and B is a pairwise disjoint union of measurable sets B_i, $i \in N$, and if $A_i \sim B_i$ for each i, then $A \sim B$. We say that A and B are equivalent by countable decomposition (modulo m) if there exist sets M and N in $\mathscr{B}$ of measure zero such that $A \Delta M$ and $B \Delta N$ are equivalent by countable decomposition. A set $A \in \mathscr{B}$ is said to be compressible in the sense of Hopf if there exists $B \subseteq A$ such that $A \sim B$ and $m(A - B) > 0$. Clearly the notion of compressibility in the sense of Hopf depends only on the σ-ideal of m-null sets in $\mathscr{B}$ and not on m itself. If μ is a finite measure on $\mathscr{B}$ invariant under σ and having the same null sets as m, then $A \sim B \pmod{m}$ implies that $\mu(A) = \mu(B)$. Moreover, whenever such a μ exists, no measurable set of positive measure can be compressible in the sense of Hopf. In particular, X cannot be compressible in the sense of Hopf whenever such a μ exists. E. Hopf[2] proved the difficult converse of this, namely, that *if X is not compressible in the sense of Hopf then there exists a finite measure μ invariant under σ and having the same null sets as m.*

In ref. 3, Hajian and Kakutani gave rather simple necessary and sufficient condition for a non-singular automorphism to admit an invariant probability measure absolutely continuous with respect to the given one. Call a measurable subset A of X weakly wandering if the sets $\sigma^n(A)$ are pair-wise disjoint as n runs over some infinite subset of positive integers. Hajian and Kakutani show that *σ admits an invariant probability measure if and only if there is no weakly wandering set of positive measure.*

The book under review deals with this problem at a much more concrete level for some special classes transformations (not necessarily invertible) on intervals of the real line. The results mentioned above, being of foundational nature, do not seem to be immediately applicable and different methods have to be devised to exhibit the existence of a finite invariant measure, absolutely continuous with respect to a given one. Since transformations considered are much more concrete (piece-wise smooth maps on intervals) interesting calculations combined with some functional analytic considerations permit one to exhibit finite invariant measure for a given transformation.

Consider a measurable map τ: $[0, 1] \to [0, 1]$ such that $\lambda \circ \tau^{-1}$ is absolutely continuous with respect to λ, the Lebesgue measure on $[0, 1]$. For $f \in L^1([0, 1], \lambda)$, the measure

$$\mu(A) = \int_{\tau^{-1}(A)} f d\lambda, \ A \subset [0, 1]$$

is absolutely continuous with respect to λ. The map P_τ: $f \to (d\mu/d\lambda)$, called Frobenius-Perron operator and referred to as the hero of the book, is positive, and a linear contraction. Further, $P_{\tau \circ \sigma} = P_\tau \circ P_\sigma$. These and other properties of P_τ are presented in chapter 3. With the aid of these properties the rest of the book embarks on a full scale discussion of dynamical and statistical properties of those τ which belong to one or more of the following or similar classes:

(i) The class $\mathscr{T}$ of those τ which (a) are piece-wise smooth and expanding in the sense that $|\tau'_i| > \alpha > 1$, where τ_i is the restriction of τ to the interval I_i where it is smooth, (b) the function $g(x) = 1/|\tau'(x)|$, (at the end point of an interval of the partition associated to τ, g is defined to be one of the one-sided derivatives which exists) is of bounded variation.

(ii) The class $\mathscr{T}_{pc}$ of piece-wise continuous and convex functions, $\tau(a_{i-1}) = 0$, $\tau'(a_{i-1}) > 0$, $\tau'(0) > 1$.

(iii) Class $\mathscr{T}_M$ of piece-wise smooth maps which are homeomorphisms of each subintervals of a partition onto a union of subintervals of the same partition. Such a τ is called Markov, in addition if these homeomorphisms are linear then we call τ a piece-wise linear Markov transformation.

It is shown, following Lasoto and Yorke (but sometimes with different proofs) that in case (i) and (ii) the τ admits an invariant absolutely continuous (w.r.t. λ) probability measure. In case (i) the theorem is rather tight since, as an example of Lasoto and Yorke shows, the theorem fails if $|\tau'|$ is < 1 even at one point. The number of absolutely continuous invariant measures is finite, and the spectrum of τ consists of finite number of eigenvalues together continuous spectrum which is of Lebesgue type. A suitable power of τ, restricted to a subset, is indeed exact. In case τ is uniquely ergodic and weakly mixing then it is a Bernouli shift, hence amenable to questions concerning central limit theorem and the law of large numbers. These are discussed as also the form and the support of invariant densities. The Markov transformations and piece-wise linear Markov transformations are amenable to matrix analysis and this is taken up in Chapter 9. The stability of invariant measure under perturbations, random or non-random, is discussed. The inverse problem, namely, of computing τ given the invariant density f is treated and the last chapter considers some applications of the existence of invariant probability measures including Kolodziej's proof of Poncelet's theorem in projective geometry.

There is a good collection of illustrative examples and also a number of exercises for each chapter. The book supplements the existing texts on ergodic theory and applied dynamics by fully discussing the dynamical properties of piece-wise smooth τ on an interval of the real line and so recommended for graduate students and more advanced scientists interested in these fields.

1. Birkhoff, G. D. and Smith, P. A., *J. Math.*, 1928, Tome VII-Fasc.IV, 345–379; *Birkhoff Collected Mathematical Papers*, 1968, vol. 2, pp. 360–394.
2. Hopf, E., *Trans. Am. Math. Soc.*, 1932, **34**, 373–393.
3. Hajian, A. B. and Kakutani, S., *Trans. Am. Math. Soc.*, 1964, **110**, 136–151.

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Diamonds in India. T. M. Babu, Geological Society of India, PB No. 1922, Gavipuram, Bangalore 560 019, India. 1998. 331 pp. Price: Rs 300.

The very mention of diamond evokes fantasies of fabulous riches. Up to the Middle Ages they were so rare and expensive that only powerful potentiates were able to afford diamonds. But in modern times even ordinary people are able to possess a few, thanks to the discovery of numerous diamond deposits on all the continents of the Earth and very large production. At one time, almost since the Vedic ages, India appears to have been the only source of this valuable gem until about the early eighteenth century when diamonds were discovered in Borneo and later elsewhere. By this time diamond mining in our country had practically ceased to exist. Although ancient Sanskrit texts mention several areas where diamonds were found, verifiable historical records are available for only a few deposits. Mining activities in southern India – which in its time was the leading producer of this gem and had yielded some of the most famous stones in history – gradually declined and had become defunct by the time diamonds were discovered elsewhere in the world. Stray diamonds are still picked up occasionally from some localities in southern India, but regular production at present comes only from a single mine and numerous shallow workings in the Panna district of Madhya Pradesh. Here also sustained mining started after Independence though desultory operations there were going on since the mid-eighteenth century.

The Geological Survey of India almost since its inception a century and a half ago kept looking for this mineral but it was quite low in its priorities. After Independence the Survey along with other government and semi-government agencies kept searching for new deposits, but it is only since the last few years that their efforts are bearing fruit in identifying new occurrences of diamondiferous rocks. Diamond mining was nationalized in 1959 and the government reserved for itself exploration and exploitation of diamond which effectively killed the possibility of private initiative in developing this mineral potential. The recent opening of the economy has dereserved diamond along with a few other minerals, and this has now attracted some leading transnational mining companies to India. But the usual procrastination by the concerned state governments in awarding licences is holding up exploration and exploitation of this core mineral. Diamond is not only an item of jewellery but its industrial applications are equally important. We imported Rs 6874 crores worth of rough diamonds in 1995–96 but sadly produced only Rs 18 crores worth during the same period. If our government had been alert after opening up the diamond sector to private enterprise we would possibly have been in a position by this time to correct this imbalance to some extent.

The last connected account of diamond deposits in India was published by V. Ball more than a century ago in 1881 and there was need to update it. The present book by Babu fills this long-felt need. Its importance also lies in the fact that it describes the several new bodies of primary host rocks of diamond in the country that have been discovered during the last decade, besides giving information about already known occurrences though most of them are now defunct but have historical value. Information about old workings is quite often valuable because they are a pointer to possible rediscovery of rich mineral occurrences. In fact, some of the present day mineral deposits in India have been brought to light – and even successfully exploited – after examination of ancient workings whose records were lost in antiquity. Several kimberlite/lamproite bodies have been discovered in Andhra Pradesh and the adjoining tracts of Karnataka, some diamond-bearing, and this book gives their first connected account. The same is true of the newly found bodies in Chhattisgarh region of Madhya Pradesh though their treatment is not so detailed. The primary bodies, particularly the Majhgawan deposit in the Panna area, receives good treatment, but the shallow workings, which make a valuable addition to the current total production, are ignored. But on the whole, the book is an important contribution on diamonds in India as it is full of facts and statistical data and makes a good reference volume.

The publication also gives a general account about diamond, its industry and occurrences in other parts of the world. In fact, this occupies about half the pages of the book. While kimberlite, the primary host rock of diamond is discussed in detail, another newly identified primary source, lamproite, gets cursory attention. Several of the large primary bodies long considered as kimberlite are now identified as lamproite, and this rock certainly deserved much greater treatment in the book. Another glaring omission is the absence of the name of Venetia in the list of leading diamond areas/mines of the world. This mine produced 3.5 million carat diamonds in 1997 and is the richest mine in the world with the fantastic incidence of 127 carats per hundred tonnes of kimberlite. The undersea deposits are also ignored. Those off the continental shelf along the Namibian coast yielded 4.8 lakh carats in 1997. There is a whole chapter on basics of diamond cutting and polishing, but the Indian industry which employs about 7.5 lakh workers and exported finished diamonds worth Rs 15,375 crores, forming 79 per cent of the total Indian exports in 1994–95, gets only passing reference. The value of the book would have been enhanced if a chapter had been devoted to the cutting and polishing industry of India, mainly concentrated in Surat and Mumbai.

On the whole the book is well-written and nicely produced though the reproductions of photographs are poor as they are dark and lack detail and contrast.

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Intraplate Deformation in the Central Indian Ocean Basin. Yu. P. Neprochnov, D. Gopala Rao, C. Subrahmanyam and K. S. R. Murthy, eds. Memoir 39, Geological Society of India, Bangalore, 1998. XIV + 250 pp. Price: Rs 550.

This volume is the outgrowth of joint Indo-Russian (former USSR) research programme under the umbrella of project B 2.3: 'The crustal structure of the Indian Ocean Floor' of Integrated Long Term Programme (ILTP). The study was carried out under the following broad headings: bathymetry; seismics; shipborne geomagnetics; shipborne gravimetry; ocean bottom heatflow study; dudging of the basement rocks and geophysical and geological investigations. Two symposia were organized: first at NGRI, Hyderabad (1988) and second at Zvenigorod near Moscow (1990) by the initiative of V. K. Gaur (Indian Earth Science Coordinator) and Academician A. L. Yanshin (Russian Earth Science Coordinator).