Black hole: Gravitational charge equal to field energy

Naresh Dadhich

Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India

In general relativity, both non-gravitational matter energy as well as gravitational field energy are sources of the field. In this note we propose that a collapsing body turns into a black hole when contributions of matter energy and field energy become equal. For measure of field energy we use the Brown–York quasilocal energy while for the gravitational charge (measure of gravitational pull matter energy produces), the Gauss–Komar integral. This is an energetics definition, similar to the classical escape velocity argument, which is intuitively and physically appealing. One of the remarkable results that immediately follows is that an extremal hole can never be formed by collapse of a dispersed state as there is no net energy to drive it. This is however well-known from other considerations, but here it follows from simple energetics of collapse.

FOR definition of a black hole, we need to confine both ordinary particles and photons inside a compact surface. Behaviour of the former is constrained by the usual gravitational potential which could be measured by adoption of the Gauss theorem to stationary spacetimes[1] while the latter can never remain at rest and hence can only respond to space curvature. It can be argued[2] that field energy produces curvature in space and thereby directs motion of photons, since the surface that does not let photons escape out would also constrain ordinary particles as well. It means that at this surface measures of gravitational charge and field energy must become equal. This is what we wish to demonstrate for stationary black holes.

The measure of energy in general relativity (GR) is inherently ambiguous owing to contribution of gravitational field energy. The remarkable feature of GR is that its contribution is automatically taken care of by the curvature of space and hence remains generally hidden in the usual considerations. For instance, while deriving the Schwarzschild solution we ultimately solve the Laplace equation instead of the Poisson equation with the field energy density on the right. It turns out that the effect of field energy is taken care of by the curvature of the space part of the metric leaving the Laplace equation unaffected[2].

For measure of field energy we shall use the recently proposed Brown–York quasilocal energy[3]. It is defined in terms of the difference of mean curvature of a closed 2-boundary embedded in the curved 3-hypersurface of a black hole spacetime and when embedded in (reference) asymptotically flat space. The quasilocal energy is the integral of this mean curvature difference over the 2-boundary. It brings out explicitly contribution of field energy and yields a very simple relation for energy enclosed by a black hole horizon, $E(r_+) = r_+$.

Let an energy distribution have total energy $E(\infty) = M$ in absolute dispersed state and as it collapses on its own under gravity it picks up gravitational field energy. The Brown–York quasilocal energy represents the sum of the field energy gained during collapse and the energy at infinity. This is the energy, $E(r)$, contained inside the radius r.

We can hence define field energy as given by

$$E_F(r) = E(r) - M. \tag{1}$$

where $E(r)$ is given by[3]

$$E = \frac{1}{8\pi}\int d^2x\sqrt{\sigma}(K - K_o). \tag{2}$$

Here σ is the determinant of the 2-metric of the boundary B and K is the mean extrinsic curvature of B isometrically embedded in the curved 3-hypersurface C and 0 refers to B's curvature when embedded in the asymptotic flat space as the reference. E is minus variation in action in a unit change in proper time separation between B and its neighbouring 2-surface. It is therefore the value of the Hamiltonian that generates unit time translations orthogonal to C at the boundary B. This is obviously the natural prescription for measure of energy. It is a covariant expression modulo reference spacetime and it also possesses additivity property.

Gravitational charge is defined by the Gauss integral for the red-shifted proper acceleration over the 2-surface[1,4–6] and it is given by

$$M_c = \frac{1}{4\pi}\int \mathbf{g}\cdot d\mathbf{s}, \tag{3}$$

where $\mathbf{g} = -N\nabla(\ln N)$, and N is the lapse function. $\mathbf{g}$ is the red-shifted proper acceleration experienced by a free particle relative to infinity.

For a charged hole, eq. (2) yields

$$E(r) = r - (r^2 - 2Mr + Q^2)^{1/2}, \tag{4}$$

while eq. (3) yields the gravitational charge as[1,5]

$$M_c(r) = M - Q^2/r, \tag{5}$$

which gives $M_c(r_+) = \sqrt{M^2 - Q^2}$ $(G = c = 1)$.

e-mail: nkd@iucaa.ernet.in

We propose that a black hole is defined by equality of gravitational charge and field energy which means

$$E_F = M_c. \tag{6}$$

In view of eqs (1), (4), (5) and (6) we obtain the characterizing expression for a charged black hole as

$$(r^2 - 2Mr + Q^2)(2Mr - Q^2) = 0. \tag{7}$$

This clearly defines the horizon, $r_+ = M + \sqrt{M^2 - Q^2}$. Thus the general characterizing relation for black hole is,

$$E(r) - E(\infty) = M_c(r). \tag{8}$$

This holds good for all coordinates, and in particular we have also verified it for isotropic coordinates. Though we cannot evaluate for a rotating black hole E and M_c at any arbitrary r, they could be done at the horizon. There they do obey the above relation as $E(r_+) - M = M_c(r_+) = \sqrt{M^2 - a^2}$. The above characterizing relation is true in general.

From eq. (4), $E(r)$ will go for large r as $E \simeq M + (M^2 - Q^2)/2r$. Note that $E(r)$ is the total energy enclosed by the radius r, while the interaction energies $-M^2/2r$ and $Q^2/2r$ lie exterior to r. Hence $E(r) = E(\infty) - E(outside\ r) \simeq M - (-M^2/2r + Q^2/2r)$. In the dispersed state $E(\infty) = M$ at infinity, as individual particles collapse on their own under gravity to form a fluid ball, field energy builds up and gets added to M to give E. The remarkable feature is that when this gain becomes equal to M, the Schwarzschild horizon is defined. That is, the horizon is being marked by equipartition of E into matter (M) and field energy (M). This is however true in general. Now when an electric charge, which introduces repulsive force, is added on the hole, the gain in E under collapse will naturally be reduced as the strength of collapsing force (attraction) is decreased by electric repulsion. The relative strength of the two opposing tendencies will be the measure $\sqrt{M^2 - Q^2}$, which will now indicate the gain over M in E at the horizon. Interestingly it is this gain which is also the measure of gravitational charge[5] enclosed in the horizon which is responsible for the Hawking temperature.

In this case matter energy (coming through energy–momentum tensor) contained inside the radius r would be $M - Q^2/2r$ because the electrostatic energy $Q^2/2r$ is exterior to r. The equipartition of E into matter and nonmatter would mean

$$E(r) - (M - Q^2/2r) = M - Q^2/2r, \tag{9}$$

which would also lead to the same relation eq. (7). That is alternatively a black hole is characterized by equipartition of the Brown–York quasilocal energy into *matter and non-matter energy*.

For the extremal hole ($M^2 = Q^2$), it can be easily seen from eq. (4) that $E = M$ everywhere and hence E_F vanishes indicating the absence of a driving force for collapse. That means, extremal black hole cannot be formed by collapse from a dispersed state. One has to do work to effect such an assembly because the net force (attractive due to M and repulsive due to Q^2) vanishes. This is yet another way of getting at the third law of black hole dynamics that extremal hole cannot be created from a non-extremal one by a finite sequence of physical processes[7]. The remarkable thing here is that it is the basic energetics that prohibits it.

In the spirit of Christodoulou's irreducible mass[8], we can define irreducible energy as

$$E_{ir}^2 = r_+^2 + a^2 = E^2(r_+) + a^2, \tag{10}$$

which will lead to the equivalent relation

$$M^2 = \frac{1}{4}\left(E_{ir} + \frac{Q^2}{E_{ir}}\right)^2 + \frac{J^2}{E_{ir}^2}, \tag{11}$$

where $E_{ir} = 2M_{ir}$. Let us try to interpret the above relation in analogy with the special relativistic conservation law. Clearly M represents the total energy, J/E_{ir} the momentum and $(E_{ir} + Q^2/E_{ir})/2$ the rest energy. The electric field contributes to increasing rest energy while rotation, as expected, also contributes to kinetic energy of the hole. E_{ir} will be bounded as $2M \geq E_{ir} \geq \sqrt{M^2 + a^2}$ or $\sqrt{2M^2 - Q^2}$. The fraction of energy available for extraction would be $(E_{ir}(\mathrm{Sch}) - E_{ir}(\mathrm{extremal}))/E_{ir}(\mathrm{Sch})$. It is 29% for the rotating and 50% for the charged black hole.

It is thus an interesting and insightful application of the Brown–York quasilocal energy in understanding the role of field energy in characterizing black hole. The fraction that can be put in field energy, $E(r) - M \leq (M^2 - a^2 - Q^2)^{1/2}$, has the value M for the Schwarzschild and zero for the extremal hole. The equality defines the horizon. Thus the field energy is bounded above by the total energy M. Further gravitational charge is conserved for the Schwarzschild hole while the Brown–York energy for extremal hole. The two indicate the two limits of collapse. Since collapse is driven by the field energy, which in contrast to the Schwarzschild case, vanishes for an extremal hole and hence it cannot be formed of gravitational collapse. Like the photon, extremal hole has to be born as such. Our definition (eq. (1)) for a black hole can in fact be established rigorously for stationary spacetimes by using the Gauss–Codacci relations[9].

Finally, this is a physically and intuitively appealing characterization of black hole based on balance between gravitational charge and field energy. An excellent example of this kind of consideration is the escape velocity argument, which though gives the right result follows from an invalid relation. It only refers to escape of particles from the gravitational field and not to one-way character of a closed 2-surface, which is the characteristic feature of the horizon. The horizon marks irresistability of gravitational pull through gravitational charge, as well as one-way character, no escape out of closed surface, through space curvature caused by the field energy. These two features must define the same surface, as the photon is the limiting case of an ordinary particle.

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The use of RAPD in assessing genetic variability in *Andrographis paniculata* Nees, a hepatoprotective drug

P. Padmesh, K. K. Sabu, S. Seeni and P. Pushpangadan

Plant Biotechnology Division, Tropical Botanic Garden and Research Institute, Palode, Thiruvananthapuram 695 562, India

RAPD analysis was done to determine intraspecific variability in *Andrographis paniculata*, a popular antipyretic and hepatoprotective drug used in traditional medicine in India. The accessions collected from parts of India and south-east Asia on molecular analysis revealed moderate variation within the species. Similarity measurement using UPGMA followed by cluster analysis resulted in 5 major groups based on geographical distribution that generally reflected expected trends between the genotypes. There were also important exceptions like AP-48, an accession from Thailand showing close resemblance to AP-38 collected from Tamil Nadu and AP-29 from Assam significantly diverse from the rest of the native genotypes. The results indicated that RAPD could be effectively used for genetic diversity analysis in wild species of prospective value as it is reliable, rapid and superior to those based on pedigree information.

INDIA being one of the twelve megadiversity centres has immense biotic wealth marked by remarkable ecosystem, species and genetic diversity. This rich biological diversity is matched equally by rich cultural diversity and health traditions. Over 7000 species out of an estimated 17,000 higher angiosperms recorded from India are reportedly used for medicinal purposes[1]. For efficient conservation and management of medicinal plant diversity, the genetic composition of species collected from different phytogeographical regions needs to be assessed. While there are a few commendable efforts to study the genetic variation within populations of selected medicinal plant species based on isozyme profile in different regions of distribution[2], a broad based analysis of genetic diversity between and within the species is still lacking. Molecular analysis of intraspecific variation in particular may find application in resolving disputes of taxonomic identities, relations and authentication of the species, developing a comprehensive database of genetic variability in the species for future reference and protection of genetic diversity of the species, identification of useful genotypes that could be developed as cultivars for field trials and sustainable utilization through formulation of standard drugs free from batch to batch variations. Since a vast majority of Indian population is dependent on traditional medicine for primary health care, and as against the recent revival of interest in plant medicines across the globe and consequent pressure on precious herbal resources, it makes sense to rationalize the use of medicinal plants through scientific screening and validation. In the present study we provide evidence through RAPD assay for the occurrence of genetic variation in *Andrographis paniculata* the Kalmegh of Ayurveda, reputed for its antipyretic and hepatoprotective properties, wide distribution and high adaptability in different phytogeographical zones of south-east Asia. The genus *Andrographis* as a whole is of potential significance to India as 25 out of 28 species in the world are distributed in India with 23 of them occurring in the peninsular region[3].

A germplasm collection of 52 accessions was organized from different parts of India, Thailand, Malaysia and Indonesia and maintained under uniform growth conditions. Of these 15 accessions, those displaying interesting morphological and phytochemical variations were selected (data not shown) for further investigations (Figure 1). Total genomic DNA from the young leaves of the plants was isolated following the modified Murray and Thompson method using CTAB. Extraction buffer contained 1.2% PVP-40T (mol wt 40,000, Sigma, USA) to remove high phenolic contaminants and double $CHCl_3$ extraction at 10,000 rpm helped to remove poly-