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Possible impact of tidal frequencies on the predictability of the earth's axial rotation

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Short-term fluctuations in earth axial rotation, which is proxy for fluctuations in length of day (LOD), represent composite response of terrestrial coupling of ocean and atmospheric dynamics and extra-terrestrial torque (sun–moon system) of tidal reverberations. We examine here the possible influence of tidal frequencies (periodic signal) on the predictability of LOD changes using the techniques from the recent development in nonlinear dynamical system theory. The second-order Kolmogorov entropy (K_2) measures the rate of loss of information in dissipative dynamical system and defines a lower bound of the entropy. We present here a comparative study of Kolmogorov entropy (K_2) of original and filtered (tidal) LOD time series data. Our analyses indicate a contrasting sensitivity in K_2 values which correspond to predictive time limit of about 6–8 and 12–13 days for original and tidally removed (filtered) LOD data respectively. This difference of about 40–50% in coherent K_2 structures (predictive time limit) of filtered and original LOD data possibly indicates that presence of tidal signal in LOD time series enhances the degree of chaoticity and thereby limits the predictive time. It may be suggested that nonlinear resonances created by the earth's seasonal cycle and various other frequencies taking place due to thermodynamical property of the atmosphere–ocean system possibly turned the underlying dynamics to chaotic route. The understanding of these physical interactions may create physical premises for theoretical modelling of coupled ocean–atmospheric LOD dynamics and thereby allowing causally related prediction of underlying dynamics.

THE planet earth continuously changes its orientation in space under the influence of internally generated torque at the core–mantle boundary and also due to the gravitational attraction of earth–moon system. The angular rotational speed of the earth (earth's axial rotation), therefore, fluctuates and is reflected in fluctuations in length of day (LOD). Fluctuations in LOD are, therefore, proxy for earth axial rotation. The LOD signal exhibits minute but complicated changes corresponding to variations of several milli-seconds in LOD (Figure 1). These changes are caused due to the effect of a wide variety of geophysical (e.g. oceanic–atmospheric) phenomena and astronomical disturbances. Fluctuations in LOD, therefore, exhibit a composite response of coupled earth–ocean–atmosphere and astronomical system (e.g. deterministic tidal signals, stochastic weather pattern and quasi-periodic seasonal terms, etc.). Recent nonlinear analyses of LOD time series have indicated possible evidence of low dimensional strange attractors (chaos)[1,2]. The other fundamental properties of chaos include high sensitivity to the initial conditions which lead to exponential growth of disturbances in attractor regions, finite and non-zero (K_2) entropy, broad-band spectral characteristics, complex phase-space trajectories, etc.[3].

Prediction of a physical phenomenon is an interesting curiosity and ultimate goal of science. An appropriate analysis of various properties of chaotic attractor renders a way of precise forecast of the state of the dynamical system. In order to make a precise statement or quantify predictive time limit it is essential to examine the effect of noise and signal in composite chaotic time signal. Recently, some studies have shown that filtering of noisy and stochastic signals from chaotic time series possibly affects the determination of nonlinear parameters (e.g. attractor dimension estimates and Lyapunov exponent). It has been noted that small scale noise shows

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up as nonconvergent picture and may change the effective attractor dimension. Brandstater and Swinney[4] suggested that low pass filtered data may lead to reducing the number of degrees of freedom. Filtering may dramatically distort/alter, in certain cases, the shape of the reconstructed attractor and scramble phase also. But basic characteristics of the divergent nature of attractor remain intact[5]. However, several other studies found difficulties in drawing any definite conclusion[6–9]. We do not know, however, what happens to those physical processes which are strongly dominated by periodic extraterrestrial tidal signals. One such example is coupled LOD system where the resultant tidal effect is quite dominant even for slight changes in the earth's ellipticity. Consequently, the earth rotation changes and conserves its angular momentum. This is an important question from the predictability point of view of the physical system because LOD time series is a coupled nonlinear dynamical system dominated by a number of periodic tidal signals. Here we examine the effect of periodic tidal signal on the behaviour of second-order entropy (K_2) which is a measure of predictive time limit. We accomplish this task by comparing the second-order K_2 entropy of original LOD data and after filtering periodic tidal terms from LOD time series. It should be noted here that the purpose of this study is to examine the sensitivity of the periodic tidal frequencies on the K_2 value of chaotic LOD time series but not exactly to evaluate actual forecast.

LOD data

The best interpretable LOD time series available here comprises 10,580 daily variation data values spanning a period of 1962–1992. The LOD data set is the time derivative of UT–AT, where UT is universal time and AT is atomic reference time (86,400 sec). The original (UT–AT) data were obtained from the Bureau International de l'Heure (BIH) (M. Feissel pers. commun.). Figure 1 shows a portion of quasi-periodic and stochastic time series drawn from the original LOD data set. The complex quasi-periodic LOD variations, displayed in Figure 1, can be classified into three spectral bands: (i) The longer period components of secular changes which are produced by tidal friction (sun–moon system) and from the internal sources such as changes in moment of inertia of solid earth, (ii) irregular 'decadal variations' (up to several milliseconds/century in the LOD) are caused possibly due to angular momentum transfer between the earth's solid mantle and fluid core, and (iii) the unpredictable and most rapid 'nontidal' variations on time scales ranging from days up to a few years. These are largely of atmospheric origin (up to about 1 ms in amplitude).

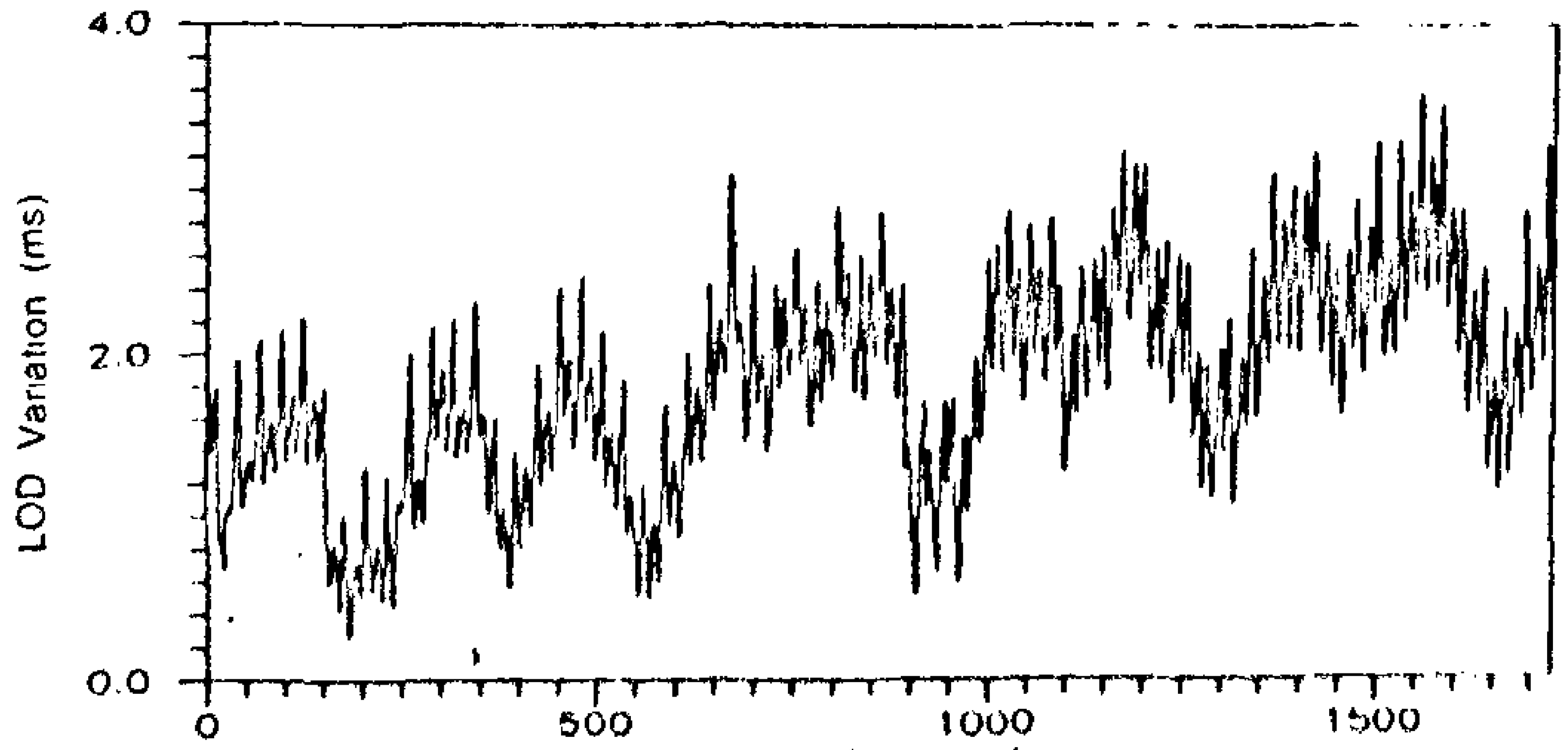

Figure 1. A representative view of LOD time series.

Analysis of second-order Kolmogorov entropy

The concept of entropy essentially provides distinction between the random and deterministic systems. The greater entropy is often associated with more randomness and less system order. The Kolmogorov entropy (K_2) allows one to classify deterministic systems by rates of information generation. Entropy has also been shown to be a parameter that characterizes chaotic behaviour[10]. The quantification of entropy may therefore have two important implications: (i) a finite entropy suggests that the data representing the underlying dynamics do not represent random system but rather chaotic one, and (ii) it renders a characteristic time scale of the system dynamics which can be used as a time limit for its predictability.

For a dissipative physical system like LOD, which is characterized by a low-dimensional strange attractor, an accurate forecast is possible only for a certain time interval. This time interval limit can be estimated from observed LOD time series say $X(t_n)$, following the method of Kolmogorov entropy (K_2) analysis. The Kolmogorov entropy (K_2) is a measure of the rate at which information about the state of dynamical system is lost in the course of time which provides the characteristic measure of chaotic motion.

There has been keen interest in the development of nonlinear algorithm in the last ten years. Several workers[9,11] have provided the method of K_2 entropy analysis and discussed the theoretical and practical aspects of realization of this method. Prior to calculating K_2 entropy, it is essential to calculate estimates of the correlation integral[9,11]. Following Takens[12] we construct an m-component 'state' vector X_i, ($m = 1, 2, 3 \ldots$) from a time series $[x(t)]$ as:

$$X_i = [X_1(t_i), X_2(t_i), \ldots X_m(t_i)],$$

where $X_m(t_i) = X[(t_i + (k-1))\tau]$, and τ is an appropriate time delay (of the order of characteristic physical time scales). The distribution of state vectors in the reconstructed phase space is directly related to the dimension. A suitable distribution-dependent quantity can be defined to examine its scaling with distance in phase space. The correlation integral (Grassberger and Procaccia[11]) for N vectors distributed in an m-dimensional space may be given as follows:

$$C_m(r) = \lim_{N \to \alpha} \frac{1}{N^2} \sum_{i=2}^{N} \sum_{j=1}^{N} \theta(r - |X_i - X_j|),$$

where θ is Heaviside step function $= 0$ if $x < 0$, and $= 1$ if $x > 0$, and r is the distance between point pairs in phase space and the distances are taken in terms of Euclidean norm. The distance distribution function will obey a power-law scaling for small r values if the number of data points is large enough. This power-law relation for the attractor dimension d is given by

$$d = \lim_{r \to 0} \frac{\log C_m(r)}{\log r},$$

where d is the correlation dimension and $C_m(r)$ is correlation coefficient. Our earlier analyses have shown evidence of low-dimensional strange attractor in LOD system with a possible dimension of 5–6 (refs 1, 2).

This dynamical quantity (K_2) is an approximation to the Kolmogorov–Sinai invariant entropy (K_2). The Kolomogorov entropy is defined as[14]

$$K_2 = \lim_{r \to 0} \frac{1}{\tau} \frac{C_m(r)}{C_{m+1}(r)},$$

where τ is sampling rate. This K_2 entropy has the property that K_2 is zero for an ordered system, and is infinite in a random system. K_2 is a finite constant > 0 for chaotic dynamics. An efficient method is proposed to compute this quantity by Grassberger and Procaccia[9,11]. In fact it is noted that finite and convergent K_2 value and strange attractor dimension have been taken almost as proof for the presence of low-dimension chaos in the dynamics of system with irregular behaviour.

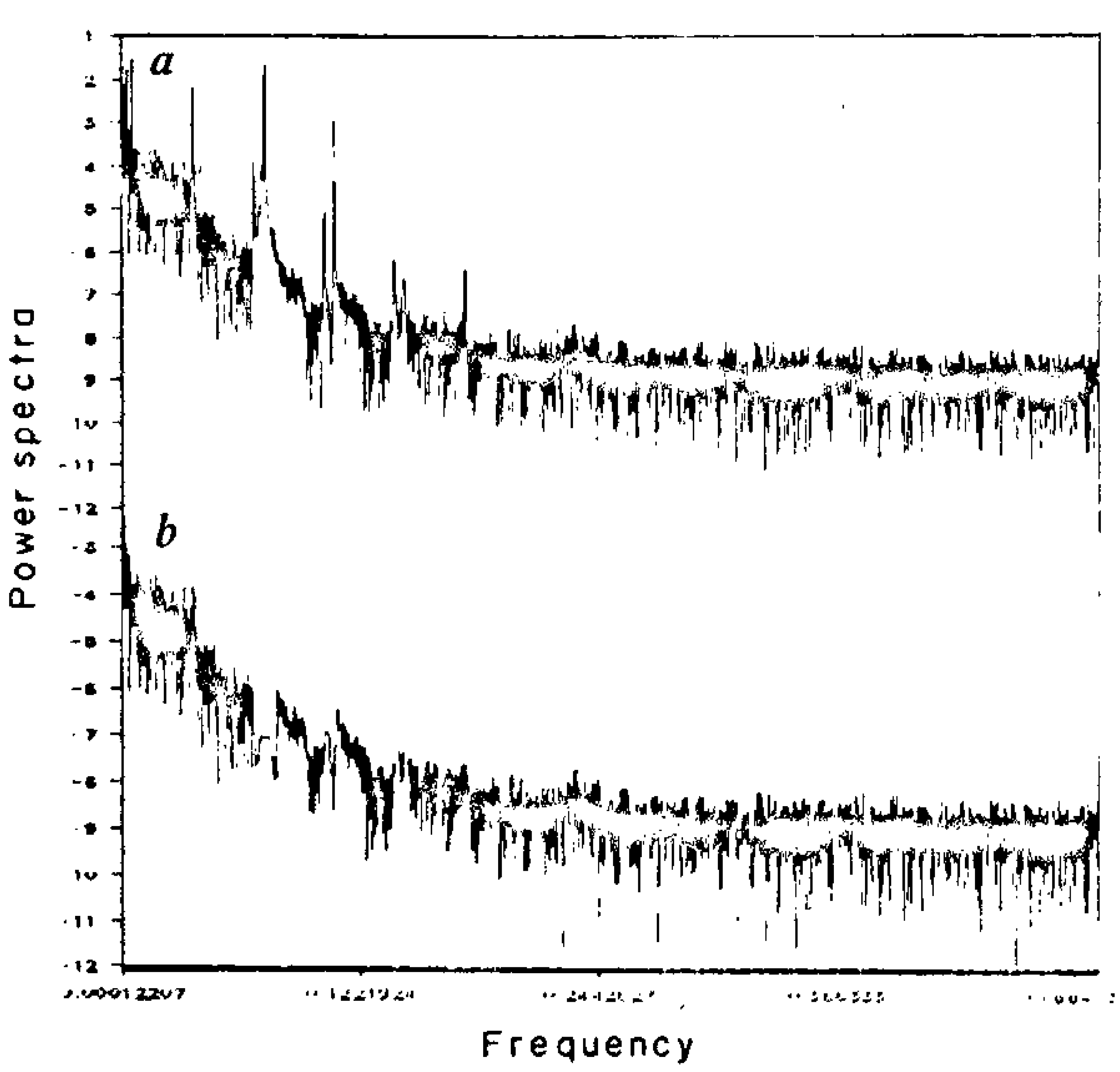

Figure 2. Comparison of power spectra of (*a*) original and (*b*) filtered LOD time series.

Results

Filtering and spectral comparison

Fourier spectra of LOD data show broad-banded and noisy (Figure 2 *a*) spectral patterns. There are evidences of several tidal components with dominant spikes clustered around 27, 13, 9, 7 days, etc., and nonrandom seasonal components at 372 and 182 days. The common practice to achieve our goal is to properly filter out these periodic tidal signals from the data series and then study the coupled nonlinear dynamics. Accordingly, all dominant periodic tidal signals are eliminated by using the method of Delache and Scherrer[15]. This is carried out by using the fast Fourier transform (FFT) method for 2^{13} data series. After obtaining FFT of the detrended original LOD time series, the Fourier coefficients of the dominant periodic are successively set to zero and inverse transform is taken. Thus the resulting time series is deprived of all periodic components. Figure 2 *a*, *b* shows the comparison of filtered and original LOD power spectra. Figure 2 *b* also shows filtered spectra which is clearly broad-banded and noisy but deprived of periodic signal.

Comparison of phase-space trajectories

Figure 3 gives the comparison of original and filtered trajectories of LOD signal in a two-dimensional phase space. Comparison of trajectories of original (left panel) and filtered (right panel) LOD data on (X, dX/dT) phase plane obviously exhibits intrinsic changes in the shape of the phase diagram. It is interesting to note, however, that the basic chaotic characteristics of attractor seems to be retained. It is remarkable to note that filter of noise or periodic signal from the chaotic time series does not significantly alter the inherent chaotic feature. However, these kinds of operations have considerable influence on predictive time limit.

Comparison of second-order Kolmogorov entropy

Using the two sets of filtered and original data, we calculated the second-order K_2 entropy following the

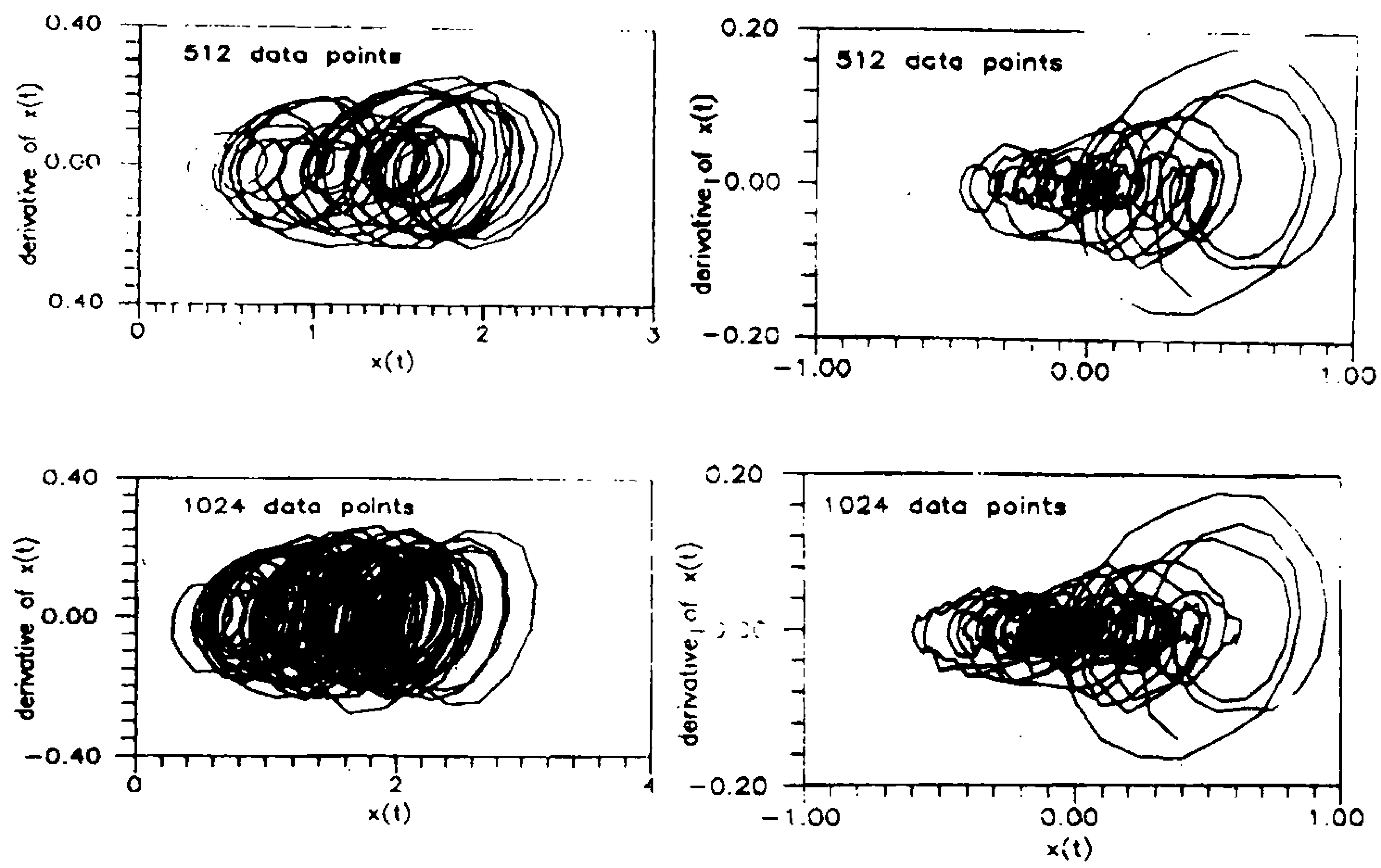

Figure 3. Comparison of phase space trajectories of original (left panel) and filtered (right panel) LOD time series in $(X, dX/dt)$ phase plane.

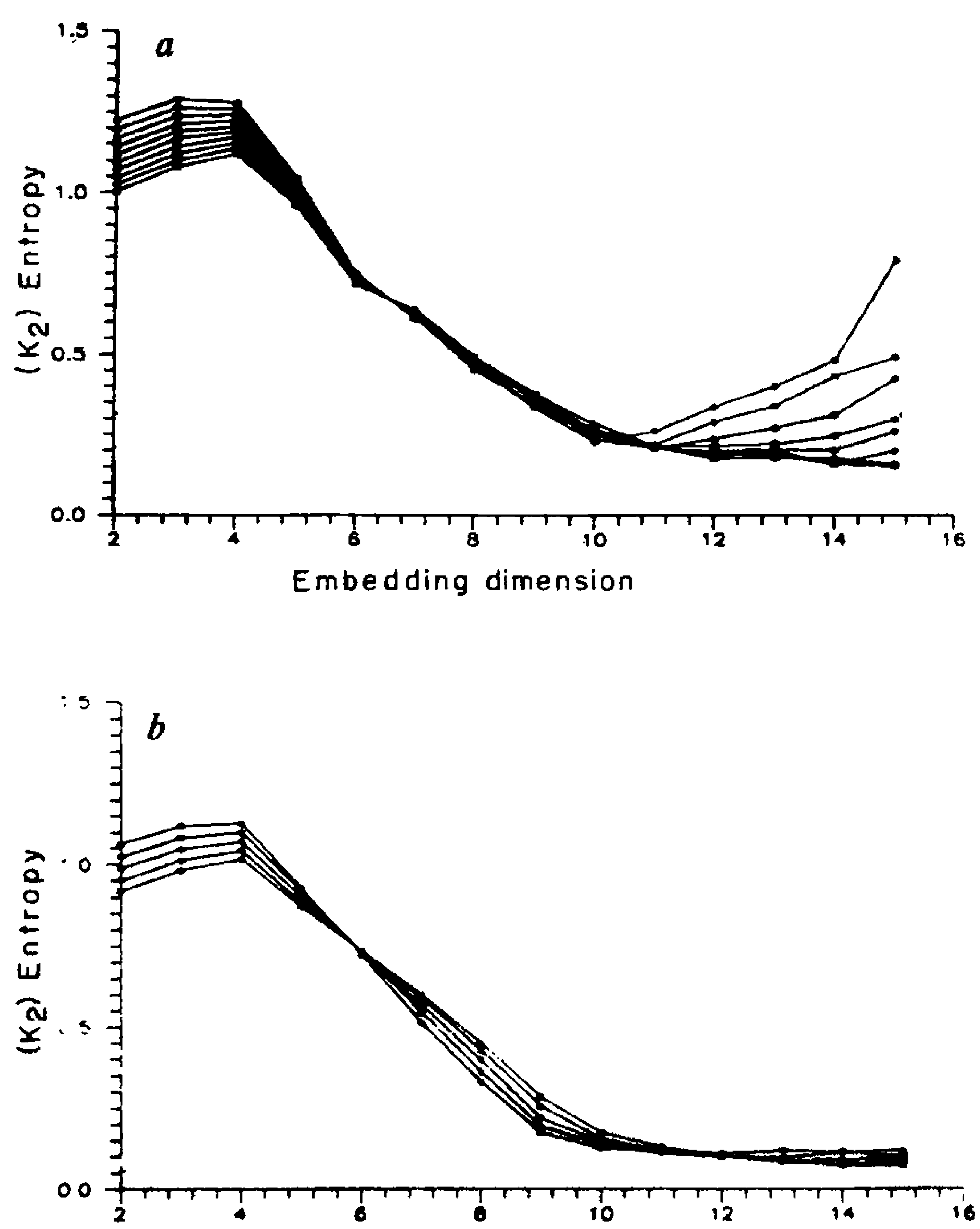

Figure 4. K_2 entropy of (*a*) original and (*b*) filtered LOD time series.

method discussed in the previous section. Figure 4 *a*, *b* shows the dependence of correlation entropy K_2 on the embedding dimension for both original LOD data and filtered time series respectively. Obviously, the value of K_2 for different values of distance r is seen to decrease with increase of embedding dimension and tend to be saturated for higher embedding dimensions. Obviously, Figure 4 *a*, *b* shows that the entropy is finite and positive ($K_2 > 0$), thus providing a strong evidence for the chaoticity of LOD time series. The saturated value of K_2 for original LOD time series is 0.17 ± 0.02 corresponding to a predictive time limit of about 7–8 days. The saturated value of K_2 for filtered LOD data gives value of 0.095 ± 0.01 corresponding to 12–13 days. The study indicates that prediction of the state of chaos in the LOD time series is affected by the presence of periodic tidal frequencies, which are considerably larger up to 40–50%. These reports have implications on the accurate LOD forecast.

Physical interactions of LOD–ocean–atmospheric dynamics

The coupled ocean–atmospheric dynamics can be modelled as low dimensional order chaotic processes[16–19]. These workers have also suggested that nonlinear interactions between seasonal cycles and inter-annual variations in coupled ocean atmospheric system can be a probable cause of irregularities/chaos in the El Nino/Southern oscillations (ENSO). Recent studies have also demonstrated that earth's rotational dynamics can be viewed as a low dimensional chaotic dynamical system[1–3]. It is known that ocean atmospheric circulations and earth's rotational dynamics are intimately linked phenomena. The major source of nonlinearity clearly arises from the response of the atmospheric to thermal forcing and from thermodynamic interactions between the atmosphere ocean and earth's dynamical state. The broad-band nature of LOD spectrum and clustering and splitting of spectral peaks apparently visible/submerged into background noise are some of the probable signatures/characteristics of the inherent nonlinearity and chaos in the system dynamics. The nonlinear interaction of various frequencies taking place affects the LOD and atmospheric dynamics. In particular, it may be suggested that the nonlinear resonances created by the seasonal cycle and inter- and intra-annual fluctuations due to thermodynamic property of the atmosphere ocean system (possibly through a series of frequency locking and overlapping of the resonances) may have eventually turned the system dynamics to chaotic route. The understanding of these physical interactions and a particular route of chaos may possibly allow causally related prediction to be made for underlying dynamical system. This is one of the several possible conjectures regarding the predictability of coupled earth–ocean–atmospheric dynamics.

Conclusions

The analysis of K_2 entropy of filtered and original LOD data reveals contrasting estimates. The predictive value for original signal is 6–8 days. Filtering the LOD data for tidal signal, however, increases the predictive limit up to 10–13 days. This obviously indicates that the presence of tidal frequencies in chaotic LOD time series considerably affects the predictability by increasing the complexities and chaoticity in underlying dynamics.

The presence of coherent time structures in chaotic LOD time series creates the physical premises for theoretically modelling coupled LOD system dynamics.

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