Functional Analysis – Spectral Theory. *Texts and Readings In Mathematics – 13.* V. S. Sunder. Hindustan Book Agency, 1997. pp. 306.

The surfeit of books on functional analysis over the years might lead one to believe that the subject has been presented from every possible perspective and in every imaginable style. The appearance of this book by V. S. Sunder will quickly dispel that attitude, for in it, the point of view taken, and the choice of ideas described, are significantly different from many typical expositions of the subject.

'The only real way to understand the spectral theorem is as a statement concerning representations of commutative C^* algebras.' Thus speaks the author in the preface and these words summarize the outlook that he espouses. The book, as the subtitle indicates, is directed largely towards a study of spectral theory, using ideas from the theory of operator algebras (the latter being a subject in which the author has made notable research and expository contributions).

Beginning with some preliminaries about normed spaces (including the Hahn–Banach theorem, Open Mapping and Closed Graph theorems, Uniform Boundedness Principle and so on), the author goes on to discuss Hilbert spaces. Here the section on orthonormal bases describes in great detail many facts which are often passed off as 'intuitively obvious'.

This is followed by a chapter on C^* algebras (especially commutative ones) which includes the Gelfand–Naimark theory as well as representation theory. It culminates in the Hahn–Hellinger theorem whose proof is laid out very meticulously. The author's enthusiasm for von Neumann algebras finds expression in the Double Commutant Theorem (as well as in the use of these algebras elsewhere, for example, in the section on unbounded operators).

The above ideas are then used in a derivation of the spectral theorem and polar decomposition for bounded operators. A couple of sections providing a fairly detailed treatment of compact and Fredholm operators, and their interplay, wrap up the chapter.

A final chapter on unbounded operators (including the spectral theorem, using the Cayley transform) introduces the reader to the various subtleties one encounters in going beyond the confines of bounded operators.

The Appendix is both voluminous and broad enough in its content to merit being called a book within a book. It opens with some linear algebra and ends with the Stone–Weierstrass and Riesz Representation theorems. In between are sandwiched a number of standard theorems... Urysohn's lemma, Tychonoff's theorem, Tietze Extension theorem, Monotone and Dominated Convergence theorems and a host of others. The section on 'Transfinite Considerations' introduces some formal aspects of set theory, including Zorn's lemma.

From definitions of elementary concepts (even matrix multiplication is defined!) to a comprehensive appendix, every attempt is made to ensure that prior knowledge assumed is kept to a minimum.

Many of the constructs used (for example, Hilbert spaces, locally convex topologies) are well motivated in the text. The proofs provided are thorough and painstakingly detailed. A liberal sprinkling of well thought out exercises, which are essential to the main thread of the discussion, ensures that this book is not directed at the casual reader.

The use of an elaborate notational style and the repeated emphasis of assumptions and attributes, by parenthetical phrases, are hallmarks of the author. Though this device occasionally makes sentences long winded, it dispels any ambiguity.

The necessity of the assumptions for various theorems is clarified by means of (counter) examples. Instances are, the fact that the Closed Graph Theorem can be false in the absence of completeness, and the fact that a non-normal operator can have spectral values which are not approximate eigenvalues.

The use of more distinctive symbols for linear operators ($L(X, Y)$) and bounded linear operators ($\mathcal{L}(X, Y)$) and a more detailed index may have been useful. The mild mismatch of symbols, that has crept into the Hahn Banach theorem, is obvious enough to be rectified even by an unsophisticated reader.

The book is pitched at the level of a serious master's student or a beginning research student. In the case of a master's student, some sections may require a particularly dedicated effort.

The Texts and Readings In Mathematics (TRIM) series has, undoubtedly, made noteworthy contributions to mathematical literature. Sunder's book adds to the stature of the series in significant measure.

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The Story of Spin. Sin-itiro Tomonaga. Translated by Takeshi Oka. University of Chicago Press, 5801, South Ellis Avenue, Chicago, IL 60637, USA. 258 pp. Price: US $50.

The period from 1920 to 1940 is truly a landmark period in the history of Physics when Quantum Mechanics matured and the areas of Condensed Matter Physics and Nuclear Physics were being developed while the area of Elementary Particle Physics was about to be born. One of the characteristics of this period was that most of the major players were very young. For example, Heisenberg was 24 when he wrote his landmark paper on matrix mechanics. Pauli was 25 when he published his famous paper on exclusion principle, Dirac was 26 when he discovered the equation known by his name while Yukawa was 28 when he wrote his famous paper on the heavy quantum. Coming to the discovery of spin, which is the main theme of this book, Kronig was barely 20 when he thought of the idea of electron spin, Goudsmit was 23 while Uhlenbeck was 25 when they wrote their paper on electron spin. Finally, Thomas, whose factor of 1/2 helped in settling the issue of electron spin, was 23 when he wrote that paper. I do not think that a parallel to this exists in any branch of science or for that matter in any branch of intellectual activity.

It is therefore, quite fitting that the developments in this period be summarized by someone who passed his scientifically formative years during that time, directly experienced part of these developments and finally himself made significant contributions. The person I am referring to here is Sin-itiro Tomonaga who shared the Nobel Prize in Physics in 1965 for his work in the development of quantum electrodynamics.

The book has been divided into twelve lectures which were originally published in Japanese from January through October 1973 issues of *Shizen*. Subsequently, Tomonaga expanded and revised them considerably and the book was published in 1974. The translation of this book in English by Takeshi Ota of Enrico Fermi Institute, Chicago, has just appeared and he must be thanked for bringing out this masterpiece before the eyes of the physics community outside Japan.

The title of the book is somewhat misleading because even though it contains the story of spin, it in fact discusses several other developments in that period including the work of Heisenberg, Fermi and Yukawa which lead to the development of the area of Nuclear Physics. Besides, it has an in depth discussion about the general relation between the spin and the statistics of elementary particles. In fact, my only regret is that Tomonaga has not covered the area of quantum statistics. As he himself has said in the book, 'how the two types of statistics were discovered and how they were incorporated into quantum mechanics is a very interesting subject in itself'.

The existence of spin, and the statistics associated with it, is the most subtle and ingenious design of nature. This property of spin of elementary particles is peculiar to quantum mechanics. The relation between spin and statistics is apparent, yet its basis is hard to understand unless one goes to relativistic quantum theory. As Feynman has said in his famous lectures, this probably means that we do not have a complete understanding of the fundamental principles involved. Tomonaga has discussed this issue at length in Lecture 8.

In this book one finds an insightful discussion about the atomic spectroscopy and how the three main actors, Sommerfeld, Lande and Pauli were groping for an understanding of multiplicity of spectra and of the Zeeman effect. In this process a new idea crystallized in Pauli's thinking in 1924, that 'the origin of the multiplicity is not the core but the electron itself'. This idea was picked up by young Kronig who first conceived of the self-rotating electron. Unfortunately, both Pauli and the people in Copenhagen were cool to the idea and the poor fellow, not being very confident, did not publish his idea!

Unaware of all this, Goudsmit and Uhlenbeck thought of the same idea six months later and published in *Naturwissenschaften*. Actually, soon after submitting their paper they too wanted to withdraw their paper in a hurry, but they were too late! Pauli, of course had his objections but then the paper of Thomas appeared who clarified the famous factor of 1/2 – after that even Pauli approved of the idea.

It is incredible, how many people have suffered because of Pauli's *sanction*. This story has a lesson for all of us and specially to young physicists: 'Believe in yourself and do not accept something just because some great man is saying it'. This story is also an illustration of how luck can play an important role in scientific discoveries.

There is also an interesting chapter about the development of Nuclear Physics. As is well known, Niels Bohr believed that quantum mechanics does not apply within the nucleus. This belief had a lot of influence on the physicists of that time. It is worth noting that Heisenberg, Fermi and Yukawa, who finally proved this prejudice to be false, were working at places farther and farther from Copenhagen where Bohr resided. Maybe it is not too bad to be away from the masters!

In this book, one finds delightful, fairly rigorous and very insightful discussions about several developments in quantum mechanics. Any serious student of physics will find it extremely enjoyable. Further, those interested in the history and philosophy of science will also find it valuable for its numerous anecdotes and reminiscences. But most of all, it is a must for all theoretical physicists because Tomonaga has explained several developments like Pauli equation, Dirac equation, Thomas factor, relation between spin and statistics, quantization of scalar and spinor fields, etc. with full mathematical rigour and physical arguments.

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