Couple stress boundary layer flow past a stretching sheet revisited

In view of growing industrial importance of couple stress boundary layer flow of non-Newtonian fluid, a large number of fluid dynamicists such as, Dandapat and Gupta[1], Bujurke and Biradar[2] and Maitra[3] tackled problems of couple stress boundary layer flow with similarity transformation, subject to different types of boundary conditions, with a view to obtaining mostly closed-form solutions. The author solved analytically in closed-form a problem[2] of this kind taking into consideration the boundary conditions, which involve the 'material constant' of the non-Newtonian fluid, and, accordingly, the boundary conditions have been slightly modified with regard to those adopted by Bujurke and Biradar[2] without sacrificing its physical significance and generality. Such a modification of the boundary conditions[2,3] fortunately leads to the closed-form solution of the fifth-order ordinary nonlinear differential equation governing the couple stress boundary layer flow. Nevertheless, in this kind of non-Newtonian flow a situation is also possible where the boundary conditions are independent of the material constant of the fluid. Writing of the present paper is evoked by successful investigation of the non-Newtonian boundary layer flow subject to the boundary conditions independent of the material constant of the fluid. Hence the closed-form similarity solutions are obtained for the non-Newtonian boundary layer flow past a stretched sheet with a new set of boundary conditions in this quest, compared to the earlier work[3] in which Bujurke and Biradar[2] did not attempt this kind of solution.

A non-Newtonian boundary layer flow is developed past a rectangular sheet stretched by applying two equal and opposite forces while keeping the origin fixed. The equations of this non-Newtonian boundary layer flow with zero-pressure-gradient are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}\frac{\partial^4 u}{\partial y^4}, \tag{2}$$

where u and v are the components of the fluid velocity in the longitudinal (x) and the transverse (y) directions respectively, σ the material constant and ρ the density of the fluid.

The boundary conditions are

$$u=u_1x,\quad v=0 \quad \text{at } y=0$$
$$u\to 0 \quad \text{as } y\to\infty,\quad u_1<0, \tag{3}$$

where u_1 is the fluid velocity at the point (1, 0). To solve the differential eqs (1) and (2) subject to the boundary conditions (3) we resort to the similarity transforms

$$\eta=(-u_1/\nu)^{1/2}y \text{ and } \psi=-(-\nu u_1)^{1/2}xf(\eta), \tag{4}$$

where ψ is the stream function giving

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{5}$$

Because of (4) and (5), the main eq. (2) reduces to a nonlinear fifth-order similarity equation,

$$f'^2-ff''+f'''=Kf^{V} \tag{6}$$

and

$$u=u_1f^1(\eta)\quad v=(-\nu u_1)^{1/2}f(\eta), \tag{7}$$

where the prime indicates derivative with respect to η and K is the constant called stress parameter defined by

$$K=-u_1\sigma/(\rho\nu^2)>0, \tag{8}$$

so that the boundary conditions (3) are reshaped as

$$f(0)=0, f'(0)=1, f'(\infty)=0 \tag{9}$$

which are independent of K.

Obtaining an analytical solution to eq. (6) ordinarily appears to be a formidable task. But the solution of such a differential equation subject to a set of congruent boundary conditions is in the theory of differential equations established as unique. This feature emboldens one to strive for the exact analytical solution in this context on trial basis. It is not unusual to infer that the way differential eq. (6) involves f and its derivatives of higher orders, its solution contains exponential function(s) of the independent variable η. However, ultimately it is reckoned that the exact solution to eq. (6) subject to the boundary conditions (9) yields

$$f(\eta)=(1-e^{-r\eta})/r \quad r>0,$$
$$f'(\eta)=e^{-r\eta}, \tag{10}$$

where the constant r is determined from the biquadratic equation

$$Kr^4-r^2-1=0. \tag{11}$$

Where

$$r=\left(\frac{1+\sqrt{1+4K^2}}{2K}\right)^{1/2}>0, \tag{12}$$

which is designated as stress factor.

Equations (10) and (12) give the exact analytical solution to the couple stress boundary layer flow.

But the expression (7) when combined with (10) and (12) gives the velocity components

$$u=-\frac{K\nu^2\rho}{\sigma}xe^{-r\eta},$$
$$v=\left(\frac{K\nu\rho}{\sigma}\right)^{1/2}(1-e^{-r\eta})/r. \tag{13}$$

Also employing (10) and (12), the dimensionless shearing stress on the wall is found out as

$$\tau=\left(\nu\frac{\partial u}{\partial y}-\frac{\sigma}{\rho}\frac{\partial^3 u}{\partial y^3}\right)_{y=0}(-u_1)^{-3/2}\nu^{-1/2}x^{-1},$$
$$=f''(0)-Kf^{IV}(0),$$
$$\tau=-\frac{1}{r}. \tag{14}$$

The negative sign of τ is obviously concerned with its formulation (14) and the relevant force. The curve suggesting the variation of the shearing stress magnitude with stress factor is a hyperbola.

Most of the investigators are devoted to obtain exact/analytical/numerical solutions for such kind of boundary-layer flow with zero pressure-gradient. Perhaps they ruled out the possibility of getting similarity equations for a couple stress flow with a nonzero-pressure-gradient. Therefore, in this last section is encountered a nonzero-pressure-gradient that gives rise to a tractable differential equation of the flow along with a set of boundary conditions in line with (9).

With inclusion of pressure gradient $\partial P/\partial x$, P being the pressure, eq. (2) changes to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}\frac{\partial^4 u}{\partial y^4}. \quad (15)$$

If the couple stress boundary layer flow is developed with a pressure gradient

$$\frac{\partial P}{\partial x}=\frac{-u_1^3\sigma}{\nu^2}xf'(\eta), \quad (16)$$

then eq. (15) reduces, or, in other words, eq. (6) modifies as

$$f'^2-ff''+Kf'+f'''=Kf^{iv}, \quad (17)$$

which is different from that with non-zero-pressure-gradient. If the flow environment is kept the same, obviously because of change of pressure gradient, the flow velocity will change. The same set of boundary conditions for two different pressure gradients is permissible in so much as the pressure gradient in the present feature has no functional relationship with the boundary conditions.

Retaining the same boundary conditions as eq. (9) and following the same technique and subtlety as earlier, eq. (17) admits of the closed-form solution

$$f(\eta)=(1-e^{-a\eta})/a,$$
$$f'(\eta)=e^{-a\eta},$$
$$a=(1+(1/K))^{1/2}, \quad (18)$$

and the velocity components become

$$u=-\frac{K\nu^2\rho}{\sigma}xe^{-a\eta},$$
$$v=\left(\frac{K\nu\rho}{\sigma}\right)^{1/2}(1-e^{-a\eta})/a. \quad (19)$$

In this case the dimensionless shearing stress on the wall in the same manner as earlier gives

$$\tau=-aK=-(K^2+K)^{1/2}, \quad (20)$$

which satisfies the equation of a rectangular hyperbola.

Boundary-layer flows fostered by similarity transformation and complicated equations in general count upon numerical techniques for their solutions; a vivid study of earlier works[1-3] and their comparison with the present one reveals that the boundary layer flows, both Newtonian and non-Newtonian, are amenable to exact analytical solutions necessitating suitable technique and artifice in envisaging the transformation equation in algebraic trial with the differential equation of flow and prescribed boundary conditions. It has been observed that in the realm of boundary-layer flows of the present kind, different types of solutions are evolved due to different sets of boundary conditions, and sometimes subject to certain restrictions on the values of some fluid parameters. The process of obtaining closed-form solution also depends a lot on the choice of similarity transformation equations. The exact analytical solution herein, unlike that reported[2,3], is not restricted to a certain range of values of any parameter.

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Fungitoxicity of some insecticides

Microorganisms which produce plant diseases have been responsible for considerable crop destruction. It is estimated that plant diseases cause an annual 10% loss in the US crop[1] and 18 to 20% reduction in the world agricultural yields[2]. In recent years, much attention has been paid to plant pathogens[3,4]. While different fungicides are available for controlling plant pathogenic fungi, reports[5] indicate that some insecticides may have additional fungitoxic properties as well. Little information exists on this aspect in respect of the newer insecticides (Durmet, Kanodane, Nuvacron, Nuvan and Cymbush, etc.). The present work was undertaken to determine the effects of these five insecticides under laboratory conditions against selected plant pathogenic fungi.

The efficacy of these insecticides, namely Durmet (20EC), Kanodone (20EC), Nuvacron (36EC), Nuvan (76EC) and Cymbush (25EC) against fungi were tested *in vitro* by the poisoned food technique[6]. The required quantity of test insecticides was weighed and dissolved in sterile water. Requisite concentrations (0.01, 0.05, 0.1 and 0.2%) of these solutions were added to potato-dextrose agar medium in a conical flask and mixed well to get various concentrations. The contents of each conical flask were poured equally into petri dishes. The medium was allowed to set under UV light in laminar flow and 8 mm discs of the fungi were transferred aseptically to the petri dishes. The petri dishes were kept in a BOD incubator at 25°C and incubated for 7 days. The diameter of the fungal colony in each petri dish was measured after 7 days. Control was maintained without fungicides, while a Bavistin was used as a standard fungicide for comparison. Fungicidal activity was expressed as per cent inhibition by standard formula[7].

From Table 1, it is evident that Durmet, Cymbush, Nuvacron and Nuvan were highly active in inhibiting the mycelial growth of *Fusarium oxysporum*, *Helminthosporium* sps., *Sclerotium*