Do quantum-like theories imply non-classical observable effects?

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We address the issue of the connection between classical and quantum descriptions suggested by the similarities of Schrödinger equation and the Hamilton–Jacobi equation, in a specific quantum-like theory of a classical ensemble of electrons in simple static electromagnetic fields. The theoretical and experimental investigation of such a physical system was undertaken earlier by Varma and Punithavelu, who have reported results which were thought to be beyond a classical description. The question we ask is whether there are any observable effects, not predicted by the standard classical theory, and predicted exclusively by such a description based on a set of Schrödinger equations. We have now good experimental evidence that there are no such non-classical effects implied by the quantum-like description, contrary to the observations made in earlier experimental and theoretical work. We show that the observed effects which mimic quantum-like effects are due to classically predicted behaviour of electrons propagating in a magnetic field.

The Schrödinger and the Hamilton–Jacobi equations

THE close connection between the Schrödinger equation of quantum mechanics and the Hamilton–Jacobi equation of classical dynamics has suggested alternative descriptions of each theory in terms of the other. A classical-like description of quantum mechanics involves the non-local quantum potential as in the de Broglie–Bohm theory. It is possible to formulate a quantum-like description of the dynamics of an ensemble of classical particles, normally described by the Liouville equation. In this alternative description, the dynamics is governed by a set of Schrödinger equations. The role of the Planck's constant, the fundamental action, is played by some elementary invariant action in the problem. As an example, for the dynamics of an ensemble of electrons injected at small angles into an axial magnetic field B, the adiabatic invariant $\mu = \frac{1}{2} m v_{\perp}^2/\Omega$, (where $v_{\perp}$ is the transverse velocity in the magnetic field and $\Omega = eB/mc$) associated with the motion of electrons is the analogue of the Planck's action.

To address the fundamental issue clearly it is helpful to start with the standard classical-like description of quantum mechanics in terms of a Hamilton–Jacobi equation with a non-local potential. This description is widely familiar due to the interest in de Broglie–Bohm theory of quantum phenomena[1]. We start with the Schrödinger equation for a single particle.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi. \tag{1}$$

This is a linear equation and many interesting aspects of quantum mechanics come about due to the fact that superposition principle is valid for the solutions ψ of this equation. If we write for the complex wave-function,

$$\psi = R\, e^{iS/\hbar}, \tag{2}$$

where R is the real amplitude and S, the phase of the wave-function, eq. (1) becomes, after separating into real and imaginary parts,

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0, \tag{3}$$

and

$$\frac{\partial R^2}{\partial t} + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0. \tag{4}$$

This set of coupled partial differential equations is completely equivalent to the Schrödinger equation. Equation (3) is of course the Hamilton–Jacobi equation, with an additional quantum potential, which is non-local since it depends on the non-local wavefunction. These transformations form the basis of the deterministic formulations of quantum mechanics with hidden variables and the hidden variables in this theory are the initial positions which define the initial boundary condition for the evolution in terms of the Hamilton–Jacobi equation. The peculiarity of the theory is that the concept of trajectories (non-crossing in phase space) is naturally built in, though it is not known whether such additional features lead to experimentally verifiable predictions[2].

Now we turn to the inverse description, writing the Hamilton–Jacobi equation for classical mechanics in terms of an equation for a wave-function. We consider a single particle, with different initial conditions defining

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different trajectories which constitute the phase space. The Hamilton–Jacobi equation and the equation of continuity for the phase space density ρ are respectively,

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x, t) = 0, \tag{5}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\frac{\rho \nabla S}{m}\right) = 0. \tag{6}$$

The Lagrangian density is

$$\mathcal{L} = -\rho\left(\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V\right). \tag{7}$$

If we define a classical wave-function ψ through $\psi = R\, e^{iS/\hbar}$, and $R = +\rho^{1/2}$, $(\rho = \psi^* \psi)$, then we can vary the Lagrangian density in terms of the new set of variables ψ and ψ^* to arrive at the quantum-like equations for classical mechanics,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi + \frac{\hbar^2}{2m} \frac{\nabla^2 |\psi|}{|\psi|} \psi \tag{8}$$

and its complex conjugate. This equation differs from the Schrödinger equation due to the presence of the last term, which is nonlinear. Therefore, the superposition principle is not valid for the solutions of this equation.

A quantum-like theory

While dealing with an ensemble of classical particles (these could be an incoherent ensemble of electrons propagating from a source to a detector kept at a macroscopic distance), the phase space approach describes the evolution of the phase space density ρ through the Louiville equation. It is possible to make a series of transformations to the Louiville equation starting with the change of variables, $\rho = |\psi|^2$, with $\psi = Re^{iS/\hbar}$, and to arrive at a quantum-like description of the statistical mechanics of the ensemble. In the specific case of electrons in a magnetic field, this exercise gives a set of Schrödinger-like equations[3,4], each labelled by a discrete index $n = 1, 2, 3 \ldots$, with μ/n in the Schrödinger equations instead of the Planck's constant $\hbar$. (The adiabatic invariant $\mu = \frac{1}{2} m v_\perp^2/\Omega$). The adiabatic potential in the Schrödinger equation is $\mu\Omega = \frac{1}{2} m v_\perp^2$. Explicitly, the set of equations are

$$\frac{i\mu}{n} \frac{\partial \psi(n)}{\partial t} = -\left(\frac{\mu}{n}\right)^2 \frac{1}{2m} \frac{\partial^2 \psi(n)}{\partial x^2} + (\mu\Omega)\psi(n). \tag{9}$$

Since these equations were obtained by a series of transformations, one would expect that the set of equations together will describe all those phenomena described by the original Liouville equation (However, we note that the equation differs in form from the earlier Schrödinger-like equivalent of the Hamilton–Jacobi equation by the absence of the nonlinear term. We have not studied this aspect yet, but this seems to single out certain kind of densities which imply specific physical consequences). Further, it is also possible that physical effects which are not easily visible from the original equation become more identifiable in the new formalism (the converse also can happen). Suppose one goes one step further and asks the question: Are there physical phenomena not contained in the original theory which are contained in the new formalism? This can happen if the physical quantities we were dealing with in the original theory (in this case, the phase space density ρ) are not the fundamental quantities and the new formalism deals with truly fundamental quantities. If ψ is the more fundamental quantity and if for some reason in a given physical configuration only a small number of the series of Schrödinger equations are manifest (say, $n = 1$, 2, 3 and not more) then one may expect quantum-like effects at the macroscopic scale dealt by the theory, since the equation contains a large action taking up the role of the Planck's constant. This would be truly astonishing and remarkable. In the theory under consideration, several such predictions were actually found for electrons propagating in simple static electromagnetic fields. These effects were also subsequently experimentally observed, fitting the theoretical predictions reasonably well[4–6]. This behaviour was thought to be unexplainable by the standard classical theory. Since it would be really remarkable if the experimentally observed effects were genuine and not explained within the classical picture, we had undertaken a set of experiments to test the quantum-like theory we briefly described[7].

The physical system which we discuss here was theoretically studied in depth by Varma[3,4] and was experimentally probed in a series of experiments by Varma and Punithavelu[5,6]. We consider only those experiments in which the results were thought to be at variance with the classical prediction but in accordance with the predictions from the quantum-like theory. These were experiments on electrons propagating in a uniform axial magnetic field in the presence of a retarding potential near the detector. A nearly monoenergetic beam of electrons is extracted, from an electron gun, with small angular divergence at the source, and they propagate nearly axially in a uniform axial magnetic field towards a detector (Faraday cup) in front of which there is a grid on which a retarding potential can be applied. The electrons are thermal at origin, and their coherence length is of the order of their quantum wavelength, which is more than a billion times smaller than the distance between the source and the detector. We do not expect any observable quantum effects in such a system. But the quantum-like theory mentioned above does predict macroscopic quantum-like effects, which were thought to be completely outside the scope of the standard classical theory. One of the main predictions was that there were discrete allowed and forbidden energy states in this configuration. Here, the term 'allowed' refers to whether the electrons can reach the detector from the source, for a given initial energy and a specific value of the retardation potential. The theory predicted that there was a set of discrete values of the retarding potential at which the current at the detector shows maxima and minima, unlike in the case of the standard classical prediction of a smoothed out step function.

The equation derived by Varma[4] from his quantum-like theory[3], for the transmission peaks in energy is

$$E_j = \frac{1}{2}\, m \left(\frac{3\Omega L}{2\pi}\right)^2 / (j + 1/4 - \phi/2\pi)^2, \qquad (10)$$

where $\Omega = eB/mc$, L is the separation between the source and the detector, and ϕ is an undetermined phase. (The formula is derived assuming one reflection at the detector plane and another at the source plane, bringing in $3L$ instead of L.) This relation describes a set of allowed discrete energy states due to quantum-like effects. Note that the energy spectrum is hydrogen-like, with inverse-square dependence on a discrete index.

Summary of results from earlier experiments

While the theory mentioned above was developed earlier to describe tunneling-like phenomena from magnetic traps for charged particles[3,8], its prediction of discrete allowed and forbidden energies was experimentally tested later at the Physical Research Laboratory (PRL). The results from these experiments fitted the theoretical predictions well. The predicted discrete allowed and forbidden energies for the electrons to reach from the source to the detector were seen, and various checks were made to make sure that these were not simple geometrical obstructions at the wire mesh grids in front of the detector. The details of these results were reported in several publications[3–6]. (See ref. 9 for some relevant comments on the theory and on the response of the scientific community.)

The experimental set-up consisted of an electron source at one end of a vacuum chamber and a current detector at the other end, with a separation of about 20–40 cm between them. (The set-up is similar, except for details and number of the different electrodes, to the one described later in this paper – see Figure 1.) The current detector could be biased at a retarding voltage ranging from zero to a voltage exceeding the average electron energy. Normally the electron energy and the separation between the source and the detector were kept fixed, though the set-up allowed sweeping the energy. Also the separation between the source and the detector could be varied. An axial, uniform magnetic field of the order of 100–200 G was applied using a set of coils and the current at the Faraday detector was measured as a function of the retardation voltage. In such a simple experiment, the classically expected current is nearly constant till the retardation voltage reaches the electron energy. Then the current rapidly falls to zero monotonically. In the experiments, strong modulations in the current were observed throughout the range of the retarding voltage, reminiscent of resonance effects in the transmission of quantum particles in a potential well.

The most important results from earlier experiments at the PRL are summarized below:

1. The current measured by the detector was not monotonous, contrary to classical expectation. It was strongly modulated in a manner described by the quantum-like equation mentioned earlier, signifying a set of allowed and forbidden energies for the electron beam to reach the detector from the source[5,6]. The peaks of these modulations corresponded to retardation voltages obeying the $1/j^2$ dependence, suggesting a set of discrete allowed energies.
2. The average number of modulations was inversely related to the separation between the source and the detector (or the width of the potential well), suggestive of a Fourier-like relation between the well width and the spatial wavelength.
3. There were observations suggesting that the retarding voltage applied at the detector affected the emission of the electrons from the source in a non-local way.

We have been studying these developments due to our primary interest in the de Broglie–Bohm formulation of quantum mechanics. Since the earlier results were fascinating and truly anomalous with no definite explanation offered to date within the classical picture, we

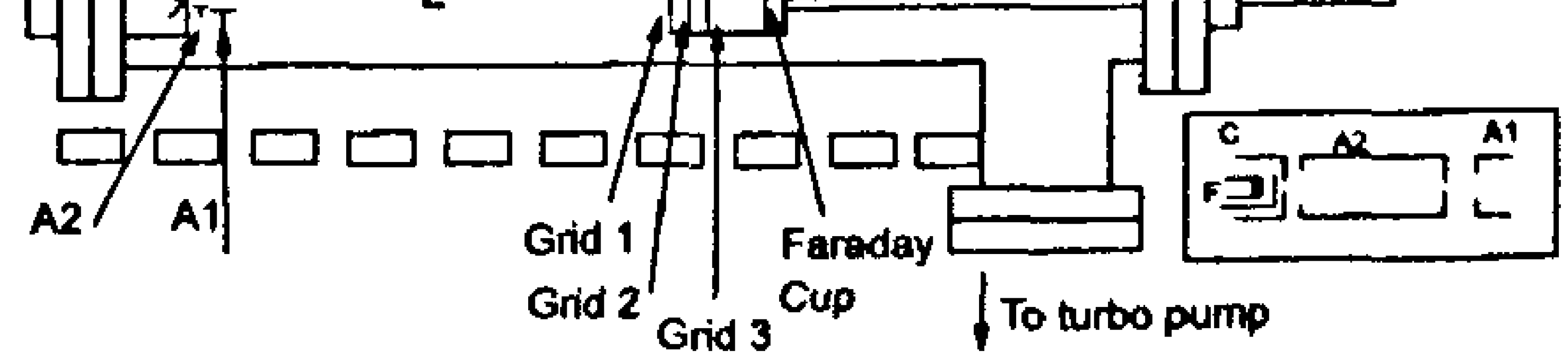

Figure 1. Schematic diagram of the experimental set-up. The detector is movable through an O-ring seal. The inset shows the structure of the electron gun. The exit hole of Anode: A2 is much smaller than the hole in A1. The filament F, the coated emitting electrode and the cathode C are all at the same negative potential which decides the energy of the beam.

decided to set-up a straightforward, but careful experiment to look for these reported effects. Our results confirm the earlier experimental results and they are also as astonishing as the earlier results, but we have, during the course of the experiments, found a simple classical explanation for the observed effects. We have pinned down the cause of these anomalous oscillations in currents and one does not need any quantum-like description to explain the observations. We have been able to derive the equation for the discrete energy states in the problem from first principles of classical dynamics of electrons in electromagnetic fields, demystifying the remarkable agreement of the data with the predictions from quantum-like theory. In fact, we will argue that the particular prediction from the quantum-like theory is just a more complicated way of deriving a fundamental property of electron beams in a uniform magnetic field.

Since our realization that the results can be explained within a classical picture emerged after observing several systematic effects in a series of experiments, we will state the development approximately chronologically, describing the experiments and results first and then the derivation of the quantum-like formula from classical considerations.

Our experiments

Experimental set-up

Our experiments were designed to make the configuration as ideal and close to the theoretical description as possible. The theory deals with a nearly monoenergetic beam of electrons with small angular divergence propagating in a uniform magnetic field in electric potential well. The formula for the discrete allowed energies is derived for a nearly rectangular potential well. This requires that the propagation region is nearly free of electric fields. But, in the earlier experiments this condition was not met and we have this feature of nearly rectangular well built into the experimental design. In most of the experiments discussed in this paper, the source of the electrons is an electron gun modified from that in a computer monitor. It is indirectly heated and has two anodes. (A third anode was removed, to avoid small multiple apertures blocking a large fraction of the beam. There are no focusing voltages applied to these anodes, since near axial propagation is ensured by the magnetic field.) Most of the electrons are collected by the second anode (A2) from the cathode region when there is no potential applied to the anodes, and a small fraction passes through the first anode (A1) into the field-free region in the tube, towards the detector. This gun has no exposed insulating parts which can get charged.

In the experiments the gun was operated well above threshold, but at low currents of a few nanoamperes to avoid any space charge effects. The beam is monoenergetic at the negative bias, applied to the filament and cathode, to about 2%. The gun and the detector were mounted in a SS vacuum chamber maintained at a vacuum of better than 2.5×10^{-7} T. The axial magnetic field, which was uniform to better than 1% was generated by a set of coils which were individually adjustable in position and polarity. The detector consisted of three high transmission gold mesh grids, followed by a copper Faraday cup coated with graphite (some special care was taken in this design to reduce the secondary electron emission from the collector since we already had a guess about the role of secondary electrons in this problem, from our own preliminary experiments). The optical transmission in each of the grids is about 85%. The ratio of the inter-wire separation of the grid to the separation between the grids is about 30, ensuring good uniformity of the electric fields. The anodes of the electron gun and the first grid of the detector were normally grounded through an ammeter, and this way we were able to get nearly field-free region between the first anode and the first grid, unlike in earlier experiments. The retarding fields appear only between the first and second grids. This is important for performing a clean experiment to test the theory, avoiding complications which could arise from extraneous factors like imperfections in the field structure, non-axial propagation, etc. The third grid which was grounded avoided curved field lines from the retarder to the Faraday cup. The detector assembly as a whole could be moved inside the vacuum chamber through an O-ring seal. All electrodes at which currents were measured were grounded through a multichannel ammeter (Keithley 6571 with low current scanner card interfaced to an IBM-PC), and currents and variations down to a few picoampere could be reliably measured. The noise in the current scanner was less than 0.01% of the average current in the detector, and the total random noise in the measured current itself was less than 1%, attributed mainly to variations in emission from the gun. The extraction voltage as well as the retarder voltage could be independently varied from zero to about 1500 V, though typical experiments were done below 1000 V. The magnetic field was between 100 and 180 G.

The experiment consisted of measuring the electron currents at the Faraday cup, grids and anodes, while the retardation voltage was scanned from zero to a maximum value exceeding the electron energy, with the magnetic field and the electron energy kept fixed at particular values. The separation between the gun and the detector was also kept fixed for a series of runs. An A/D card was programmed to generate discrete voltages between 0 and 5 V to control the analogue input control of a high voltage supply which supplies the retardation voltage. The voltage was held constant while a scan of appropriate number of channels connected to the current meter was completed and the data transferred through the serial port to the computer. Then the high voltage was increased to the next value (in steps of about 4 V) and the channel scan was repeated, after a small delay for the high voltage and currents at the electrodes to stabilize. Typically the data consisted of the scan number (proportional to the high voltage) and currents at grid 1, grid 3, Faraday cup, A1 and A2. We have looked for evidence in our data for any of the anomalous behaviour listed earlier by varying all the parameters at hand.

Patterns in the data

The most important set of data in these experiments pertains to currents measured at several electrodes with only the retarding voltage at the second grid varying from zero to beyond the electron energy, with the beam energy, magnetic field (B or Ω) and the separation (L) kept fixed. Different runs with different fixed values for these parameters were also done. Here we present only the representative data, to bring out the main features.

The curves in Figure 2 show currents at the various electrodes. The first observation is that of the systematic modulations of the electron currents at some of the electrodes. In this case, the front grid and the front anode show these modulations, and the Faraday cup and the grid immediately in front of it do not show any modulations. The pattern of the modulations is in general very similar to the pattern observed in earlier experiments

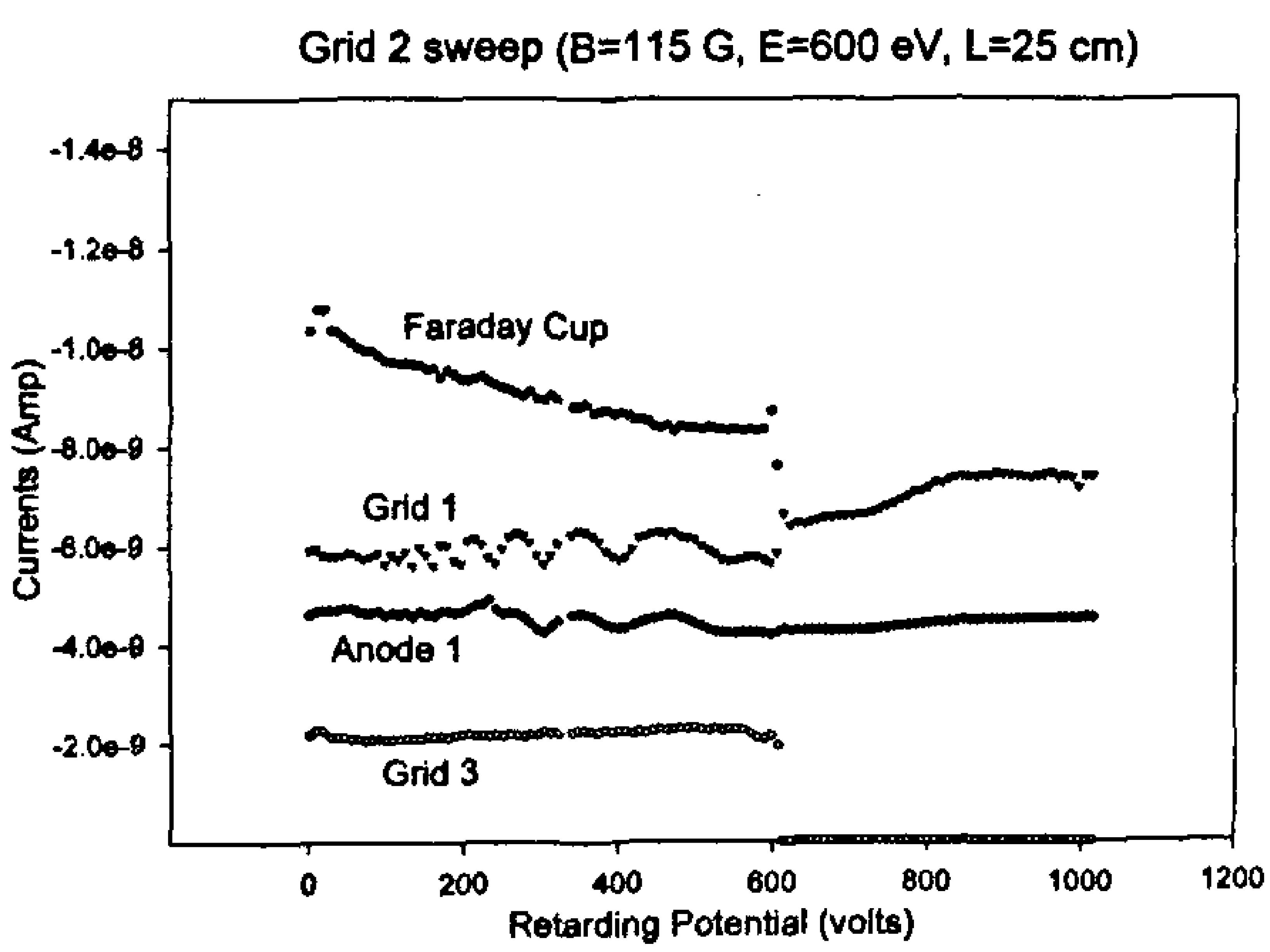

Figure 2. Currents measured at various electrodes as the retarding potential applied to the grid 2 is varied. The oscillatory behaviour of the current at the front grid and front anode (shifted along Y axis for clarity) is very clear and the peaks fit the quantum-like equation well. There is no evidence for appreciable modulations in the current measured at the Faraday cup or at grid 3.

at PRL. This is a regular and stably *reproducible feature* of all the data we have. Figure 3 is a comparison of the currents for two different magnetic fields, and Figure 4 compares the currents for two different separations, at the same magnetic field. There are also data for different electron beam energies, for retarding potentials applied to grid 1, grid 3, Faraday cup, etc. selectively, for wider range of magnetic fields, and at wider range of retarding potentials (the total number of runs are about 50 exploring various combinations). The basic features are the same as what is represented in the plots presented here. Some of the experiments were done after we pinned down the cause of the current modulations, to check the predictions of a model we arrived at, as will be discussed later in the paper.

The peaks or valleys of the modulations in all these data fit the quantum-like equation reasonably well, with the factor $\Omega L/2\pi$ instead of $3\Omega L/2\pi$ in eq. (1). The dependence on the separation and the magnetic field also is similar to what was observed in the PRL experiments. This shows that the patterns observed in the PRL experiments were due to genuine physical effects. The question we then ask is whether these features are due to some new physical phenomenon beyond the standard classical description or due to some effect within the classical paradigm. To answer this question we depend fully on the data itself and look for further features and systematics. There are no modulations in the currents in the Faraday cup or the grid just in front of it. Note that each grid intercepts only around 15% of the current passing across it, and if the modulations were in the primary electron current we should have seen comparatively large modulations in the current at the Faraday cup, which collects about 50% of the total current. So, one definite conclusion is that the modulations are seen only at electrodes where electrons of energy lower than the retarding potential can reach, namely the anodes and the grids in front of the grid on which the potential is applied. This has been verified in a number of runs and two examples are given later in this paper. Since the primary electron energy is larger than the retarding potential in the parameter region of interest in the experiment, the electrons which are responsible for the modulations are necessarily secondary electrons of lower energy. There have been other diagnostic experiments to verify this fact independently, as we will mention in a later section. So, while there is no evidence for any discrete energy states in the primary current, there is very good evidence for modulations on the secondary current and systematic modulations are seen in the currents at the front grid and the front anode, where secondary electrons can reach. The electrodes at which there are no modulations are more or less inaccessible for the secondary electrons since there is a retarding potential on the middle grid. The modulations seen in the currents on the first grid and first anode matches the equation derived from the quantum-like theory without the factor 3 in the product $3\Omega L$. If the peaks in the current are labelled by integers j, they fit the equation

$$E_j = \frac{1}{2}\, m \left(\frac{\Omega L}{2\pi}\right)^2 / j^2 . \qquad (11)$$

(Actually the fit is very good only when an additional constant parameter between 0 and 1, specific to each run, is added to the 'quantum number' j. So, instead of j, we need to use $(j+\delta)$. Once a value for δ is chosen, the peaks can be fitted within a few per cent by integer values of j. This is true for the earlier data from PRL also.)

We have done a set of experiments to look for the apparently 'non-local' effects described by Varma and Punithavelu[6]. Even before doing these experiments we had noticed that linking their observations to any non-local effect is misleading since the potential gradient in their experiment is not localized in a region. It extends from near the detector all the way to electron source and the electrons can feel the field right near the source point. In any case, our experiment saw no evidence for the large variations in the anode current observed in their experiment. Figure 5 shows the relevant plots of the currents, for retarding voltage far exceeding the

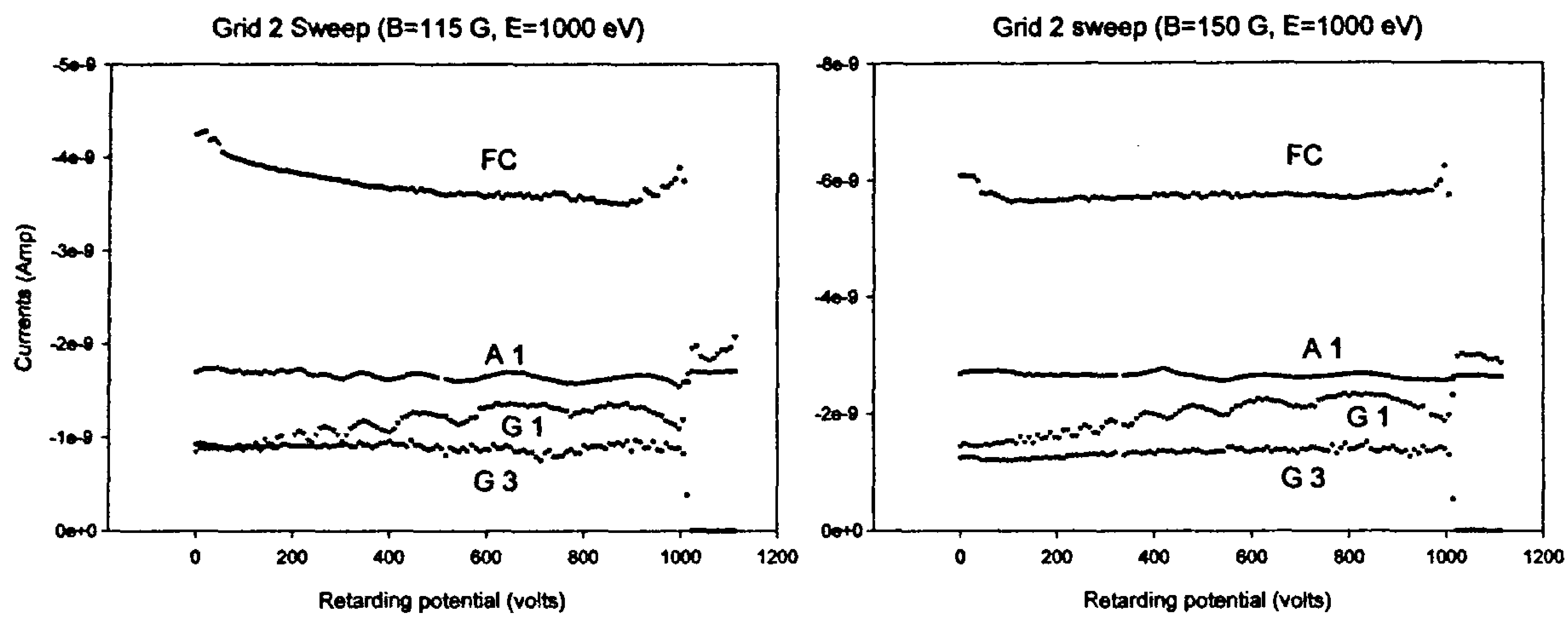

Figure 3. Plots of currents at two different magnetic fields. While the total number of modulations increase with magnetic field, the sharpness of the dips seems to decrease in higher field. FC: Faraday cup, G1: grid 1, G3: grid 3. The increase in the average number of peaks in higher field is in accordance with the quantum-like equation.

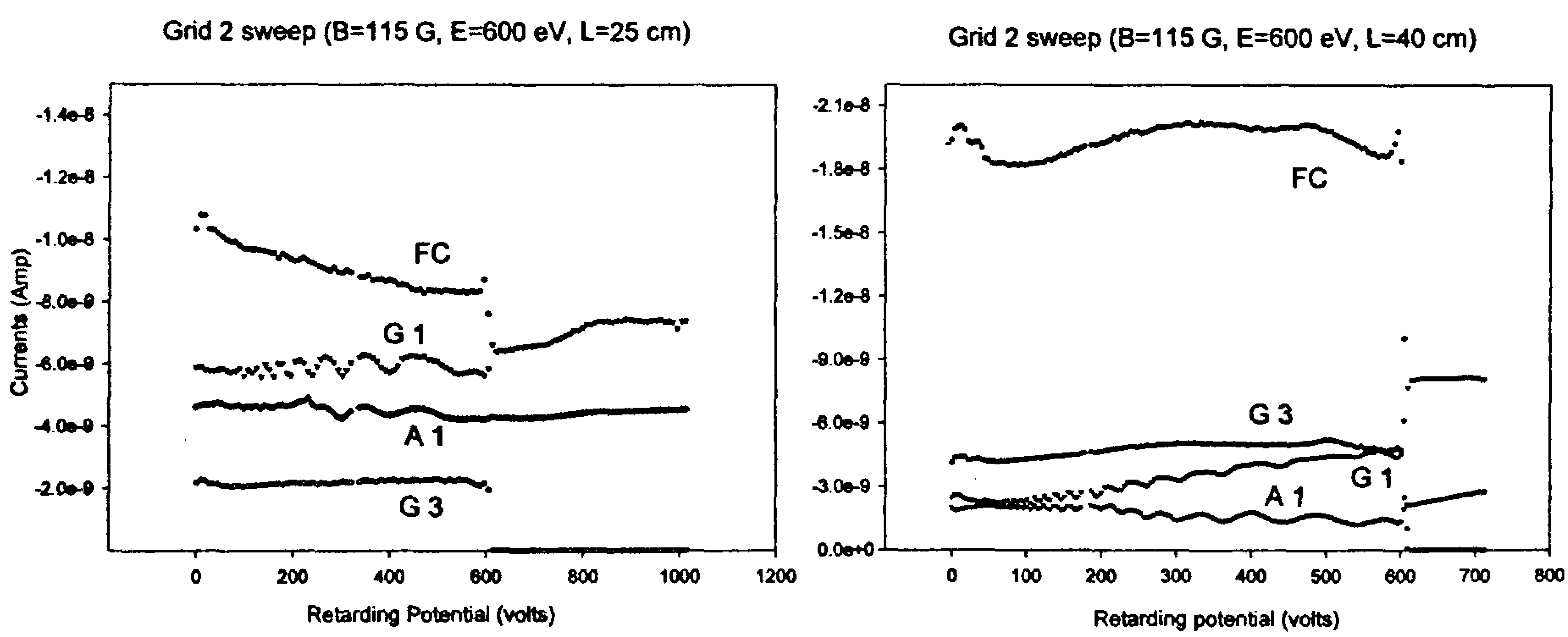

Figure 4. Currents measured for two different separations between the electron source and the detector. Note the inverse dependence of the average inter-peak distance on the separation. There are more number of modulations for larger separation. This is reminiscent of a 'Fourier pair'.

primary electron energy. As evident from the plots, we do not see any modulations when the retarding voltage is larger than the electron energy, signifying that there are no nonlocal effects and that there are no tunneling-like effects over the electrostatic barrier. In fact, the variations in currents are within a few per cent, except for the uniform drift-like changes, over the entire range from 400 V to 1200 V, in contrast to the several, nearly 100% modulations reported in ref. 4. If the earlier observations were genuinely fundamental we would have seen several dips of the current to zero for retarding voltages above the electron energy. (The classical model we present later for explaining the modulations is able to suggest a reason for these observations, but we do not discuss these details here).

The conclusion presented in the previous paragraph by itself does not preclude a quantum-like description, especially if the observed modulations do not have a description within classical physics. Even if only a small fraction of the electrons behave in a way described by the quantum-like equation, the modulations could be seen as new physics unless there is a classical reasoning for deriving eq. (2). In the next section, we show that indeed there is such a classical description of the quantum-like equation (eq. (2)). There is enough reason to suspect a classical origin for eq. (2), when we note that the similarity to the hydrogen-like spectrum is not good enough due to the absence of the parameter μ, which took the role of the Planck's constant in the quantum-like theory. We have been able to derive this equation simply, from first principles of electron propagation in electric and magnetic fields.

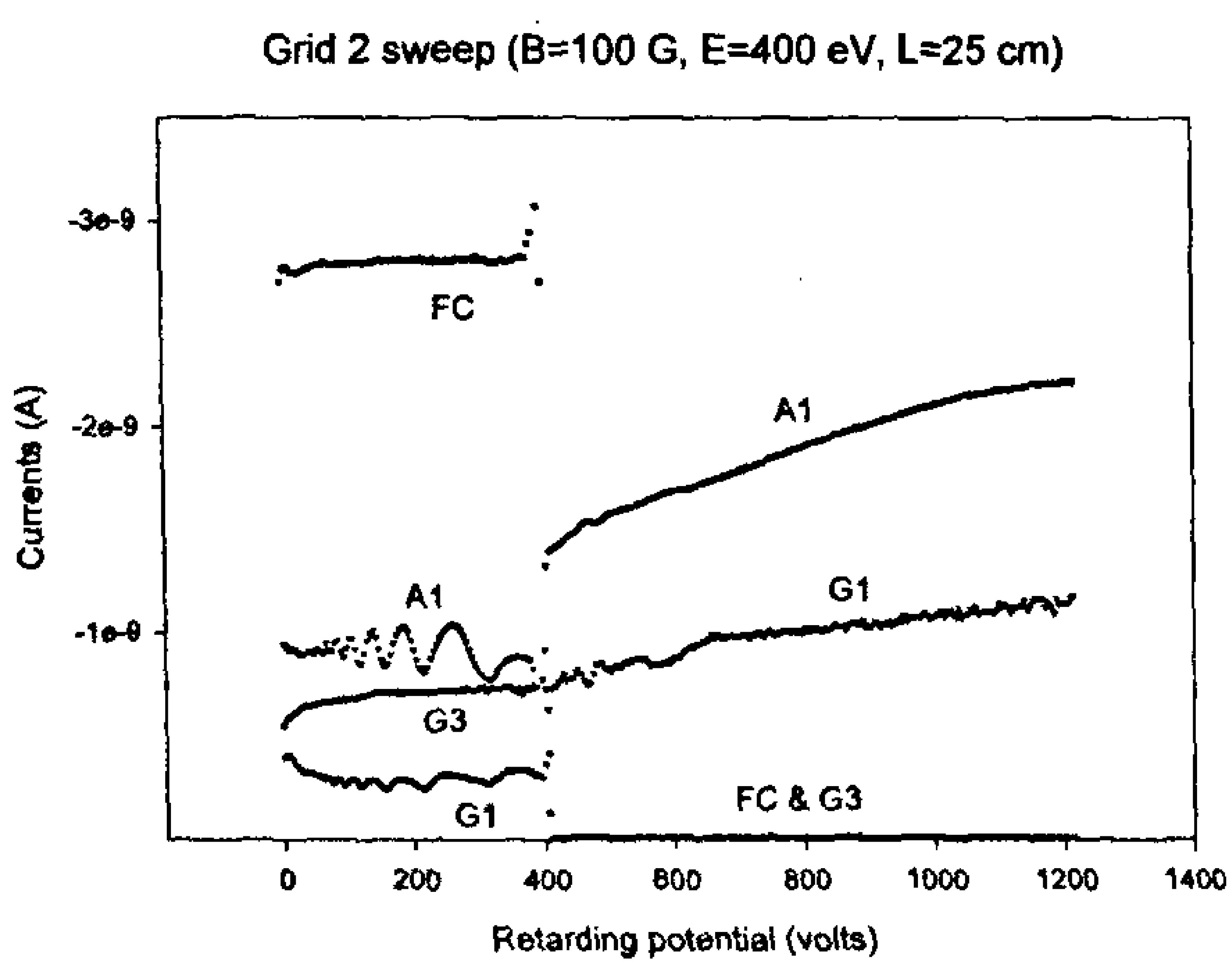

Figure 5. Currents with voltage sweeps on grid 2 far exceeding the electron energy. There is no evidence for any sharp dips or oscillations in the anode current for voltages above the beam energy (400 V in this case).

Discussion

Classical derivation of the quantum-like equation for allowed 'states'

The equation derived[4] by Varma from his quantum-like theory[3], for the transmission peaks in energy is

$$E_j = \frac{1}{2}\, m \left(\frac{3\Omega L}{2\pi}\right)^2 / (j + 1/4 - \phi/2\pi)^2, \tag{12}$$

where $\Omega = eB/mc$, L is the separation between the source and the detector, and ϕ is an undetermined phase. (The formula is derived assuming one reflection at the detector plane and another at the source plane, bringing in $3L$ instead of L. This is equivalent to three passages in the space between the source and the detector and Varma and Punithavelu needed this factor to fit their data. However, in their system since the region between the source and the detector was not field-free, the electrons gain energy over an extended region and there is no fundamental significance to this factor of 3; it is introduced empirically.) This relation is supposed to describe a set of allowed discrete energy states in the configuration, due to quantum-like effects. The interpretation of this formula was that electrons can access the electrode at a distance L from the source only for a discrete set of energies given by E_j. Also, the separation between these energy values varies as $1/j^2$, for a fixed separation and magnetic field.

The first feature we notice is that the equation resembles the equation for classical kinetic energy, without the adiabatic invariant appearing anywhere, and can be written as $E_j = \frac{1}{2} m \cdot (v_j^2)$, where v_j represents a discrete set of velocities. Our aim is to see whether the physical problem at hand suggests the existence of such a set.

For an ensemble of electrons with a specific energy E (with small width in the distribution) originating at the gun with a small spread in their injection angles into the axial magnetic field, focusing would occur at distances $L_{f,j} \simeq 2\pi j v/\Omega$. The axial velocity $v \simeq \sqrt{2E/m}$, since the transverse velocity is much smaller than the axial velocity. For electrons in the energy range of 100 eV, $L_{f,1}$ is of the order of 1 cm, in a magnetic field of around 200 G.

If L and B are fixed at particular values in the experiment, $jv = L\Omega/2\pi$, which is a constant. Substituting for v,

$$v = \sqrt{2E/m} = L\Omega/2\pi j \equiv v_j. \tag{13}$$

Note that this equation describes a *set of values for the velocities (or energies)* at which focusing would occur at the detector plane. Thus we have found the set of v_j we were looking for. We get

$$E_j = \frac{1}{2}m\left(\frac{L\Omega}{2\pi}\right)^2\frac{1}{j^2}, \qquad (14)$$

which is the same as eq. (1), if we ignore the irrelevant phase factor in eq. (1). The 'hydrogen-like' energy spectrum[4] arises due to the *pseudo wavelength* (focal length, or more appropriately, focus-to-focus length) involved in the problem, which is linear in velocity, and due to the quadratic dependence of the energy on velocity. According to our interpretation, there are no quantum-like effects, but only secondary effects generated by the dependence of the focusing on the energy, magnetic field and the separation between the source and the detector. All other dependences seen in the earlier experiments by Varma and Punithavelu[5,6] are qualitatively explained within our interpretation. Clearly the correct model for interpreting the results quantitatively will have to convert the spatial modulation of the electron beam due to multiple focusing into the actual currents reaching the electrodes. This may be done, for example, by invoking some amount of geometrical blocking, the amount of which depends whether a focal point is near an aperture or not. A detailed quantitative modelling will have to take into account the exact geometry of the source and the detector, peculiarities in secondary electron emission, etc. In general, simple geometrical reasoning seems sufficient to explain the major features reported in earlier experiments as well as in our experiments. In any case, the classical derivation of the quantum-like formula weakens the claim that the quantum-like theory had predictions beyond the classical paradigm and takes away any mystery associated with the discrete nature of the energies at which peaks in current were observed.

Some additional experiments and further insights

We have also done experiments with the detector and all the grids grounded through the electrometer, and by ramping the primary electron energy from zero to a maximum value (typically 600–1000 V) at a fixed magnetic field. Now both the primary and secondary electrons can reach all the electrodes at the detector assembly. Again characteristic oscillations are seen in all electrodes. In another set of experiments we have connected various low voltage biases to the electrodes where currents are measured, and we observed that the modulations in the current can be enhanced or depressed by changing these bias voltages, typically less than 30 V. This is a clear evidence for the fact that it is the low energy electrons which are responsible for the modulations and the primary electron current is behaving as expected classically, without showing any anomalous behaviour. (Such an experiment has been recently done by Punithavelu, in which no retarding potential was applied, and currents were measured at the detector electrodes on which small bias voltages were applied. When the primary electron energy is varied, characteristic oscillations were seen which disappeared progressively with an increase in the bias current, vanishing completely for a bias of about 10 V, clearly indicating that it is the secondary electrons which are responsible for the modulations.)

A definite model for the modulations

Now we discuss a possible model which explains the main features in both our experiments and in the earlier experiments done in a different configuration. It may be noted that this model does not attempt to take care of all observed features since these features do depend on the exact experimental configuration, design of the electron gun, electrodes, etc. There could also be additional factors which may enhance or suppress these oscillations in individual experiments. For example, the amplitude of the modulations can depend on the secondary electron emission properties of electrodes, heating of the electrodes due to focusing, etc.

When the primary electrons come through the grids at the detector assembly, they will generate secondary electrons at the grids and at the Faraday cup. The retarding potential is applied to the second grid and for all potentials below the electron energy, primary electrons will cross the second and third grids and will reach the detector. The secondary electrons generated at the first and second grids get reflected into the space between the detector and the gun, guided by the magnetic field. (There may be a small fraction getting towards the last grid and the Faraday cup.) Secondary electrons from the third grid and the Faraday cup have no access beyond the second grid due to the retarding potential, and get pushed back. The secondary electrons generated at the second grid gain energy from the second grid, equal to the potential applied to the second grid. As they travel to the anode of the gun, they will undergo several focusing and defocusing, creating a pseudo wave. At the anodes, these electrons can pass into the gun, but the fraction of electrons which get in and get reflected and the fraction which gets absorbed will depend on whether the beam is focused at the aperture of the gun. Therefore the fraction of electrons which get reflected back again by the cathode voltage into the space between the gun and the detector depends sensitively on where the focusing occurs near the gun. Since the focusing length depends on the magnetic field, retarding potential (same as the secondary electron energy after reflection at the second grid) etc., it is clear how we could get oscillations in the current obeying the formula we derived, which is the same as the equation derived in the quantum-like theory we tested. So, the modulations could be explained purely from geometrical considerations. A serious misalignment can actually reduce the oscillations since the secondary electrons then get absorbed at the first obstruction.

Another important fact to be noted is that some fraction of the secondary electrons from the second grid, after passage to the source and a reflection at the anodes, can in principle get through all the grids and reach the detector, since they have just sufficient energy to cross the second grid on which the retarding potential is applied. Whether this will actually happen may depend on the exact geometry, grid properties, etc. In our case we do not see any strong modulations in the current at the Faraday cup, though some of the plots do indicate very small modulations. In the earlier experiments at PRL, strong modulations were observed in the current at the Faraday cup also. There are considerable differences in the two experiments with regard to the grid properties, grid spacing and electric field structure.

This model has several predictions, most of which have been experimentally verified. One prediction which already was indicated in the data presented earlier is that the phase of the modulations must be the same for currents at grid 1 and at A1, since the modulations are due to the aperture at A2, which then act as a common cause for modulations at the other electrodes. This implies that the modulations of the currents at A2 (which we have not shown in the plots due to the much larger absolute value of the currents at A2) will be out of phase with that in A1 and grid 1. This also has been verified in all the runs. Another prediction, based on the picture of modulations of the secondary electrons is that if the retarding potential is applied to the grid 3, instead of grid 2, we should start seeing modulations at both grid 1 and grid 2, in phase with modulations at A1. The plot in Figure 6 verifies this expected behaviour. Ideally, if the retarding potential is applied to the Faraday cup directly, we should see modulations at all the grids and A1 in phase. Since the Faraday cup was designed to minimize secondary emission, by coating with graphite, we do not see much modulations in the currents when the retarding potential is applied to the Faraday cup. Instead, if we apply the retarding potential to grid 1, then we see modulations only at A1, and a comparison of the cases when the voltage is swept at grid 3 and grid 1 shows that the oscillation present at grid 2 in the former case is absent in the latter, supporting the model we have presented. On the whole there is a very clear evidence that the modulations are not present in the primary electron current.

In summary, we list the main results from our experiments and then we briefly restate the conclusions.

1. We confirm the essential features of modulations in currents seen by Varma and Punithavelu in earlier experiments, but many details of our data are considerably different from their observations.
2. There is no appreciable oscillation in electron current seen at the detector electrode or at any other electrode *inaccessible to secondary electrons.*
3. Clear oscillatory behaviour is seen in the electron current observed at electrodes which are accessible to secondary electrons, and the retardation energy at which there are peaks and dips in these oscillations fit the equation

$$E_j = \frac{1}{2} m \left(\frac{L\Omega}{2\pi}\right)^2 \frac{1}{j^2},$$

where j is an index labelling the peaks.

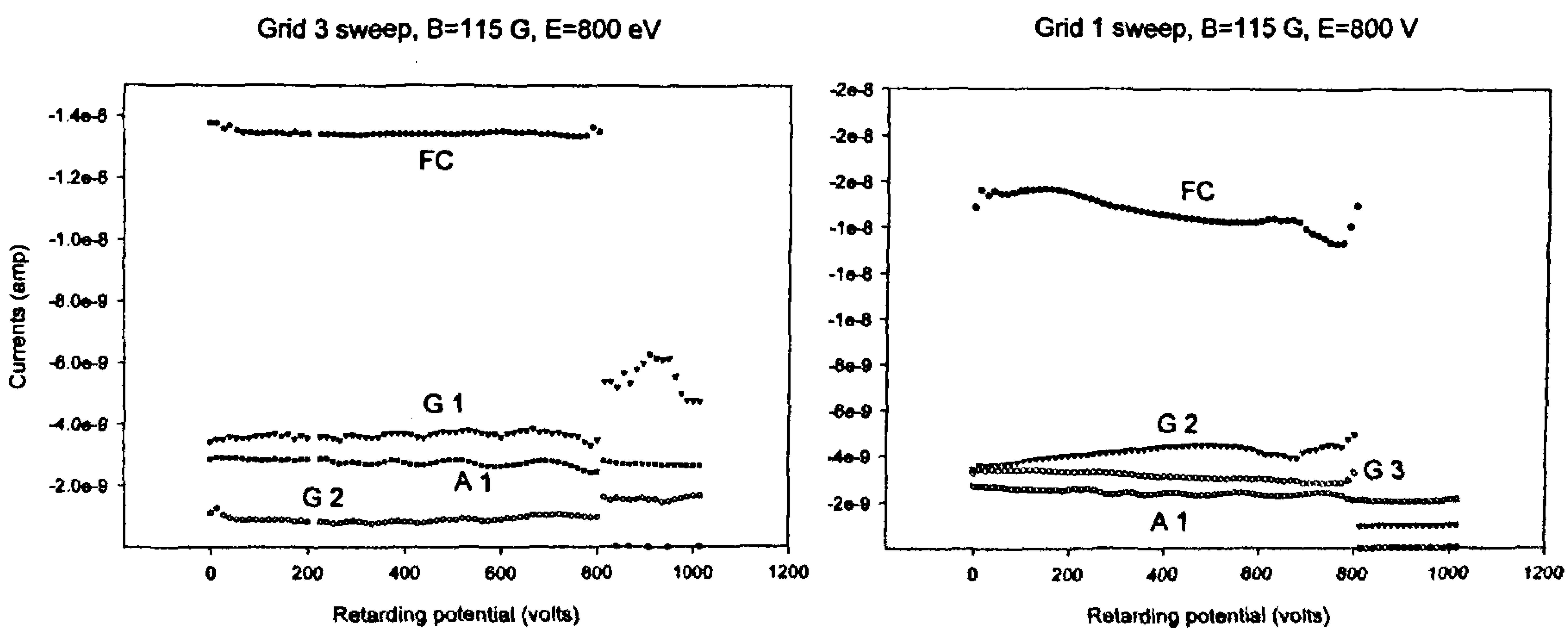

Figure 6. A comparison of data when the retarding potential is applied at different electrodes (grid 3 and grid 1 respectively). The modulations in the currents are in phase and are visible on electrodes which are not screened by the retarding potential.

4. Insight from the experiments leads to a classical derivation of the formula

$$E_j = \frac{1}{2} m \left(\frac{L\Omega}{2\pi} \right)^2 \frac{1}{j^2},$$

from the consideration of focusing of divergent electron beams in a uniform magnetic field. This demystifies all the main oscillatory features seen in our experiments, and in earlier experiments which were ascribed to quantum-like phenomena.

5. We do not see any apparently non-local behaviour in currents at any electrode when the retardation potential is larger than the primary beam energy. This is in sharp contrast to results from earlier experiments. So, the effects ascribed to Aharanov–Bohm kind of phenomena in earlier experiments do not point to any fundamental new effect.

6. Several experiments with bias voltages, small compared to the primary electron beam energy applied at various electrodes, showed that the electrons which are responsible for the oscillatory signals are of much lower energy than the primary electrons.

7. It was possible to model the current oscillations as resulting from geometrical obstructions at small electrode apertures at the electron source, and the currents depend sensitively on the amount of focusing the beam undergoes at these apertures. This model was inspired by our classical derivation of the equation mentioned above.

Conclusion

We have completed a set of experiments which show that the apparently anomalous oscillatory behaviour in electron currents seen in some earlier experiments is due to modulations of secondary electron currents, which could be modelled within a classical picture. Though the earlier experiments had impressive data which pointed to quantum-like phenomena like interference, beats etc., and which strongly suggested potentially new phenomena, insights from the experiments presented here seem to be sufficient to rule out the necessity for any description beyond classical physics. The basic source of quantum-like phenomena is the pseudo-wave like behaviour of classical electron beams in classical electromagnetic fields arising from multiple focusing along the length of propagation. No quantum-like explanation is necessary to account for the experimental results. In effect, the quantum-like theory under discussion seems to be another formalism which reproduces the classical results. Of course, there may be situations in which the quantum-like theory may have predictions which are not very transparent in the classical formalism, but we have no reason so far to think that there would be a case when the quantum-like theory would give a prediction which is not contained in the classical theory.

Apart from clarifying certain long-standing unexplained features which apparently supported predictions of a quantum-like theory, the experiments discussed here have indicated that quantum-like formulations of classical physics in general describe phenomena within the classical regime, without implications outside its range of validity, in spite of the introduction of a complex wavefunction. The question may be asked whether it is true for classical-like formulations of quantum mechanics that the description deals with only those features which are strictly within the corresponding quantum formalism, or whether there are any experimentally accessible new features outside the scope of the standard theory. This in some way is equivalent to the question whether quantum mechanics is complete as it is. Our present interest is in probing this aspect in the context of the de Broglie–Bohm theory[2] and there are some indications that the feature of non-crossing trajectories present in the theory, which is absent in the quantum formulations, might provide a way of experimentally addressing the issue.

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