RESEARCH COMMUNICATIONS

Thermodynamic model of global warming

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A thermodynamic model of global warming (greenhouse effect) has been developed to calculate the rise in global temperature and sea level due to increasing concentration of atmospheric greenhouse gases relative to the pre-industrial era (= 1800 AD). The growth rate of various greenhouse gases in future decades has been taken as per IPCC-1996 technical data. Accordingly, the mean global temperature is projected to increase by about 1.9 K during 1800–2100 AD out of which 1.3 K will be in the next 100 years (i.e. twenty first century). Also, the mean sea level is projected to rise by about 86 cm during 1800 to 2100 AD out of which 60 cm will be in the next 100 years. It is the thermal expansion of oceans which accounts for about 95% of the rise in sea level and the rest comes from the melting of ice in greenland, glaciers and mountain caps.

It is now well accepted that the increasing accumulation of greenhouse gases like carbon dioxide, nitrogen oxides, methane and chlorofluorocarbons in the atmosphere due to burning fossil fuels, agricultural activities and rapid industrialization has been producing global warming1-3. Rise in the global or earth's surface temperature even by a few degrees can produce drastic climatic impacts such as long spell of dry weather, reduction in food production, melting of arctic ice, rise in sea level, flooding of coastal areas, etc. which may adversely affect all forms of life including plants, animals and humans. Therefore, the phenomenon of global warming has received attention from different groups including scientists, technologists, environmentalists and policy makers. The Montreal protocol of 1987 and London amendment of 1990 devised regulatory methods to control the production of chlorine and bromine containing gases, specially long-lived chlorofluorocarbons and other halocarbons, which cause ozone depletion besides being agents for global warming. This has brought the matter into sharp focus and attempts have been made to project the extent of global warming by 2100 AD and beyond which has been described prominently in the recent IPCC-1996 Report4. The present work is also a step in that direction.

The quantitative projection of global warming has been made by developing energy balance (EB), radiative convection (RC) and general circulation models (GCMs)5-5. The GCM models used for climatic predictions are complicated in the sense that they analyse the detailed thermodynamic state of the atmosphere throughout the globe using equations related to the conservation of mass, energy and momentum. These equations are solved by numerical methods requiring heavy computations. The RC and EB models based upon the gross macroscopic properties of earth and atmosphere are relatively simple and acceptable. All these models recognize the fact that the temperature of the earth's surface depends basically on (i) the amount of energy received by the earth from solar radiation flux, (ii) the outgoing black body thermal radiation emitted by the earth and (iii) the absorption properties of the lower atmosphere or the troposphere (i.e. the atmosphere up to a height of 10–12 km from the earth's surface). A significant change in the molecular constituents of the atmosphere alters its absorption property for radiation, which affects the energy balance equation with time and produces a change in the mean global temperature. The atmospheric response to these radiations is described briefly.

The solar radiation extends from the ultraviolet to the infrared regions with wavelength 0.2 μm to 3.5 μm (1 μm = 10,000 Å) and its intensity distribution follows Planck's law of radiation from a black body at 6,000 K (the estimated temperature of the sun's outer surface) which shows the maximum of the peak around 0.5 μm. Most of the integrated energy flux is contained within the visible and the near-infrared regions (0.3 μm to 2.0 μm) - known as shortwave radiations. In contrast, the black body radiation from the earth's surface at a temperature around 300 K lies in the infrared region 5 μm to 60 μm with the maximum of the peak around 10 μm in the corresponding Planck's intensity distribution curve. These longwave thermal/terrestrial radiations are localized mostly between 4 to 40 μm (refs 5, 6). The resonant (or selective) absorption of radiation by atoms/molecules is a quantum mechanical process. Nitrogen, oxygen and argon, which together make up 99.88 volume percent of the atmosphere, hardly absorb any longwave thermal radiation; whereas the main absorption is produced by very small components like water vapour, carbon dioxide, nitrogen oxides, methane, ozone and chlorofluorocarbons (CFCs), etc. The main absorption bands of water vapour are centered around 6.3 μm and 15 to 20 μm. The carbon dioxide has intense absorption bands around 4.3 μm and 15 μm (12.9 to 17.1 μm). Their overlapping bands around 15 μm effectively absorb the longwave thermal radiation5-6. Since the tropospheric concentration of water vapour is only about 10⁻⁵ times that of carbon dioxide, it is not separately considered in a quantitative model. Ozone has large infrared absorption bands at 9.6 μm and 14.1 μm, but because of extremely small concentration in the lower atmosphere its role in global warming is only marginal. The interval 7 to 13 μm with no major absorption band is known as 'atmospheric window' through which most of the longwave radiations escape
into space. However, nitrogen oxides (NO, N₂O, N₉O₄, N₂O₅), hydrocarbons (CH₄, C₂H₆, C₃H₈), CFCs (CFC-11, CFC-12, CFC-113, etc.) and other halocarbons (HCFC-22, CF₄, etc.) have intense infrared absorption bands within 5 to 14 μm and hence they are quite effective greenhouse gases. The relative global warming potentials (GWP's) of carbon dioxide, methane, nitrous oxide, CFC-11 and CFC-12 over their life time are close to 1, 27, 300, 4,000 and 8,000 respectively. These gases re-emitting radiations, depending upon their local atmospheric temperature (usually cooler than the earth's surface), some of which escapes into space while the rest comes downward leading to a net trapping of infrared radiation and consequent global warming. The transmission coefficient of the atmosphere for outgoing longwave radiations decreases with increase in the concentration of greenhouse gases. The incoming shortwave solar radiation suffers a general attenuation while passing through the atmosphere and a part of it is reflected back into space because of molecular scattering, aerosols, clouds, ice patches, finite reflectivity of the earth's surface, etc. The planetary albedo represents the fraction of the solar flux reflected back into space and the rest is absorbed by the earth. These competing processes determine the mean temperature at the earth's surface, and if there were no atmosphere its value would be about 255 K rather than the comfortable 288 K found today (1990s).

The steady state models may be valid for a short time interval (when changes significantly) whereas a non-equilibrium or time dependent state is more realistic over long periods due to the changing nature of the atmosphere. The earth's surface temperature (Tₛ) is greater than its effective temperature (Tₑ) in the absence of the atmosphere, and hence

\[ Tₛ = Tₑ + T'(t), \]  

where \( T'(t) \) is the contribution due to atmosphere and it depends on the presence of greenhouse gases, time and other factors. The time-dependent thermodynamic equation will have the form

\[ Cₑ(dTₑ/dt) = SA(1 - \alpha)/4 - \beta a Tₑ^4, \]  

where \( Tₑ \) is the temperature of the earth's surface at any time, \( Cₑ \) is the heat capacity of the earth-atmosphere system, \( A \) is the surface area of the earth, \( S \) is the solar constant, \( \sigma \) is the Stefan-Boltzmann constant, \( \alpha \) is earth's albedo for solar radiation, and \( \beta \) is the transmission coefficient for longwave radiation.

The division by 4 in the first term takes into account the effective earth's surface area normal to the solar flux. In this macroscopic model, the various parameters represent their mean values despite the fact that they might differ in different regions of the earth or atmosphere. We assume that at initial time \( t = t₁, Tₛ = T₁, \) \( \alpha = \alpha₁, \beta = \beta₁ \). Then eq. (2) takes the form

\[ Cₑ(dTₑ/dt) = SA(1 - \alpha₁)/4 - \beta₁ a T₁^4. \]  

Similarly, at time \( t = t₂ \) after several years the other parameters become \( Tₛ = T₂, \alpha = \alpha₂, \beta = \beta₂ \) because of atmospheric changes and then eq. (2) will take the form

\[ Cₑ(dTₑ/dt) = SA(1 - \alpha₂)/4 - \beta₂ a T₂^4. \]  

From eqs (3) and (4) one obtains

\[ Cₑ(dTₑ/dt)(T₂ - T₁) = - SA(\alpha₂ - \alpha₁)/4 - \sigma A(\beta₂ T₂^4 - \beta₁ T₁^4). \]  

Writing \( \Delta T = T₂ - T₁ = T'(t₂) - T'(t₁) \) as the rise in the global temperature during time interval \( dt = t₂ - t₁ \), this expression may be expressed as

\[ -Cₑ(d\Delta T/dt) = SA(\alpha₂ - \alpha₁)/4 + \sigma A(\beta₂ - \beta₁)T₁^4 + 4\sigma A\beta₂ T₁^3(\Delta T) + 4\sigma A\beta₁ T₁^3(\Delta T)^2. \]  

where,

\[ T₂^4 = (T₁ + \Delta T)^4 = T₁^4 + 4T₁^3(\Delta T) + 4T₁^2(\Delta T)^2. \]  

Integrating (6) with respect to time gives

\[ Cₑ \int [a + b(\Delta T) + c(\Delta T)^2] dt = \frac{-1}{d} \]  

where,

\[ a = SA(\alpha₂ - \alpha₁)/4 + \sigma A(\beta₂ - \beta₁)T₁^4, \]

\[ b = 4\sigma A\beta₂ T₁^3, \]

\[ c = 4\sigma A\beta₁ T₁^2. \]

As \( T₁ \) is about 290 K and \( (\Delta T) \) can be a few degree K at the maximum, the last term in the denominator can be neglected compared to the second term and then the integral simplifies to

\[ Cₑ \int [a + b(\Delta T)] dt = -d. \]  

That is,

\[ (Cₑ/b) \ln(a + b(\Delta T)) = -t + k. \]  

The initial time (or year) can be so chosen that \( \Delta T = 0 \) when \( t = 0 \), that is, the time interval and rise in global temperature are calculated with respect to this very year. Then one obtains

\[ (Cₑ/b) \ln a = k. \]  

and finally,

\[ \Delta T = \frac{a/b}{\exp(-bt/Cₑ) - 1}. \]
This is the formula which gives the rise in global temperature. The year \( 1800 \) AD is regarded as the initial year \( (t = 0) \), that is, the rise in temperature is measured with respect to the pre-industrial era.

The evaluation of \( \Delta T \) requires various parameters, some of which are standard physical parameters and some are to be estimated.

(i) The standard physical parameters are: earth's radius \( R = 6.37 \times 10^6 \) cm, earth's surface area \( A = 5.1 \times 10^{18} \) cm\(^2\), solar constant \( S = 1.35 \times 10^6 \) erg cm\(^{-2}\) s\(^{-1}\), Stephen-Boltzmann constant \( \sigma = 5.67 \times 10^{-5} \) erg cm\(^{-2}\) s\(^{-1}\) K\(^{-4}\).

(ii) The earth's surface temperature \( (T_s) \) in the initial year \( 1800 \) AD is required. Reliable meteorological data on surface-air temperatures in both the hemispheres are available since 1860 AD and their analysis up to 1990 shows an increase of about 0.5 K in the mean global temperature during the last 100-125 years (refs 9–11). The statistical tests of time series data on global temperature from 1867 to 1990 using a linear function also confirm the rise in global temperature of about 0.5 K during this period rather than random variations\(^{12,13}\). Though measured data are not available, it should be realistic to assume that during 1800–1860 (the first 60–70 years of industrialization) the global temperature rose by about 0.1 K. This gives \( T_1 = 287.4 \) K in 1800 AD (288 K in 1990s).

(iii) In a steady state model \( dT/dt = 0 \) and hence the heat capacity of the earth does not enter into the calculation, rather it remains a notional parameter only. However, in the present time-dependent model the heat capacity of the earth–atmosphere system is an explicit and important parameter which has to be estimated. The estimation of the effective heat capacity of the earth is complicated. It is believed that the earth’s crust and ocean water within the first 70–100 meters of depth are primarily important for weather changes over short periods, but it may not be sufficient when considering heat transfers over hundreds of years. About 70% of the earth’s surface is covered by oceans (depth 2–10 km, mean depth = 3.7 km) while only 30% is covered by land mass. The specific heat of water is quite high, rather maximum, and so its heat capacity is also quite substantial. As convection currents and tidal waves in oceans can transfer heat from the surface to a considerable depth and vice-versa, it may be reasonable to consider the mass of water up to a depth of 3 to 4 km. The earth’s crust up to a similar depth can also be included. On this basis the heat capacity \( (C_v) \) of the earth is given by

\[
C_v = (4\pi R^2 d)(0.70 \rho_0 s_0 + 0.30 \rho_1 s_1),
\]

where \( \rho_0 \) = density of water, \( s_0 \) = specific heat of water, \( \rho_1 \) = density of the earth’s crust, \( s_1 \) = specific heat of the earth’s crust, \( R \) = radius of the earth, \( d \) = depth of the earth-crust layers (= 3 km).

The density of the earth’s crust depends upon its depth. While the mean density of the earth is 5.41 g/cm\(^3\), its value is about 3.0 g/cc for depths up to a few km and the same is used here (the density at the earth’s surface is 2.67 g/cc). The upper earth’s layers are mainly made of the oxides of silicon, iron, manganese, aluminium, etc. and their specific heat lies around 0.20 cal g\(^{-1}\) K\(^{-1}\). The mean specific heat of the earth crust will also be close to this value. Using these numbers the expression \( (13a) \) yields

\[
C_v = 5.62 \times 10^{31} \text{ erg K}^{-1}.
\]

The heat capacity of the troposphere can be easily estimated. The mass of the air up to a height of 15 km will be about \( 9.8 \times 10^{21} \) g (assuming density as on the earth) which has a heat capacity of \( 10^{21} \) cal K\(^{-1}\) or \( 4.2 \times 10^{28} \) erg K\(^{-1}\) as per Dulong-Petit’s law. This is about one thousand times smaller than the earth–ocean heat capacity and hence negligible.

(iv) The earth’s albedo depends mainly on the atmosphere, clouds and texture of its surface. The atmospheric albedo due to molecular scattering is maximum (0.29) for ultraviolet and minimum (0.03) for infrared radiations. The non-selective albedo of clouds is high (0.23) while it is small (0.02) for the earth’s surface. The mean albedo for the integral solar spectrum is taken as 0.34 and according to satellite explorer VII data its value is 0.33 (ref. 6). We take its initial value as 0.32 in 1800 AD. Rao and Chakravarty have used a value of 0.30 (ref. 2).

(v) The coefficient \( \beta \) is not a measured parameter but its values in the initial and later years are needed. The (initial) value \( \beta_1 \) in 1800 AD can be back calculated assuming the steady state condition during a short time-interval around this initial year, that is,

\[
\beta_1 = S(1 - \alpha_1)/4.
\]

For \( \alpha_1 = 0.32 \) and \( T_1 = 287.4 \) K, one obtains \( \beta_1 = 0.5932 \).

(vi) Using the known parameters the variables in eq. (12) may be expressed as

\[
a = 1.72 \times 10^{24} (\alpha_2 - \alpha_1) + 1.97 \times 10^{24} (\beta_2 - \beta_1),
\]

\[
b = 2.74 \times 10^{22} \beta_2,
\]

\[
a/b = [62.69 (\alpha_2 - \alpha_1) + 71.85 (\beta_2 - \beta_1)].
\]

Their evaluation requires the knowledge of \( \alpha_2 \) and \( \beta_2 \) in later years. For this one has develop \textit{ab initio} the relations which can represent the variation of these parameters with the concentration of greenhouse gases and the rise of global temperature.

(vii) The earth’s albedo is not likely to change due to addition of greenhouse gases because a CFC molecule
may not be different from a nitrogen or oxygen molecule so far as scattering of solar radiations is concerned. But it may affect the albedo indirectly through a rise in the global temperature which enhances the content of water vapour in the atmosphere. This is because the cloud/water vapour contributes substantially to it. For one degree Kelvin rise in the global temperature the heat required to vaporize one g of water will be less by one calorie, and this extra one calorie will cause the evaporation of 0.00161 g of water, that is, the water vapour in the atmosphere will increase by 0.161%. Therefore, one may have the empirical relation given by

\[ \alpha_2 = \alpha_1 + 0.23 \times 0.00161 (\Delta T), \]  

(16)

where \( \Delta T \) is the rise in global temperature and the mean albedo of water vapour/cloud is taken 0.23 as described earlier.

(viii) The value of \( \beta_2 \) in later years depends on the rise in the concentration of greenhouse gases and a self-consistent approach is used to estimate it. During 1800 to 1994 AD (\( t = 194 \) years), \( \Delta T = 0.60 \) K which is reproduced by eq. (12) for \( \beta_2 = 0.5871 \). During this period the total normalized concentration of greenhouse gases increased by 122 units (see the next section and Table 1) and \( \beta \) decreased by 0.0061. Therefore, one may use the simple relation

\[ \beta_2 = \beta_1 - (0.0061/122)(\Delta c), \]  

(17)

where \( \Delta c \) is the increase in the total normalised concentration of the greenhouse gases in the atmosphere. Though past changes and future projections may not be linearly related, the relation of this nature will be correct in the first approximation.

The concentration of atmospheric greenhouse gases in the pre-industrial era has been obtained by analysing air samples trapped in deep ice cores in the Arctic and Antarctic regions through gas chromatography methods\textsuperscript{14,15} which are used to analyse air samples regularly. The data for \( \approx 1800 \) AD (the pre-industrial era) and 1994 AD and their current growth rates as per IPCC-96 technical summary\textsuperscript{4} are given in Table 2. The growth rates for carbon dioxide, methane and nitrous oxide are averaged over the decade 1984–94 while those for halocarbons are based on recent years (1990s). Similar values (though not updated) have been reported elsewhere\textsuperscript{8,16}. The GWP of a gas is an important parameter which changes over time-horizon depending mainly on chemical reaction, life time and its distribution profile in the atmosphere. For example, the GWP of methane is 56 over 20 years time-horizon and 21 over 100 years time-horizon. A mean value of 27 has been taken for the present calculation. Similarly, the GWP of CFC-11 is 4900 over 20 years time-horizon and 3800

### Table 1. Global warming for projected growth of greenhouse gases

<table>
<thead>
<tr>
<th>Year</th>
<th>1800</th>
<th>1994</th>
<th>2000</th>
<th>2025</th>
<th>2050</th>
<th>2075</th>
<th>2100 AD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>381.4</td>
<td>503.4</td>
<td>513.9</td>
<td>567.8</td>
<td>618.5</td>
<td>670.8</td>
<td>722.8</td>
</tr>
<tr>
<td>Earth's albedo</td>
<td>0.32</td>
<td>0.2022</td>
<td>0.32022</td>
<td>0.32024</td>
<td>0.32036</td>
<td>0.32047</td>
<td>0.32064</td>
</tr>
<tr>
<td>Transmission</td>
<td>0.5932</td>
<td>0.5871</td>
<td>0.5866</td>
<td>0.5839</td>
<td>0.5813</td>
<td>0.5787</td>
<td>0.5761</td>
</tr>
<tr>
<td>Time (t, years)</td>
<td>0</td>
<td>194</td>
<td>200</td>
<td>225</td>
<td>250</td>
<td>275</td>
<td>300</td>
</tr>
<tr>
<td>Rise in global temperature (( \Delta T, K ))</td>
<td>0</td>
<td>0.60</td>
<td>0.66</td>
<td>0.97</td>
<td>1.27</td>
<td>1.72</td>
<td>1.92</td>
</tr>
<tr>
<td>Mean global temperature (K)</td>
<td>287.4</td>
<td>288.0</td>
<td>288.06</td>
<td>288.37</td>
<td>288.67</td>
<td>289.12</td>
<td>289.32</td>
</tr>
</tbody>
</table>

*The sum of volume concentration in ppmv multiplied by GWP for each gas.

### Table 2. Atmospheric greenhouse gases and relevant parameters

<table>
<thead>
<tr>
<th>Gas</th>
<th>CO\textsubscript{2}</th>
<th>CH\textsubscript{4}</th>
<th>N\textsubscript{2}O</th>
<th>CFC-11</th>
<th>CFC-12</th>
<th>HCFC-22</th>
<th>CF\textsubscript{4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) in 1800</td>
<td>280 ppmv</td>
<td>700 ppbv</td>
<td>275 ppbv</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(ii) in 1994</td>
<td>358 ppmv</td>
<td>1720 ppbv</td>
<td>312 ppbv</td>
<td>268 ppbv</td>
<td>503 ppbv</td>
<td>110 pptv</td>
<td>72 pptv</td>
</tr>
<tr>
<td>Growth rate:</td>
<td>1.5 ppmv</td>
<td>10 ppbv</td>
<td>0.8 ppbv</td>
<td>0</td>
<td>7.5 ppbv</td>
<td>5 ppbv</td>
<td>1.2 pptv</td>
</tr>
<tr>
<td>per year</td>
<td>(0.42%)</td>
<td>(0.6%)</td>
<td>(0.25%)</td>
<td>(0.5%)</td>
<td>(5%)</td>
<td>(1.7%)</td>
<td></td>
</tr>
<tr>
<td>GWP</td>
<td>1</td>
<td>27</td>
<td>300</td>
<td>4000</td>
<td>8000</td>
<td>1500</td>
<td>1800</td>
</tr>
<tr>
<td>Life time (yrs)</td>
<td>200</td>
<td>12</td>
<td>120</td>
<td>50</td>
<td>100</td>
<td>12</td>
<td>50000</td>
</tr>
</tbody>
</table>

ppmv, part per million by volume; ppbv, part per billion by volume; pptv, part per trillion by volume. pre-industrial era \( \approx 1800 \) AD.
over 100 years time-horizon and hence a weighted mean value of 4000 has been taken, etc.17. The future global warming commitment of a greenhouse gas over a chosen time-horizon can be estimated by the product of its concentration and GWP. One can use different GWPs over different time-horizons, but such a rigorous exercise may not improve the results because of a typical uncertainty of about ±35% in the best estimated values of GWPs. There are other ozone-depleting halocarbons (CFC-113, CFC-114, HFC-23, SF₆, etc.) which have sufficiently high positive global warming potentials, but their current atmospheric concentration is extremely small and hence not included in the calculation17. However, they may gain importance in future. In recent years aerosols, specially sulphate aerosols, have been recognized as important agents which can affect the global warming phenomenon by scattering of radiations. The aerosols produce a small cooling effect though there is large uncertainty. Because of their short atmospheric lifetime, the aerosol concentrations are largest near their source region and they tend to produce strong regional climatic changes even if their global mean effect is relatively small and hence ignored in the present calculation.

The projection for the amount of greenhouse gases in future has been made assuming current linear growth rate, though it may change depending upon the development scenario (including pace of industrialization, population growth, agricultural activities, etc.) specially in Asian and African continents. For the current growth rate, the concentration of CO₂, CH₄, N₂O and CFC-12 becomes 517 ppmv, • 2780 ppbv, 396.8 ppbv and 1298 pptv respectively in 2100 AD, etc. Since the growth rate of CFC-11 is zero, its concentration has been retained at the constant level of 268 ppbv up to 2050 AD and then reduced to zero in 2075 AD and beyond assuming that the present content is entirely depleted during its life time. While calculating the rise in global temperature one considers the total normalized concentration which is ‘the sum of the product of volume concentration and GWP for each gas’. Thus the total normalized concentration of greenhouse gases is equal to 280 × 1 + 0.7 × 27 + 0.275 × 300 = 381.4 units in 1800 AD and similarly 503.4 units in 1994 AD (increase of 122 units). The rise in global temperature has been calculated using eq. (12). The albedo (αₛ) in 1994 AD is obtained by taking ΔT = 0.60 K and this is used to calculate ΔT in 2000 AD, likewise the albedo in 2000 AD is used for obtaining ΔT in 2025 AD, etc. The results are given in Table 1.

The rise in global temperature depends on the concentration of greenhouse gases and the time taken in reaching this level. The contributions from different gases is not additive in the rigorous sense because the formula for ΔT is a complicated function. The global temperature is likely to increase by 1.30 K in the next hundred years from now which is similar to but significantly smaller than the projected range of 2 to 2.5 K by many workers as per IPCC92 emission scenario4. The ocean–atmosphere interaction is quite important in climatology and a qualitative reasoning suggests the delaying of global warming by a few decades because of the enormous heat capacity of the oceans3,18. In fact the atmosphere–ocean coupled general circulation models (AOGCMs) have been used. While the range of projections for temperature is reasonably good, it is not so for the thermal expansion of the oceans4,18. The present model includes the effective heat capacity of the earth–ocean system as an explicit parameter and the same mean depth of the oceans has been retained while calculating the rise in sea level (see next section). The projections for temperature and sea level are quite reasonable indicating a mutually consistent approach. The strength of the present model lies not simply in making these projections but in its physically consistent approach based upon the principles of physics in a natural manner.

The rise in sea level can inundate coastal regions/cities endangering lives of the people and hence it has drawn wide attention. On the basis of recent analysis of sea level data there is a broad agreement among climatologists and geologists that the sea level has risen by about 18 cm during the past 100 years, which is within the 10–20 cm range given earlier in IPCC 1990 Report. This rise is being attributed to global warming and hence its projection in future has assumed significance. The rise in sea level occurs mainly due to (i) thermal expansion of sea water, (ii) melting/caving of large ice sheets of Antarctica and Greenland, (iii) melting of mountain glaciers and ice caps, and (iv) possible changes in surface and ground water storage, etc. It is desirable to discuss each of them separately.

(i) The increase in the volume of sea water (∆V) because of thermal expansion is given by

\[ \Delta V = V_1 \gamma (\Delta T), \]

where \( V_1 \) is the initial volume, \( \gamma \) is the coefficient of cubical expansion and \( \Delta T \) is the rise in temperature. Assuming that the surface area of the sea water remains unchanged, the rise in sea level (\( \Delta SL \)) is given by

\[ \Delta SL = d \gamma (\Delta T), \]

where \( d \) is the initial mean depth of the oceans which was taken as 3 km while estimating the heat capacity of the earth. For water \( \gamma = 15 \times 10^{-3} \) K⁻¹ at 15°C and then \( \Delta SL = 0 \) in 1800 AD and 27.0 cm in 1994 AD, etc. The rise in sea level due to thermal expansion depends on the initial volume of sea water which has to be estimated in a consistent manner. The effect of salinity variation on a regional/global scale has been ignored.
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(ii) Antarctica contains 90% of the world's ice spread over an area of about $12 \times 10^8 \text{ km}^2$ with temperature varying from 0 to $-80^\circ \text{C}$ from summer to winter besides spatial variation (interior is colder than the periphery)$^{19}$.

The temperatures in Antarctica are so low that little surface melting takes place and the loss is mainly by iceberg caving which is determined by long-term dynamic processes. The contribution of the Antarctic ice sheet to the rise in sea level is very little, rather it expected to change by $-1$ to $-5 \text{ cm}$ by 2100 AD as per recent estimates$^{20,26}$. The Greenland ice sheet has an area of about $1.7 \times 10^8 \text{ km}^2$ with temperature varying between 10 and $-30^\circ \text{C}$ during the year and it responds to temperature signals giving rise to surface melting and iceberg caving (i.e. runoff) as well as accumulation. The geological survey data indicates very similar runoff and accumulation rates per year and so the contribution of Greenland ice sheet to sea level rise is quite small but still significant in view of global warming.

(iii) The thinning of glaciers and mountain ice caps since mid 1950s has become obvious in many parts of the world. Rapid thinning has been observed in this century in the Alps, south-central Alaska and more than fifty other glaciers which are monitored by concerned agencies (World Glacier Monitoring Service, UNESCO, etc.). The melting of glaciers and ice caps can make a significant contribution to sea level rise and so it is important to ascertain the effect of global warming on this process. Melting of ice at any location requires absorption of extra energy from its surrounding, and one may consider the problem in the light of thermodynamic laws for the emission/absorption of longwave radiations by the two systems – the ice sheet and the atmosphere at different temperatures. Here we are interested in calculating the excess of melting due to global warming, that is, whatever ice used to melt before – 1800 AD (the initial reference year) will be taken as background effect and neglected.

The extra energy ($\Delta E$) radiated by the atmosphere because of global warming during time $t$ can be expressed as

$$\Delta E = 4\varepsilon_4 \left( T_1 + \Delta T/2 \right)^4 - T_1^4 \right) t,$$

where $\varepsilon_4$ is the emissive power of the atmosphere (unity for black body). The net extra energy absorbed by mountain glaciers and ice caps can be given by

$$E = 4\varepsilon_4 \left( A_4 A' \right) T_1^3 \left( \Delta T/2 \right) t,$$

where $A'$ is the surface area of the ice exposed to atmosphere and $A_4$ is its absorption coefficient (unity for black body). Since longwave atmospheric transmission coefficient ($\beta$) is $\approx 0.6$ its absorption coefficient should be $\approx 0.4$ ignoring small reflections (specially from cloud patches), and following the rule that 'good absorbers are good emitters' one may assume that emissive power $\varepsilon_4 \approx 0.4$ as well. The ice surface has a high reflection coefficient of about 0.93 for ordinary light (like polished silver or silvered glass)$^{21}$ and so its absorption coefficient $\alpha_4 \approx 0.07$. There might be some variation for infrared radiations but that is ignored.

The energy absorbed by ice is used in raising its temperature to 0$^\circ \text{C}$, melting it to water at 0$^\circ \text{C}$ and warming the water to about 16$^\circ \text{C}$. Since the specific heat of ice is quite low ($\approx 0.50 \text{ cal g}^{-1} \text{ K}^{-1}$) relative to its latent heat of fusion (80 cal g$^{-1}$ K$^{-1}$), the initial temperature of ice (whether $-10$ or $-5^\circ \text{C}$) is not very important in calculating the quantity of ice melted. The heat required in melting 1 g of ice at 0$^\circ \text{C}$ and warming the water to 16$^\circ \text{C}$ is equal to 90 cal or roughly 100 cal taken for the present calculation. The amount of ice melted can be calculated by using eq. (21).

The estimated area of mountain glaciers and ice caps is about $0.70 \times 10^8 \text{ km}^2$ having temperatures several degrees below the freezing point. Between 1800 and 1994 AD ($t = 194$ years, $\Delta T = 0.60 \text{ K}$), the net energy received by glaciers and ice caps comes to $E = 1.94 \times 10^{27} \text{ erg}$ which causes the melting of $4.6 \times 10^{17} \text{ g}$ of ice or produces $4.6 \times 10^{17} \text{ cm}^3$ of water. The surface area of oceans (70% of earth's surface area) multiplied by the rise in sea level is equal to this volume of water. This gives $\Delta SL = 0.13 \text{ cm}$ in 1994 and 0.64 cm in 2100 AD assuming that the entire water from the melting of ice flows uniformly into the oceans.

The Greenland ice sheet has an area of about $1.7 \times 10^8 \text{ km}^2$ and the amount of ice melting can again be calculated by using eq. (21). Though the mean temperature of Greenland may be lower than that of the glaciers and ice caps with somewhat different response to global climatic changes, we presume a similar behaviour. This gives $\Delta SL = 0.32$ and 1.55 cm in 1994 and 2100 AD respectively due to melting of Greenland ice. This type of calculation always involves some inherent uncertainties but despite that it provides a physically acceptable numerical approach.

The estimates made through mass balance studies using available data on net gain or loss of ice at the surfaces suffer from uncertainties due to paucity of data. The more accepted GCM method has its own shortcomings because it hardly evaluates the changes in the water mass at the surface of ice sheets which is governed by thermodynamic processes. Its projection of global temperature is reasonably good but not so for the thermal expansion of oceans$^{18,26}$. The results of the present thermodynamic calculations appear much different but they are physically consistent.

Changes in the terrestrial water budget can affect sea level. The main factors include (1) pumping out underground water for irrigation whose major fraction eventually reaches the oceans through runoff and evapo-
ration/condensation, (2) filling of surface reservoirs involving transfer of water from sea to land, (3) deforestation and desertification causing loss of biomass water, soil moisture and atmospheric vapour, (4) loss of wetland, etc. The second factor may reduce the sea level whereas the other factors tend to increase it and the net effect could be either positive or negative which is hard to estimate because of large spread and lack of suitable numerical methods. Recent works of Chao and Roden-berg suggest that the sum total may be close to zero. In any case the change in sea level due to these processes in the long run could be only marginal.

The recent estimates by Barnett, Douglas, Tanner and Gornitz on the basis of measurements done at a large number of sea stations over many years indicate that the mean sea level has risen by about 18 cm during the past 100 years. However, the data between 1930 and 1980 indicates a faster rise of about 23 cm per century that might be due to global warming. The present calculation indicates that because of thermal expansion the sea level has risen by about 27 cm between 1800 and 1994 during which the global temperature increased by 0.6 K. It implies that during the first hundred years (∆T = 0.2 K) the sea level increased by about 9 cm and in the next hundred years it rose by about 18 cm, which agrees very well with the observed data. It shows the viability of the present model. The projection based on this model indicates a rise in sea level of about 61 cm in the next century. A faster rise in later years occurs due to enhanced global temperature.

Several workers have projected a sea level rise of 35 to 50 cm by the year 2100 (relative to 1990s) with contributions from thermal expansion, glaciers and ice caps, Greenland and Antarctica. For example, Titus and Narayanan’s best estimates are 21, 9, 5 and -1 cm respectively with a total of 34 cm within a range of 5 to 77 cm. The IPCC96 projections are 28, 16, 6 and -1 cm respectively with a total of 49 cm within a range of 20 to 86 cm, etc. These estimates depend upon the methodology for calculating the rise in global temperature and sea level. Also their wide range indicates substantial uncertainties. The present study shows that the contributions from glaciers and Greenland are much smaller than the above projected values. The thermal expansion of oceans depends on the product of its initial volume and rise in temperature (which itself depends on the former through the heat capacity of the earth) and hence a mutually consistent approach is required.

The thermodynamic model provides an alternative method to estimate the extent of global warming and rise in sea level and it is physically consistent and conceptually viable. The projections for the rise in global temperature and sea level by the year 2100 have been made keeping in mind that the past results are also reproduced. The methodology involves many concepts and parameters and it is not unusual if models differ in their predictions. The present model is fairly exhaustive and its projections are expected to be reliable within reasonable limits.


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