write down the inequalities satisfied by the different coefficients.

Most practical applications require the manipulation of matrices of high rank. This normally formidable task became especially simple with the advent of digital computers and the invention of computer languages such as Fortran. A vast branch of numerical analysis is today concerned with writing fast algorithms for matrix manipulations (calculation of determinant, inverse, eigenvalues, etc.) on both serial and parallel computers.

The first impression after reading the book by Kwak and Hong is favourable. The authors have set down their lecture notes on linear algebra in a systematic way, leading to a logical development of the subject. All the important theorems and results are discussed in terms of simple worked examples. The student's understanding of a result is tested by problems at the end of each subsection. The applications of a particular theorem in other fields are discussed (unfortunately, a physicist will miss a reference to quantum mechanics!). Every chapter ends with a variety of exercises. This reviewer is especially happy that the authors have even typeset the manuscript of their book; it has been clearly a labour of love. There appear to be very few misprints and ambiguities. The book is certainly a valuable reference for an expert. But a novice is likely to find it difficult to use the book in its present form. Before recommending this book for general consumption, therefore, it seems appropriate to make a few suggestions.

Every theorem, lemma, corollary, definition, example, problem and exercise carries the same pattern of numbering in the form of $\alpha.\ \beta$, where α is the chapter number and β an integer. The book is replete with *recalls* to earlier results. Locating a particular theorem or lemma in an earlier chapter becomes difficult owing to the overexuberant numbering that follows the same pattern for all entities. One way out of this difficulty would be for the authors to indicate the page number with every recall. A better solution would be to reduce the burden of results in each chapter and resort to renumbering of some entities. All problems can be regrouped at the end of the chapter. At the end of a section, the student can be directed to tackle the relevant problems. Definitions and examples can be identified by single numbers like β which start with one in each chapter. A corollary of theorem $\alpha.\beta$ can be tagged with $\alpha.\beta.\gamma$ where γ is an integer.

The index can be expanded to yield a more complete representation of the results. There are no references to other books. The authors employ nomenclature that is difficult to find in available books. For instance, *the fundamental theorem of algebra* is well known. *Two fundamental theorems* (of linear algebra) are not mentioned in other books. Is the Cayley–Hamilton theorem known as the Hamilton–Cayley theorem? The authors must check all nomenclature in their book to bring it in line with some standard reference, say, *The Encyclopedia of Mathematics* (Kluwer). If a theorem is known by a particular name in another work, a reference to that work will also be useful to the interested reader.

There are a few misprints (p. 25, p. 343) and ambiguities (p. 259, below definition 7.4). Some definitions are either assumed tacitly or not prominently set down (commutative diagram on p. 137; the ~symbol on p. 155).

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Interpreting Bodies: Classical and Quantum Objects in Modern Physics. Elena Castellani (ed.). Princeton University Press, 41 William Street, Princeton, New Jersey 08540. 1998. 329 pp. Price: US$ 19.95.

This book is the outcome of a project to understand the nature of physical objects in the light of modern physics. The editor has brought together physicists, philosophers and logicians in an attempt to probe this question from different points of view.

The volume contains essays by well-known physicists of the past like Born, Heisenberg and Schrödinger, as well as writings contributed by present day physicists. It also contains a blend of old and new writings by philosophers and logicians.

Being a theoretical physicist by training, I was naturally inclined to read the physics articles with greater interest. The articles by Max Born, Erwin Schrödinger and Werner Heisenberg which have been reprinted in this volume are classics. Present-day physics students will certainly derive inspiration from these articles. Here, I summarize the key issues addressed in these articles. Max Born deals with the notion of reality. The main point he makes is 'The idea of invariant is the clue to a rational concept of reality, not only in physics but in every aspect of the world'. The idea of an invariant is an old one in mathematics dating back to Sylvester, but has come to the fore in twentieth-century physics in the theory of elementary particles. He reinforces his point of view by giving several examples from our everyday experience. For instance, if we look at the shadow of a circle cut out from a piece of cardboard and observe its shadow on the wall, we find that the shadow, in general looks like an ellipse and one can change the lengths of the axes of the ellipse by turning the cardboard piece around. However, if we see several such shadows at different places, we can reconstruct the original circle and determine its radius. Thus, the radius is an 'invariant' of these transformations. He emphasizes that 'The feature which suggests reality is always some kind of invariance of a structure independent of the aspect, the projection. This feature, however, is the same in ordinary life and in science...'. Schrödinger deals with the question of what one means by an elementary particle. He discusses the dual identity of an elementary particle – an amalgamation of particle and wave identities and emphasizes that all observations point to the fact that microscopic particles are indistinguishable and do not have clearly defined identities as individuals. Heisenberg offers the interesting view (which was quite popular during the fifties): 'There is no difference in principle between elementary particles and compound systems. This is probably the most important experimental result of the past fifty years.'

Elena Castellani's article deals with a group-theoretic approach to the problem of physical objects. In this approach, one views objects as 'sets of invariants' which emerge from certain symmetry properties of groups. Max Born's article serves as a conceptual foundation for Castellani's article. This article is informative, well written and mathematically quite precise.

Gian Carlo Ghirardi's article addresses the relation between the worlds of microscopic and macroscopic objects. This issue, which is closely related to the problem of measurement in quantum mechanics has been analysed within the framework of dynamical reduction theories. Diederik Aerts' article is also along similar lines. He compares and contrasts his 'creation–discovery' view with other alternative points of view. Giulio Peruzzi gives an overview of contemporary particle physics experiments in the light of present day theoretical particle physics.

Now, I give a brief summary of the philosophy articles compiled here. Hans Reichenbach's article on the genidentity of quantum particles has been reprinted here. The central point that he makes is that in the quantum domain, all our common sense notions of identification of particles break down and we need to 'replace an individual examination of particles by inferences based on statistical properties of an assemblage of particles'. Peter Mittelstaedt tries to accommodate the notion of a quantum object within the purview of Kantian philosophy. Seen in this light, quantum mechanical objects are 'incomplete' or 'unsharp' objects, while classical objects are completely determined. Giuliano Toraldo di Francia's article gives a historical overview of the general issue of the identity of an individual object. He gives a sketch of the evolution of ideas in this field from ancient times to the more modern developments in the realm of foundations of quantum physics. David Lewis has touched upon the relation between the constituents of an object and the physical identity of the object. Tim Maudlin addresses similar issues. He emphasizes the difference between a 'reductionist' approach and a 'holistic' approach and points out that in the quantum domain because of entanglement, it is not always clear how to assign properties to 'individual' systems which are 'parts' of composite systems. Bas van Fraassen has dealt with the notion of permutation invariance of indistinguishable particles in the context of quantum physics. Steven French