

Stochastic differential equations and applications

A Discussion Meeting on Stochastic Differential Equations and Applications, sponsored by the Indian Academy of Sciences, was held at Orange County, Coorg, during 2–7 December 1998. Seventeen probabilists from different parts of the country participated in this meeting. The core of the meeting was several short series of intensive lectures on topics of great current interest in stochastic processes.

Shrikant Iyer (Indian Institute of Technology, Kanpur) gave a series of lectures on *Superprocesses*, an area of intense activity in recent years. He introduced superprocesses as scaled limits of branching Brownian motions, also mentioning in passing the 'martingale' formulation and the connections with a class of nonlinear p.d.e.s. He went on to describe Dynkin's scheme for calculating associated moments, leading to existence of local times and self-intersection local times in certain dimensions. Finally, he indicated how interaction among the branching particles can be introduced by means of a change of the underlying probability measure and mentioned several current issues, including his own ongoing work.

R. L. Karandikar (Indian Statistical Institute, New Delhi) gave a series of lectures on *Probabilistic methods in mathematical finance*. By means of a simple discrete caricature, he illustrated how the simple requirement that there be no arbitrage opportunities translates into the mathematical condition that the discounted value be a martingale process under an equivalent probability measure. He introduced call and put options, and the European and American variations thereof (respectively, payable at the terminal time of the contract or any time in between). The condition that the market be complete then translates into a unique choice of the equivalent measure, leading to the

celebrated Black–Scholes formula. He pointed out the need to remove the implicit oversimplifications involved in the modelling, on one hand unrealistic assumptions such as a frictionless market, on the other, the use of Gaussian distributions instead of appropriate heavy-tailed distributions that model better the prevalence of large shocks. The mathematical issues these lead to and the current status thereof were briefly outlined.

S. Ramasubramanian (Indian Statistical Institute, Bangalore) gave a couple of lectures on the *Skorohod problem*. Starting with the problem as a purely deterministic one, he pointed out its connections with reflected Brownian motion and in turn, its connections with Leontief systems in economics. He then introduced a more general formulation allowing for general drift and diffusion and discussed the mathematical issues concerning its well-posedness.

B. Rajeev (Indian Statistical Institute, Bangalore) gave a series of lectures on *Ito formula for distributions*. He began with the inter-relationship between Tanak's formula and the classical Ito's formula, followed by a brief survey of the stochastic calculus for Hilbert space-valued semimartingale processes as developed by Metivier and Pelloumail. Casting the Schwartz space and the space of tempered distributions as intersection (resp., union) of a countable family of nested Hilbert spaces and developing some key estimates in this set-up, he derived the Ito formula for distributions, a mathematical tool of great potential utility.

Rahul Roy (Indian Statistical Institute, New Delhi) gave a series of talks on *Percolation*. Beginning with the two dimensional case, he developed the basic tools like FKG and Berg–Kesten inequalities, exponential decay estimates, etc. and proved existence and

uniqueness of infinite clusters for appropriate parameter ranges, also characterizing the 'critical' probability in the process. He then outlined more advanced topics like oriented percolation and continuum percolation, mentioning in particular a counterexample to a well-known conjecture by Anish Sarkar (who also happened to be one of the participants). He concluded by listing several open problems.

In addition to these, there was a tutorial talk on the theory of distributions by M. K. Ghosh (Indian Institute of Science, Bangalore) and a guest lecture by V. Mandrekar (Michigan State Univ., East Lansing) on Kolmogorov strong law of large numbers and series convergence under permutations.

The audience participation throughout was extremely enthusiastic. 'Spirited' discussions went on late into the evenings by the poolside, sometimes to the accompaniment of a bonfire. At times the excitement would reach such pitch that an odd honeymooning couple or two co-inhabiting the resort would cast curious glances at the discussants and wonder what a Hilbert space might be.

At the conclusion of the meeting, it was unanimously felt that this format of short intensive lecture series accompanied by intensive discussions was far better than that of a conventional conference and the audience enriched their knowledge base immensely in the process. It was agreed that the kindred spirits scattered across the country that had managed to get together in course of this meeting should strive to meet thus periodically and keep the interaction alive.

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