Integers and fractions

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The 1998 Nobel Prize in Physics has been awarded jointly to Robert B. Laughlin, Stanford University, USA, Horst L. Störmer, Columbia University and Bell Labs, USA, and Daniel C. Tsui, Princeton University, USA, for 'their discovery of a new form of quantum fluid with fractionally charged excitations' appearing in the fractional quantum Hall effect. The quantum Hall effect has two versions, the integer (which won an earlier Nobel Prize) and the fractional. These two versions share the remarkable feature that an experimentally measured quantity, characterizing one aspect of a complicated many-particle system, stays perfectly fixed at some simple and universal values even though many different parameters of the system vary from sample to sample (in fact, some parameters like disorder vary quite randomly). In addition, the fractional quantum Hall effect has the astounding property that the low-energy excitations of the system carry a charge which is a simple fraction of the charge of an electron, even though the system is composed entirely of particles like atoms, ions and electrons, all of whose charges are integer multiples of the electronic charge.

In 1879, Edwin Hall discovered that when a two-dimensional layer of electrons (such as in a thin gold plate) is placed in a magnetic field perpendicular to the layer (pointing, say, in the $\hat{z}$-direction), an electric field along one direction in the layer (say, the $\hat{y}$-direction) causes a current flow along the $\hat{x}$-direction which is at right angles to both the electric and the magnetic field. The ratio of the current density J_x to the electric field E_y is now called the Hall conductivity σ_{xy}. This quantity depends on the strength of the magnetic field, and the density and sign (negative for electrons and positive for holes) of the charge carriers. Classically, the Hall effect can be understood as being caused by the Lorentz force which acts on a charged particle moving in a magnetic field; this force acts in a direction perpendicular to both the velocity and the magnetic field and is pro-

portional to the product of the two. The Hall conductivity is to be distinguished from the usual conductivity σ_{xx} which denotes the ratio of the current density to the electric field along the *same* direction.

Hall performed his experiments at room temperatures and at magnetic fields of less than a Tesla (10^4 Gauss). By the end of the 1970s, semiconductor technology had progressed to the extent that experimenters could perform similar measurements at the interface of a semiconductor and an insulator (such as a Si–SiO$_2$ device called a MOSFET) at very low temperatures of around 1 K in magnetic fields of around 10 Tesla. The two-dimensional interface acts as a quantum well which is very narrow (typically, 5×10^{-7} cm wide) in the direction perpendicular to the interface. This freezes out the motion of the electrons (or holes) in that direction, thereby constraining them to move only along the two dimensions parallel to the interface. The density of carriers in these systems is about 10^{11} cm^{-2}, while the mobility of the carriers is typically 10^4 cm^2/V-s. We recall here that the mobility denotes the drift velocity achieved by a carrier when an electric field of unit strength is applied. The finite value of the mobility in the above systems is due to the various imperfections and impurities which cause the carriers to get scattered every 10^{-12} s or so.

In 1980, the German physicist Klaus von Klitzing and his collaborators observed in such systems that the Hall conductivity σ_{xy} does not vary linearly with the strength of the magnetic field (as the classical argument would suggest) but rather shows plateaus, i.e. σ_{xy} does not vary at all for certain ranges of the applied magnetic field. Further, its value at the plateaus is quantized at the remarkably simple values

$$\sigma_{xy} = r\frac{e^2}{h},\qquad(1)$$

where r is a small integer like 1, 2, 3,... (whose value depends on the density of

the carriers and the magnetic field), e is the charge of an electron, and h is 2π times the Planck's constant $\hbar$. (The value of σ_{xx} is extremely close to zero wherever σ_{xy} has a plateau.) Amazingly, the quantized values of σ_{xy} do not depend at all on any other parameters of the system such as its shape and size, the effective masses of the carriers, the strengths of their interactions with each other and with impurities, and the density of impurities. Finally, the plateau values of σ_{xy} have an experimental uncertainty of only about one part in ten million which makes it one of the most accurately measured quantities in all of science. For this reason, the integer quantum Hall effect (IQHE), so called because r is an integer, is now used to define the value of e^2/h which is a fundamental constant of nature. The 1985 Nobel Prize in Physics was awarded to von Klitzing for his discovery of the IQHE.

The IQHE can be explained by invoking some facts from one-particle quantum mechanics. A magnetic field leads to the formation of energy bands called Landau levels which are separated from each other by gaps. The number of states in each Landau level is directly proportional to both the area of the system and the strength of the magnetic field. Due to the Pauli exclusion principle for fermions, each state can accommodate only one electron; at very low temperatures, all states up to a maximum energy called the Fermi energy are occupied. If the density of electrons is such that exactly an integer number of Landau level is filled, then the Hall conductivity is given by that integer times e^2/h. Further, the existence of a finite energy gap to the next (unoccupied) Landau level would lead to the vanishing of the usual conductivity σ_{xx} at a plateau at zero temperature; if the temperature is finite but much smaller than the gap, then σ_{xx} is exponentially small.

The above simplified discussion has to be modified in a major way to take into account the imperfections and impurities which are randomly located in

the system; we will refer to all these point objects as disorder. In an ideal system with no disorder, each Landau level would be infinitely narrow and all the states would be spatially extended. In a real system, the disorder broadens each Landau level, and also converts a number of states from extended states (which can carry current) to localized states (which cannot carry current). We thus get alternating and non-overlapping bands of extended and localized states, with the extended bands staying close to the energies of the ideal Landau levels. The existence of localized bands initially led people to worry that the Hall conductivity may depend sensitively and unpredictably on the amount of disorder. However, using some clever arguments based on gauge invariance and topology, theorists were able to prove the amazing result that the extended band in any given Landau level contributes *exactly* the same amount to the Hall conductivity as that entire Landau level would have contributed if there had been no disorder and all the states had been extended. This means that if the Fermi energy lies within a band of localized states, and there is an integer number of filled bands of extended states below that energy, then the Hall conductivity is simply equal to that integer times e^2/h. Thus, although the localized states do not contribute to the Hall conductivity, they are crucial for the finite widths of the conductivity plateaus; this is because the Hall conductivity does not change at all if the Fermi energy moves by small amounts (due to changes in the carrier density or magnetic field), as long as it stays within a given localized band.

In 1982, Störmer, Tsui and collaborators working on some other systems discovered additional plateaus in σ_{xy} of the same form as in eq. (1), but with the parameter r now equal to some simple odd-denominator fractions like 1/3, 2/3, 1/5, 2/5, 3/7,....; simultaneously, σ_{xx} vanishes. Note that r denotes the number of Landau levels occupied by electrons, hence a value of r less than 1 means that the lowest Landau level is only partially filled. Although this *fractional* quantum Hall effect (FQHE) sounds similar to the IQHE, both its discovery and its theoretical understanding involved much greater levels of difficulty and subtlety than the IQHE.

Experimentally, the carrier mobility required to observe the FQHE is many times larger, i.e. about 10^5–10^6 cm²/V-s. This required the use of a new class of systems such as *GaAs-AlGaAs* heterostructures. A technique called modulation doping molecular beam epitaxy was used to produce interfaces which are extremely sharp, almost free of defects and well-separated from the impurities (namely, the dopant atoms which act as donors or acceptors). This vastly reduces the rate of scattering of the carriers which reside at the interface and therefore leads to much higher mobilities. The FQHE also requires much lower temperatures going down to about 0.1 K and higher magnetic fields up to about 30 Tesla (since the filling r is inversely proportional to the magnetic field).

The discovery of the FQHE raised the theoretical question of why there should be a gap above the ground state (as is suggested by the vanishing of σ_{xx}) when the lowest Landau level is only partially occupied by electrons. Clearly, this requires an explanation which is very different from the one for the IQHE. Within a year, Laughlin came up with a many-body explanation for the existence of a gap. In doing so, he also discovered a very accurate variational quantum mechanical wave function for the system which explained all the observed phenomena and predicted some new ones. Laughlin's key idea was that the Coulomb repulsion between the carriers (assumed to be electrons from here on) must play a crucial role in the FQHE unlike the case of the IQHE. This must lead to the electrons 'avoiding' each other as best as they can, subject to the constraint that they are all in the lowest Landau level. Using some general properties of states in the lowest Landau level, Laughlin introduced a variational many-electron wave function of the form

$$\Psi(z_j, z_j^*) = \prod_{j<k} (z_j - z_k)^m \exp\left[-\sum_j z_j z_j^*\right]$$

(2)

where $z_j = x_j + iy_j$ and $z_j^* = x_j - iy_j$ are complex variables denoting the position of electron j in the $x-y$ plane; to simplify the notation, we have scaled the coordinates in some way so as to make

them dimensionless. For m equal to an odd integer, this wave function is antisymmetric (as the wave function of several identical fermions should be; the spin degree of freedom plays no role here since the electrons are completely polarized in the lowest Landau level), vanishes very rapidly as any two particles approach each other (thereby ensuring that they stay away from each other and minimize their Coulomb repulsion), and, very importantly, describes a state with an uniform density corresponding to the special value of Landau level filling given by $r = 1/m$. (For other filling fractions which are not of the form $1/m$, Laughlin and others suggested related variational wave functions using similar arguments.) The state defined by eq. (2) describes a peculiar kind of quantum liquid which has no long-range positional order (unlike the atoms in a crystalline solid), but does have a long-range phase correlation; this is related to the fact that the wave function picks up a phase of $2\pi m$ when any particle j is taken in a loop around any other particle k, no matter how large the loop is.

After showing that the wave function in eq. (2) perfectly explains the properties of the ground state such as the conductivity quantization, Laughlin went on to study the nature of the low-energy excitations called quasiparticles. He showed that these are separated from the ground state by an energy gap of about 1 K, and that the quasiparticle size is typically about 5×10^{-7} cm which is substantially smaller than the average distance between two electrons (about 3×10^{-6} cm). There are two types of quasiparticles, called quasielectrons and quasiholes; in the neighbourhood of these objects, the local electron density is slightly larger or smaller, respectively, than the special value of density (or filling) given by the Laughlin wave function. Since each quasiparticle costs a finite energy, the system tends to oppose any attempts to change its electron density away from one of the special fillings of $1/m$; this property is called incompressibility and it is one of the striking features of the fractional quantum Hall system, sometimes called a Laughlin liquid. If a small change in filling is forced upon the system by varying either the number of electrons or the magnetic field, the system reacts by producing a small number of qua-

sielectrons or quasiholes which then go and bind to one of the points of disorder; this allows the rest of the system to remain at the special filling, the Hall conductivity thereby remaining exactly at the quantized value. Thus the finite widths of the Hall plateaus again depend on the presence of a small amount of disorder.

Laughlin also made the extraordinary prediction that the quasiparticles should carry a charge which is a fraction of the electron charge, e.g. the quasielectron and quasihole charges should be equal to $\pm e/m$, respectively, if $r = 1/m$. This prediction has been directly verified in recent experiments which study tiny fluctuations (called shot noise) in the current flowing across a narrow region of a fractional quantum Hall system. The amount of shot noise is directly proportional to the charge of the carriers, and the experiments conclusively proved that these are fractional. (Although the quasiparticles cannot move through the bulk of a quantum Hall system because they get pinned by impurities, they do move freely along the edges of such a system, and they can jump from one edge to a nearby edge as shown by the shot noise experiments.)

For those who are uneasy with the idea of fractional charge, we should emphasize that the quasiparticles in a fractional quantum Hall system are not elementary particles (like electrons or photons), but are collective excitations involving a large number of electrons. It

was already known in another many-body system (polyacetylene chains) that composite objects may carry fractional charge. The FQHE has provided us with a new and wonderful example of objects with a fractional quantum number. Incidentally, it has been suggested by some theorists that these quasiparticles are neither fermions (obeying Fermi–Dirac statistics) nor bosons (obeying Bose–Einstein statistics), but that they obey a novel kind of quantum statistics (called fractional or anyonic statistics) which lies in between the two possibilities; however this has not yet been proved experimentally.

Laughlin's wave function has led to new ways of thinking about strongly interacting many-body systems. For instance, if $r = 1/m$ where m is odd, it is natural to think of a quantum Hall system as being made up of 'composite bosons' which are combinations of electrons and m elementary quanta of magnetic flux. These 'bosons' effectively see a zero magnetic field, since the entire magnetic field has been 'absorbed' in the way these strange particles are defined. Now one can view the FQHE as resulting from a Bose–Einstein condensation of these bosons at low temperatures, in the same way that Cooper pairs of electrons condense in superconductors or the atoms of He^4 condense to form a superfluid at low temperatures. This idea of composites of electric charge and magnetic flux has turned out to be remarkably useful in explaining many other aspects of

quantum Hall systems, such as the behaviour of the system at the filling $r = 1/2$ (which does *not* show a conductivity plateau, but has some other interesting properties which are well-explained by the notion of 'composite fermions').

Before ending, one should appreciate that the theoretical and experimental work on the quantum Hall effect is a reflection of the high level of both industrial and university research and the close collaboration between the two in the West, particularly in the USA.

1. Tsui, D. C., Störmer, H. L. and Gossard, A. C., *Phys. Rev. Lett.*, 1982, **48**, 1559–1562.
2. Laughlin, R. B., *Phys. Rev. Lett.*, 1983, **50**, 1395–1398.
3. Halperin, B. I., *Sci. Am.*, April 1986, **254**, 40–48.
4. Eisenstein, J. P. and Stormer, H. L., *Science*, 22 June 1990, **248**, 1510–1516.
5. Kivelson, S., Lee, D.-H. and Zhang, S.-C., *Sci. Am.*, March 1996, **274**, 64–69.
6. Anderson, P. W., *Phys. Today*, October 1997, **50**, 42–47.
7. Collins, G. P., *Phys. Today*, November 1997, **50**, 17–19.
8. Daviss, B., *New Scientist*, 31 January 1998, **157**, 36–40.

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Nitric oxide as a cell signaling molecule: The 1998 Nobel Prize in Medicine

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Cell signaling is such an important area of current research in biology, that the 1998 Nobel Prize in Medicine is the third Nobel Prize that has been awarded in this area of research during the present decade. Earlier in 1992, the Nobel Prize for Physiology or Medicine was awarded to Edwin G. Krebs and

Edmond H. Fischer for their pioneering work on reversible phosphorylation as a biological regulatory mechanism; the 1994 Nobel Prize in Medicine was awarded to Martin Rodbell and Arthur G. Gilman for their work on the role of G proteins in signal transduction, and this year, the 1998 Nobel

Prize for Medicine has been awarded to the following three US scientists for their work concerning nitric oxide as a signaling molecule in cardiovascular system: (1) Robert F. Furchgott of the State University of New York in Brooklyn, (2) Louis J. Ignarro of the University of California-Los Ange-